ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering

THE CROSS DISPLACEMENTS FIELD OF A LINEAR ELASTIC KINEMATIC ELEMENT WITH UNIFORMLY ROTATIVE MOTION, CONSTITUTIVE PART OF A LINKAGE

Adina CATANEANU, Dan Georghe BAGNARU
University of Craiova, Faculty of Mechanics, adinavc@mecanica.ucv.ro

Abstract. By using Hamilton's principle, it was deduced in [1] the mathematical model of vibrations in case of a linear elastic kinematical element having a rotative motion with constant angular velocity. Now, considering this element to be the crank of a parallelogram mechanism, using Laplace and Fourier integral transforms, the cross displacements field under a time and position function is determined. Finally, the plots of this function are presented.

1. THE MATHEMATICAL MODEL FOR THE VIBRATIONS IN CASE OF A LINEAR ELASTIC CRANK OF A PARALLELOGRAM MECHANISM

Let's consider the OA crank of the parallelogram mechanism from the figure 1.

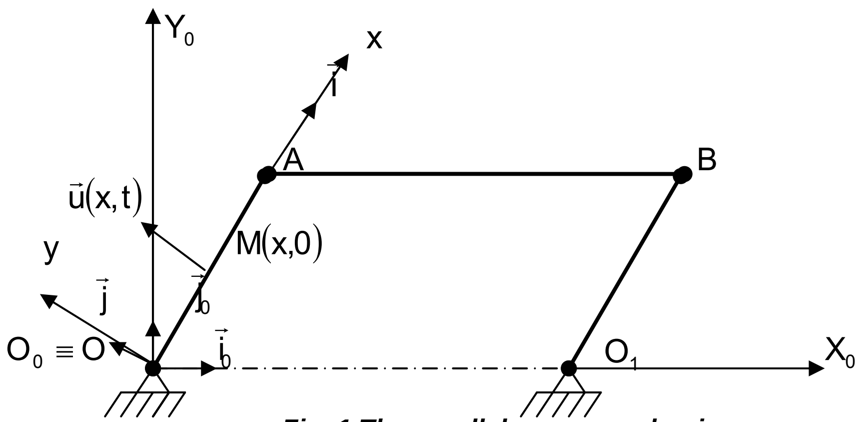

Fig. 1 The parallelogram mechanism

The mathematical model for the vibrations of an OA kinematical element with uniformly rotative motion, for lack of disturbing forces and moments, is:

$$c^2 \cdot \frac{\partial^4 u(x,t)}{\partial x^4} + \frac{\partial^2 u(x,t)}{\partial t^2} - \omega_0^2 \cdot u(x,t) = 0 \ , \qquad (1)$$

where:

$c^2 = \dfrac{E \cdot I}{\rho \cdot A}$,

$u(x,t)$ - the cross linear elastic displacement;

ω_0 - the angular velocity of the crank OA;

ρ - the density mass of the OA bar;

$A = b \cdot h$ - the plan area of the OA bar;

E - the Young modulus;

$I_{zz} = I = \frac{bh^3}{12}$ - the geometrical moment of inertia of the cross section of the OA bar, in relation to the Oz axis;

b - the width of the cross section of the OA bar;

h - the thickness of the cross section of the OA bar;

r - the length of the OA crank.

2. THE ANALYTICAL EXPRESSION OF THE CROSS DISPLACAMENTS FIELD OF THE OA KINEMATIC ELEMENT

By applying in (1) the unilateral transform Laplace in relation to time followed by the finite transform Fourier in sine (the crank OA being double jointed), it results an algebraic equation having unknown the cross displacement $\tilde{u}(n,s)$ in Laplace and Fourier images. Elementary solving this equation, it results:

$$\tilde{u}(n,s) = \frac{s \cdot f(n) + g(n)}{s^2 + c^2\alpha_n^4 - \omega_0^2} \quad , \tag{2}$$

where:

$$f(x) = u(x,0);\ g(x) = \frac{\partial u(x,0)}{\partial t};\ f(n) = \int_0^r f(x)\sin(\alpha_n x)dx;\ g(n) = \int_0^r g(x)\sin(\alpha_n x)dx;\ \alpha_n = \frac{n\pi}{r}.$$

For the function f(x), is adopted the definition law of the center warped fiber of the OA bar with the length r, simply supported, at the middle stressed by a force with the intensity F_0, normal to the bar axis, that is:

$$f(x) = \frac{F_0 x}{12 \cdot E \cdot I}\left(\frac{3r^2}{4} - x^2\right) + \frac{F_0\left(x - \frac{r}{2}\right)^3}{6 \cdot E \cdot I} \cdot H\left(x - \frac{r}{2}\right),$$

where:

$H\left(x - \frac{r}{2}\right)$ is the Heaviside function.

This way, the finite Fourier transform in sine of the function f(x) results under the form:

$$f(n) = \frac{F_0 r^4}{n\pi E \cdot I}\left[\frac{(-1)^{n+1}}{12} + \frac{(-1)^{n+1}(6 - \pi^2 n^2)}{12\pi^2 n^2} + \frac{(-1)^n}{2\pi^2 n^2} + \frac{1}{n^3\pi^3}\sin\left(\frac{n\pi}{2}\right)\right]. \tag{3}$$

By inverting, in (2), the Laplace transform and the finite in sine Fourier transform, it results the definition law of the cross displacement:

$$u(x,t) = \frac{2}{r}\sum_{n=1}^{\infty} u(n,t)\sin(\alpha_n x), \tag{4}$$

where:

$$u(n,t) = \frac{1}{\omega_n}[g(n) \cdot \sinh(\omega_n t) + \omega_n \cdot f(n) \cdot \cosh(\omega_n t)], \tag{5}$$

$$\omega_n = \sqrt{\omega_o^2 - c^2\alpha_n^4} \tag{6}$$

otherwise, with:

$$\Omega_n = \sqrt{c^2\alpha_n^4 - \omega_0^2}, \tag{6'}$$

$$u(n,t)=\frac{1}{\Omega_n}\left[-g(n)\cdot\sin(\Omega_n t)+\Omega_n\cdot f(n)\cdot\cos(\Omega_n t)\right].\qquad (5')$$

3. THE CROSS DISPLACEMENT DIAGRAMS

It is considered the concrete case with:
r=0,04[m]; ω_0=523,3[rad/sec]; b=0,018[m]; h=0,005[m]; E=$2,1\cdot10^{11}$[N/m^2]; ρ=7800[Kg/m^3]; F_0=1000[N].

With the above data, with g(n)=0 and f(n)=0 given by (3), the plot of the function (4), where u(n,t) has the definition law (5'), is given in figures 2, 3 and 4.

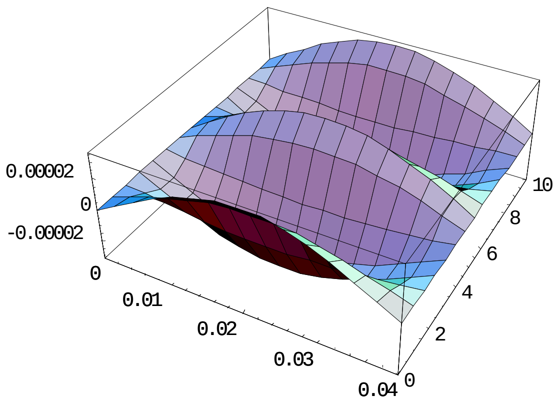

Fig. 2 u =u (x, t)

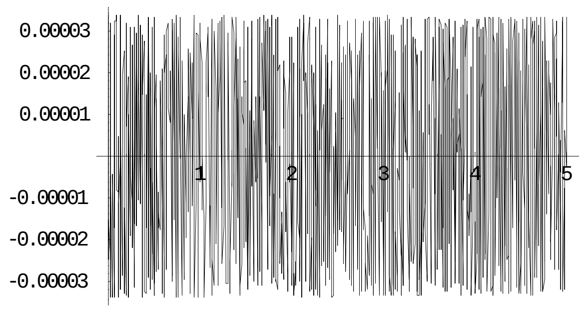

Fig.3 u=u(0,04/2, t)

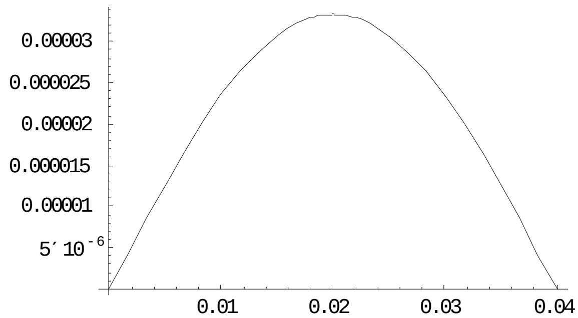

Fig.4 u=u(x, 10 sec)

For a bigger length of than crank OA, with r=0,300 [m], it result the diagrams from figures 5 and 6.

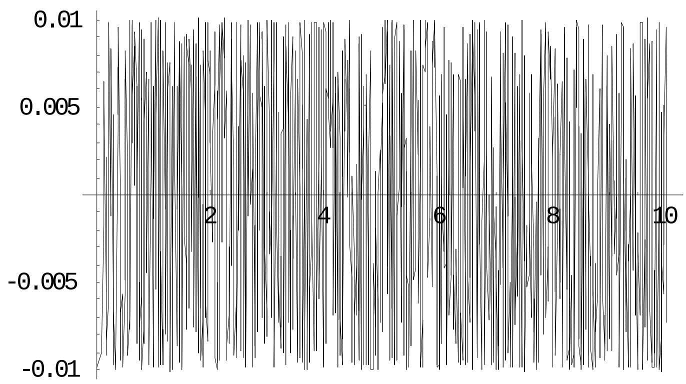

Fig.5 u =u (x, t)

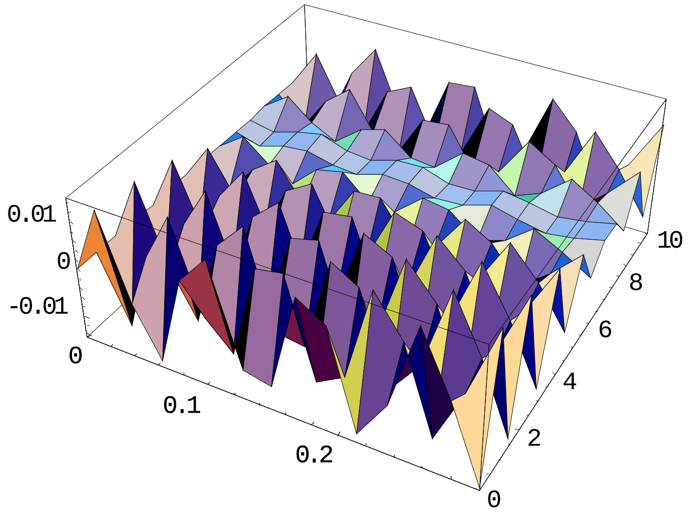

Fig.6 u=u(0,300/2, t)

4. CONCLUSIONS

In the research work and designing, the displacements determination, as a result of the vibrations, depending on the kinematical parameters of the motion, is essential. This precedes the determination of the components of specific strains tensor and of the stresses tensor, which are necessary to the dimensioning and testing calculus specific for this activity.

BIBLIOGRAPHY

1. Bagnaru, D., Rinderu, P., Vibratiile sistemelor mecanice, Editura LOTUS Craiova, 1998, ISBN 973-98290-6-6.
2. Bagnaru, D., Rizescu, S., Bolcu, D., Vibratiile sistemelor elastice, Editura Didactica si Pedagogica, Bucuresti, 1997, ISBN 973-30-5907.
3. Bagnaru, D., Cataneanu, A., Campul deplasarilor transversale pentru un element cinematic aflat in rotatie uniforma, Buletinul Universitatii "Petrol -Gaze", Ploiesti, 1999, ISSN 1221-9371,Vol.LI, nr.1, pag.135-138.
4. Ditkine, V., Proudnikov, A., Transformation integrales et calcul operationnel, Ed. Mir,Moscou,1978.
5. Nowacki, W., Dinamica sistemelor elastice, Editura Tehnica, Bucuresti, 1969