MODELING AND SIMULATION OF UNDERWATER ROBOTS MOVEMENT IN SPACE

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The evolution of an underwater robot in real conditions supposes the execution of spatial movements according to six degrees of freedom (elementary translation movements in the direction of the three axes of coordinates). Description of such movements supposes the relation to a system of axes, favourable for the movement analysis.
The evolution of the submerged underwater robot will be observed in relation with the position equations, when this moves at low speed, in order to finally find an appropriate position for it at the work site.

Chapter 1. INTRODUCTION

Having in view the mission requirements and its operational needs, taking into account the present achievements in civil and military robotics, along with the role of underwater robots in contemporary man's life, the present paper aims at presenting the position equations, in space, of the underwater robot vehicle (URV) cinematics.
Practically, the starting point in the study of a URV cinematics is the propulsion system and the propeller positioning, so as the robot remain stable, not being dependant on propulsion. The robot is supposed to move safely (at low speeds), in vertical and horizontal planes, towards the intervention area, there follows the determination of the robot's, or its arm's movement towards the working site and back. The paper makes reference to the evolution of the submerged robot and calculates the position equations of the robot's cinematic.
Observing the behaviour of a self-ruling underwater robot in immersion, in lifelike conditions, and determination of position equations of the robot, all aim at stating the design and function properties, as well as efficiency growth and vitality of the robot.
This paper intends to show how to state the position equations of the underwater robot's cinematics, in space, while the robot is submerged on demand. We should have in view the safe vertical and horizontal movement of the robot, as well as an equilibrium control of the robot under continuous changing conditions of the environment in which the robot moves.
The evolution of the submerged underwater robot will be observed in relation with the position equations, when this moves at low speed, in order to finally find an appropriate position for it at the work site.

Chapter 2. THE EVOLUTION OF A SUBMERGED UNDERWATER CRAFT

2.1 Generalities concerning

Underwater robots are vehicles that operate at low speeds. i.e. 1÷ 4 Nd.
To control or supervise the immersion means to maintain steady immersion quota of immersion, during the vertical and horizontal movement of the robot, for a certain extent of time. In order to maintain a proper steering one should study this operational parameters in their attempt to study and analyze the underwater robots' cinematic.

Steering is the robot's capacity to alter and maintain its ordered course when operated by rudders and propellers. Steering is influenced by a series of factors, some of which being related to the mini submarine, and others to outside factors. [3, pag.32-37].
Thus, the factors that influence the underwater robot's steering are:
- related to the robot, i.e. main dimensions, C_G and C_F, positions, UR shape and rudders dimensions, propellers (place and location angle, dimensions and their number, propeller's pitch), running speed, and direction;
- external factors: wind direction and force, waves and their direction (wave characteristics in shallow waters), underwater currents and their direction, depth (when passing from deep waters to shallow waters the shape and endurance of the waves is changed).

It is necessary to observe the action of the autonomous underwater vehicle (AUV) in immersion, in as similar to reality conditions as possible, so as not to affect the robot's functioning in general and its vitality in particular.
Anchoring, weighing anchor in immersion, immersion in constant depth between waterways, vertical downward running, non-operative position of the robot on the bottom of the sea, the uplift to a certain height, then travelling above bed sea at a constant height, avoiding obstacles in the way, are only a few essential aspects that create good reasons for calculating the vehicle's position equations from the starting point to the work site.

2.2 Notions about stability of underwater robots

By stability of underwater robot we refer to a certain characteristic of the robot that helps to go back to the initial position even in the situation when the robot is no longer submerged, and the external cause that produced its inclination does not act on it any longer. Thus, stability is the equilibrium characteristic of the underwater robot.
Depending on the size of the inclination angle we can distinguish: initial stability, stability at small inclination angles, and stability at wide inclination angles.
Fundamental for the theory of initial stability is the hypothesis of very small inclinations, and the moment of getting to the initial position, has a laniary variation, depending on the inclination angle. At big inclinations, the hypothesis of coming back to the initial position is no longer taken into account. [2, pg. 82]
By static stability we refer to that property of the underwater robot of setting against external static acts, which tend to tilt it with a very small angular acceleration, which can be easily not taken into consideration.
By dynamic stability we refer to that property of the underwater robot to set against the external dynamic acts, which carries out its full effect, producing an angular acceleration that should be taken into consideration.
The inclination of the underwater vehicles around a longitudinal axis is called transverse inclination, which corresponds with transversal stability, while the inclination around a transverse axis is called longitudinal inclination, which in its turn corresponds with longitudinal stability.

2.3 Determination of position equations

The determination of position equations is necessary for the cinematic analysis of robots, according to which the positioning in sites or areas of intervention is calculated. In cinematics movement is considered independent of masses and forces; only the geometrical aspect is taken into account [2].
Among the problems that can be solved through the cinematic analysis of the robots, the most difficult, and most important is the one related to position of robots.
The transformation matrices $r_{i,i+1}$ and $r_{i+1,i}$, dependent on the vectors of the two systems are:

$$r_{i,i+1} = \begin{bmatrix} i_{i+1}.i_i & j_{i+1}.i_i & k_{i+1}.i_i \\ i_{i+1}.j_i & j_{i+1}.j_i & k_{i+1}j_i \\ i_{i+1}.k_i & j_{i+1}.k_i & k_{i+1}.k_i \end{bmatrix} \quad (1)$$

$$r_{i+1,i} = \begin{bmatrix} i_i.i_{i+1} & j_i.i_{i+1} & k_i.i_{i+1} \\ i_i.j_{i+1} & j_i.j_{i+1} & k_i.j_{i+1} \\ i_i.k_{i+1} & j_i.k_{i+1} & k_i.k_{i+1} \end{bmatrix} \quad (2)$$

It is observed that for the rectangular systems (systems used in this paper), the matrix $r_{i+1,i}$ is inverse in relation to matrix $r_{i,i+1}$; consequently, it can be written:

$$r_{i+1,i} = \left(r_{i,i+1}\right)^{-1} = \left(r_{i,i+1}\right)^{T} \quad (3)$$

Matrix $r_{i,i+1}$ characterizes the relative elementary rotation movement of elements *i* (in this case the depot ship) and *i+1* (the underwater robot vehicle), and represents the transformation matrix of a point's coordinates from the mobile system URV to the stable one (the depot ship).

It is considered that the mobile system URV with an angle ά around the permanent system formed by the base ship. By convention, the counter clockwise direction is considered positive for the variation of the pitch angle *θ*, rolling angle *ψ* and swing angle *φ*.

The base ship is the permanent element while the robot is the mobile element; the element "n" is solitary with the hand of the robot's handler. Dependant on this, the permanent and the mobile systems of coordinates are selected. The symmetrical axes of the vehicle are taken into account, and dependant on the direction of the underwater robot, the sense of the Cartesian system axes of coordinates is selected.

In order to "shape" the movement of the robot while this is at work underwater, we suppose that this consists of a successions of rigid solids interconnected through joints (usually they are joints with a single degree of freedom) [1, pg. 37].

We consider:

To = the permanent reference system, solitary with the base ship;

Ti = the mobile reference systems (all the other robot systems, manipulator arm, hand, etc.);

Ri = matrix that characterizes the relative rotation movement.

Usually, these reference systems are chosen in accordance with their origin which should be either in the centre of the joint of two, or in the elements, or in the load centre C_G of the element the respective reference system is solitary with. [1]

The relative matrix of two consecutive reference elements, in our case the base ship, and the URV, is described by the variation of a number of parameters (conventionally named-generalized coordinates), which is equal to the number of degrees of freedom of the joint (in this case of the element), which interconnects the elements with which the respective reference system is solitary with.

The derivatives of these coordinates in relation to time are the generalized speeds, while their derivatives in relation to time of the generalized speeds are the generalized accelerations. The generalized coordinates can be of rotation, materialized by angles *φ, θ, ψ* of two elements, or of translation, materialized in the linear drift (a, b ,c) of the two elements.

A random rotation can be obtain as a consequence of three successive rotations: one around axis X, with an angle *ψ*, another one around axis Y, with an angle *θ*, and a last one around axis Z, with an angle *φ*.

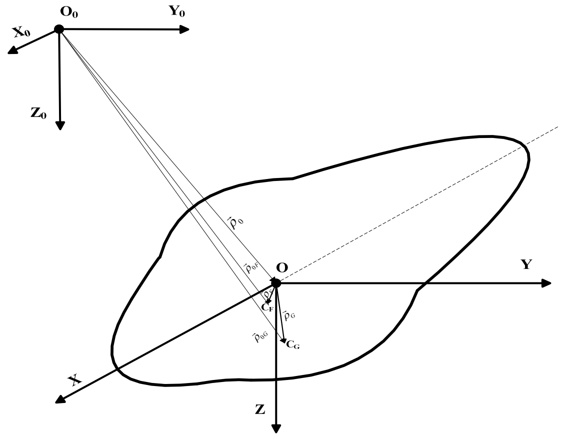

Fig. 1 Selection of the systems of coordinate axes of the underwater robot vehicle

The permanent system of coordinate axes is attached to the base ship.
The URV has a diametrical axial plane. The mobile system of axes, attached to the vehicle, uses the advantages of this axial plane, in the following way: two of the axes are selected in the axial plane and the other perpendicular on the plane.
Axis Ox is selected as longitudinal in the axial plane, positive towards the robot's bow (in onward direction), parallel with the surface of calm water.
Axis Oy is selected as transverse in the axial plane of the robot, positive towards starboard, parallel with the surface of calm waters.
Axis Oz is vertical in the axial plane of the robot, positive downwards.

2.3.1 Determination of transformation matrix of the coordinate axis in space
The relations can be extended to the particular rotations of the trirectangular systems around axes Ox, Oy, Oz, as follow:

a) rotation around axis Ox:

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (33) \qquad \text{or} \qquad (r^0) = \left[R_{0x}^{\alpha}\right] \cdot (r), \quad (34)$$

b) rotation around axis Oy:

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\beta \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (35) \qquad \text{or} \qquad (r^0) = \left[R_{0y}^{\beta}\right] \cdot (r), \quad (36)$$

c) rotation around axis Oz:

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (37) \qquad \text{or} \qquad (r^0) = \left[R_{0z}^{\gamma}\right] \cdot (r). \quad (38)$$

In order to obtain the projections of point M on the axes of system Oxyz, for the particular rotations presented, knowing its projections on the axes of system $Ox_0y_0z_0$ we have the relations:

$$(r) = \left[R_{0x}^{\alpha}\right]^T \cdot (r^0) \quad (39)$$

for rotation around axis Ox;

$$(r) = \left[R_{0y}^{\beta}\right]^{T} \cdot (r^{0}) \tag{40}$$

for rotation around Oy;

$$(r) = \left[R_{0z}^{\gamma}\right]^{T} \cdot (r^{0}) \tag{41}$$

for rotation around axis Oz.

Position and movement of vehicle, in relation to the fix Cartesian system $Ox_0y_0z_0$, correspond to the position and movement of a robot Oxyz attached to the respective rigid frame. The six degrees of freedom of the robot will be determined by the position vector $\bar{\rho}_0$ of the origin O and by the position of mobile vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ of axes Ox, Oy, and Oz (fig. 2)

Position vector $\vec{r}$ of point M in relation to the mobile system represented by the underwater robot is:

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k} \tag{42}$$

where:

$$\bar{i} = \alpha_{11}\bar{i}_0 + \alpha_{12}\bar{j}_o + \alpha_{13}\bar{k}_0 , \tag{43}$$

$$\bar{i} = \alpha_{21}\bar{i}_0 + \alpha_{22}\bar{j}_o + \alpha_{23}\bar{k}_0 , \tag{44}$$

$$\bar{k} = \alpha_{31}\bar{i}_0 + \alpha_{32}\bar{j}_o + \alpha_{33}\bar{k}_0 . \tag{45}$$

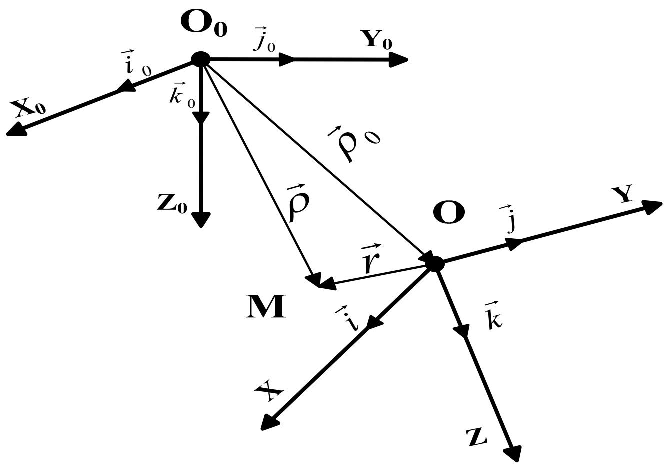

Fig. 2

Replacing previous relations, we obtain the following expression for vector $\mathbf{\bar{r}^0}$:

$$(r^{0}) = (\alpha_{11}x + \alpha_{21}y + \alpha_{31}z)\bar{i}_0 + (\alpha_{12}x + \alpha_{22}y + \alpha_{32}z)\bar{j}_0 + (\alpha_{13}x + \alpha_{23}y + \alpha_{33}z)\bar{k}_0 . \tag{46}$$

Projections of vector $\mathbf{\bar{r}^0}$ on the axes of the permanent system can be deduced by adding the corresponding dot products (α_{11} is practically $\cos\alpha_{11}$):

$$(r^{0}) = \begin{bmatrix} (i_0)^T(r) \\ (j_0)^T(r) \\ (k_0)^T(r) \end{bmatrix} = \begin{bmatrix} \alpha_{11}x + \alpha_{21}y + \alpha_{31}z \\ \alpha_{12}x + \alpha_{22}y + \alpha_{32}z \\ \alpha_{13}x + \alpha_{23}y + \alpha_{33}z \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{47}$$

where:

$$\begin{array}{lll} \alpha_{11} = (i_0)^T(i); & \alpha_{12} = (j_0)^T(i); & \alpha_{13} = (k_0)^T(i); \\ \alpha_{21} = (i_0)^T(i); & \alpha_{22} = (j_0)^T(j); & \alpha_{23} = (k_0)^T(j); \\ \alpha_{31} = (i_0)^T(k); & \alpha_{32} = (j_0)^T(k); & \alpha_{33} = (k_0)^T(k); \end{array} \tag{48}$$

Observations:
In the rotation of coordinate axes in space nine angles interfere (direction cosines), three for each new axis in relation to each of the three initial axes, which are fixed. These nine direction cosines are not independent among themselves, they are linked through fundamental relations related to a direction in space:

$$\begin{aligned} &\alpha_{11}^2+\alpha_{12}^2+\alpha_{13}^2=1\\ &\alpha_{21}^2+\alpha_{22}^2+\alpha_{23}^2=1\\ &\alpha_{31}^2+\alpha_{32}^2+\alpha_{33}^2=1 \end{aligned} \tag{49}$$

and

$$\begin{aligned} &\alpha_{11}^2+\alpha_{21}^2+\alpha_{31}^2=1\\ &\alpha_{12}^2+\alpha_{22}^2+\alpha_{32}^2=1\\ &\alpha_{13}^2+\alpha_{23}^2+\alpha_{33}^2=1 \end{aligned} \tag{50}$$

There are also other two groups of relations which express the conditions of perpendicularity of every two axes of the same system:

$$\begin{aligned} &\alpha_{11}\alpha_{21}+\alpha_{12}\alpha_{22}+\alpha_{13}\alpha_{23}=0\\ &\alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32}+\alpha_{23}\alpha_{33}=0\\ &\alpha_{31}\alpha_{11}+\alpha_{32}\alpha_{12}+\alpha_{33}\alpha_{13}=0 \end{aligned} \tag{51}$$

and

$$\begin{aligned} &\alpha_{11}\alpha_{12}+\alpha_{21}\alpha_{22}+\alpha_{31}\alpha_{32}=0\\ &\alpha_{12}\alpha_{13}+\alpha_{22}\alpha_{23}+\alpha_{32}\alpha_{33}=0\\ &\alpha_{13}\alpha_{11}+\alpha_{23}\alpha_{21}+\alpha_{33}\alpha_{31}=0 \end{aligned} \tag{52}$$

There results that the nine direction cosines which determine the positions of the system Oxyz, in relation to permanent system $O_0x_0y_0z_0$, are not independent; they have to demonstrate and verify the above-mentioned relations.
These relations determine an octagonal transformation.
Since there are six distinct relations in the direction cosines system Oxyz, axes, we deduce that only three of these nine cosines are enough to determine the position of trihedron Oxyz in relation to system $O_0x_0y_0z_0$.
The six relations can be reduced to:

$$\alpha_{m1}^2+\alpha_{m2}^2+\alpha_{m3}^2=1 \qquad (m = 1, 2, 3), \tag{53}$$

$$\alpha_{m1}\alpha_{n1}+\alpha_{m2}\alpha_{n2}+\alpha_{m3}\alpha_{n3}=0 \qquad (m, n = 1,2,3; m \neq n). \tag{54}$$

The direction cosines expressed in relation 48 can form the orthogonal quadric matrix:

$$[a]=\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & \alpha_{22} & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}, \tag{55}$$

where:

$$\det[a]=1. \tag{56}$$

As a result, vector $(\mathbf{r}^0)$ can be expressed as follows:

$$(r^0)=[a]^T(r)=[a]^{-1}(r), \tag{57}$$

where :

$[\mathbf{a}]^T$ is the transpose matrix and $[\mathbf{a}]^{-1}$ is the inverse matrix.

If we multiply the previous relations with matrix $[\mathbf{a}]$, on the left, we get the following relation:

$$(r) = [a](r^0), \tag{58}$$

because:

$$[a]\cdot[a]^T = [a]\cdot[a]^{-1} = [a]^T \cdot [a] = [a]^{-1} \cdot [a] = [E] \tag{59}$$

Observations:

Matrix $[\mathbf{a}]$ is called the pass matrix from permanent system $O_0x_0y_0z_0$ into the mobile system Oxyz.

Matrix $[\mathbf{a}]^T$ is named pass matrix from mobile system Oxyz into the permanent system fix $O_0x_0y_0z_0$.

The matrix of the two transformations, $[\mathbf{a}]$ and $[\mathbf{a}]^T$ are non-singular matrices which operate as transition operators. Their determinants are different from zero, because their columns or lines are linearly independent.

2.3.2 Determination of matrix of the RSA position at random movement in space

We take the position of point M defined by position vector $\vec{r}$, in relation to mobile system Oxyz, and by $\vec{\rho}$ in relation to the permanent system $O_0x_0y_0z_0$ (fig. 3)

Three fundamental notions are defined:

a) Absolute movement of point M, considered in relation to system $O_0x_0y_0z_0$.
b) Relative movement of point M considered in relation to mobile system Oxyz.
c) Transport movement, considered as movement of a point M' soldiery with mobile system Oxyz (the robot in our case).

Position of point M in relation to mobile system Oxyz is given by the matrix which is associated with position vector $\vec{r}$.

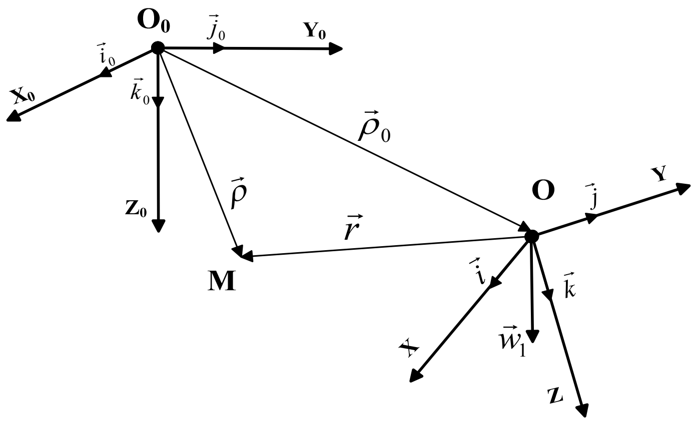

Fig. 3

$$(\mathrm{r}) = [\mathrm{x} \quad \mathrm{y} \quad \mathrm{z}]^T \tag{60}$$

where:

x, y, z are projections of vector $\vec{r}$ on the axes of mobile reference system Oxyz.

Position of mobile reference system Oxyz (of the robot) in relation to the permanent system $O_0x_0y_0z_0$ (of the base ship) will be given by the position of origin O of the mobile system by coordinates $\rho_{0x}, \rho_{oy}, \rho_{0z}$ and by Euler angles ψ, θ and φ, formed by axes Ox, Oy, and Oz with the axes of the permanent system. These parameters form the following matrices:

$$(\rho_{0x}) = \begin{bmatrix} \rho_{0x} \\ \rho_{oy} \\ \rho_{0z} \end{bmatrix}, \qquad (\lambda) = \begin{bmatrix} \Psi \\ \theta \\ \varphi \end{bmatrix} \tag{61}$$

Position of point M, in relation to the fixed point, is given by the matrix equation:

$$(\rho) = (\rho_0) + [a_t]^T (r) \tag{62}$$

where:

$$[a_t] = [a_\varphi] \cdot [a_\Theta] \cdot [a_\Psi]; \tag{63}$$

$$[a_\Psi] = \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{64}$$

$$[a_\Theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}; \tag{65}$$

$$[a_\varphi] = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{66}$$

After doing the products, matrix $[a_t]$ appears as a orthogonal matrix, consisting of the following elements:

$$\begin{aligned}
\alpha_{11} &= \cos\Psi\cos\varphi - \sin\Psi\cos\theta\sin\varphi; \\
\alpha_{12} &= \sin\Psi\cos\varphi + \cos\Psi\cos\theta\sin\varphi; \\
\alpha_{13} &= \sin\theta\sin\varphi; \\
\alpha_{21} &= -\cos\Psi\sin\varphi - \sin\Psi\cos\theta\cos\varphi; \\
\alpha_{22} &= -\sin\Psi\sin\varphi + \cos\Psi\cos\theta\cos\varphi; \\
\alpha_{23} &= \sin\theta\cos\varphi; \\
\alpha_{31} &= \sin\Psi\sin\theta; \\
\alpha_{32} &= -\cos\Psi\sin\theta; \\
\alpha_{33} &= \cos\theta.
\end{aligned} \tag{67}$$

Chapter 3. CONCLUSIONS

The evolution under immersion and maximum stability of the underwater robot is important for the accuracy of the previously stated tasks. The base ship at anchor is considered fixed during the robot's movement from and to the work site.

Taking into account the position of the intervention ship (which has all the facilities for the robot's safe transportation, launch, and recovery, currently named „base ship"), off which the launching is made, we can state the underwater robot's trajectory, as well as the correct positioning of the robot around the work site. The robot will remain in this position all through the work period of the manipulator arm and of the mechanic hand. The final, fixed position of the robot will be convenient for the robot to reach the respective object until the work is finished.
From this moment on, the minisubmarine will become the base submarine. In this case, the system of mobile coordinate axes will be attached to the mechanical hand of the manipulator arm, while the calculus will be made according to the same algorithm, and taking into account the new systems of coordinate axes .The manipulator arm should be able to work continually, at high parameters, all through the necessary work period, at the underwater structure.
When it comes to the study of the underwater robot's cinematics, in order to state what the movement equations are, it is necessary to choose the right systems of coordinate axes.
The cinematic system influences the construction, manoeuvrability and functional parameters of the robot. The structural, cinematic lay-out determines the functional and cinematic potential of the robot. In order to determine the matrix of the underwater robot's position in the case of some movement in space, the following determinations were made:

- determination of position equations for successive rotations;
- determination of position equations for successive translations.

The matrices for the transformation of the coordinate axes system of the position vector in a plane for translation were determined, and then the same was determined for the rotation of the coordinate axes. After these calculuses were done, the determinations for transfer matrices of the coordinate axes in space were also done.
Since knowing the movement of an underwater vehicle means the determination, at a certain moment, of the movement of its position vector, the calculuses were done for the determination of the position matrix of the underwater robot, taking into account any movement in space.
After that, the form of the movement equations in space was determined, with six degrees of freedom, and finally, the general movement equations that develop in vertical plane, were determined. The evolution in immersion is the basic movement of any underwater vehicle.
In conclusion, by determining the position equations, this paperwork solves one of the essential and difficult problems of the cinemaric analysis of the underwater robot, which, after reaching the work site, it is anchored and becomes a fixed system (base robot, with a well-determined position), for the cinematic calculuses and for the mathematical model of the manipulator arm which is attached to it, and which starts working from this moment on.
All these equations can become a mathematical model which can evaluate the movement in immersion of a underwater robot.

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LIST OF ACRONYMS USED

AUR = autonomous underwater robot;
AUV = autonomous underwater vehicle;
ROV = remotely operated vehicle;
T-R = translation-rotation;
UR = underwater robot;
URS = underwater robotic systems;
URV = underwater robot vehicle;
UUV = unmanned underwater vehicle.

SYMBOLS USED

C_F = flotability centre;
C_G = centre of gravity;
C_p = local pressure coefficient;
C_x = total drag coefficient expressed either by the dead flat part of the ship (the largest part of the UR), and marked with $C_{x(D^2,}$ or with volume representing the cube of 2/3 and marked with $C_{x(V^{2/3})}$;
$C_{x,f}$ = the drag coefficient based on the friction of the limit layer or drag coefficient of friction;
$C_{x,p}$ = the drag coefficient based on the action of pressure forces or shape forces;
$(C_x)_M$ = the aerodynamic drag coefficient of the robot model;
$(C_x)_N$ = the aerodynamic drag coefficient of the robot prototype;
r_i = matrix that characterizes the relative rotation movement;
θ = pitch angle or longitudinal trim;
Ψ_c = constant course angle;
Ψ = roll angle or transversal trim;
$[a]^{-1}$ = the inverse matrix;
$[a]^T$ = the transposed matrix;
$[I]$ = the unity matrix;
$\bar{i}, \bar{j}, \bar{k}$ = the mobile system of coordinates unit vectors and $\bar{i}_0, \bar{j}_0, \bar{k}_0$ the fixed system of coord. unit vectors;
$\vec{p}$ = the rolling angular speed;
$\overrightarrow{\rho_F}$ = the position vector of hull centre with the mobile system of coordinates;
$\overrightarrow{\rho_G}$ = the position vector of the gravity centre in relation to the mobile system of coordinates;
$\overrightarrow{\rho_0}$ = position vector in relation to the general (fixed) system of coordinates;
$\overrightarrow{\rho_{OF}}$ = the position vector of the hull centre in relation to the fixed system of coordinates;
$\overrightarrow{\rho_{OG}}$ = position vector of the gravity centre in relation to the fixed system of coordinates;
$\vec{q}$ = the pitch angular speed;
$\vec{r}$ = the swing angular speed;
$\vec{u}, \vec{v}, \vec{w}$ = projection of translation speeds for axes Ox, Oy, Oz;