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THE COMPARATIVE STUDY BETWEEN THE EXACT AND THE APPROXIMATE METHODS FOR DETERMINATION OF THE DISPLACEMENT FIELD IN CASE OF KINEMATICAL ELEMENTS BEING IN VIBRATION

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Abstract. At the beginning, the paper presents the mathematical models of the vibrations of a linear elastic connecting rod from a parallelogram mechanism, using the mechanical model with concentrated masses and the continuous medium type mechanical model. Then, the transversal displacement field is determined for the two cases. Finally, in order to compare the results obtained both by the precise method and by approximate one, for a particular case, the authors use the variation diagrams of the transversal dependant on time and on point distribution.

1. MATHEMATICAL MODELS OF THE VIBRATIONS OF THE LINEAR ELASTIC CONNECTING ROD FROM A PARALLELOGRAM MECHANISM

Figure 1 illustrates the mechanical model with two degrees of freedom for the free transversal vibrations of the linear elastic connecting rod OA from a R (RRR) mechanism and in figure 2 is presented the continuous medium type mechanical model for the vibrations of the same kinematical element.

In the relations (2) and (3), the motion equations were deducted for a rectilinear linear elastic kinematical element by concentrating the total mass m_t of the kinematical element on "n" points P_i and by using the method of the influence coefficients.

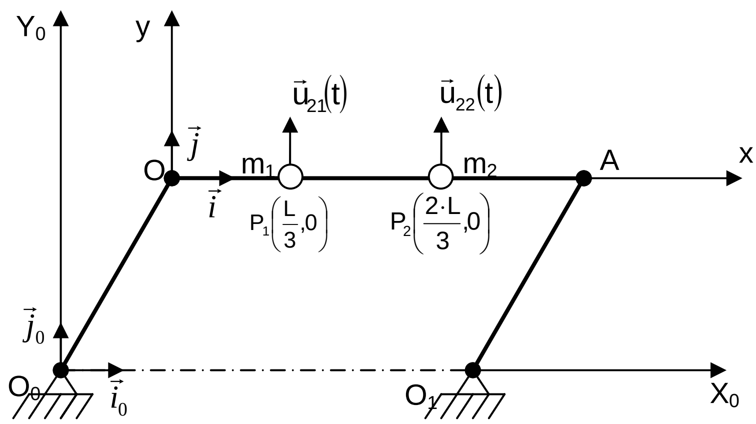

Fig.1 The mechanical model with concentrated masses

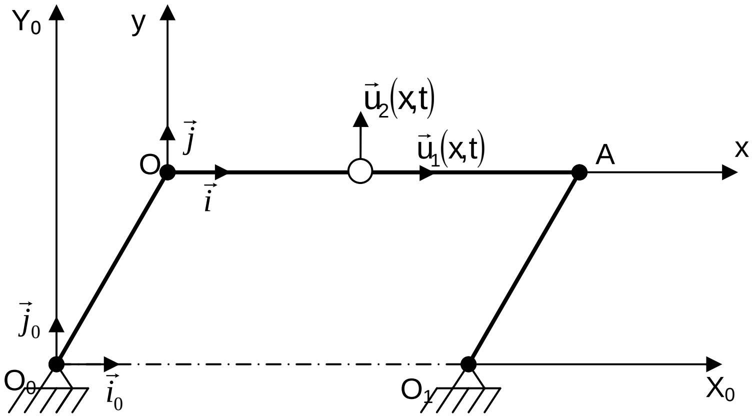

Fig.2 The continuous medium mechanical model type

The m_t mass of the connecting rod OA is considered to be concentrated on P_1 and P_2 points (figure1) so that $m_1=m_2=m=m_t/2$. Using the previous mentioned method, it results the mathematical model in its approximate method manner which uses the mechanical model of the vibrations with a finite number of degrees of freedom:

$$\begin{aligned} \alpha_{11} m_1 \ddot{u}_{21} + \alpha_{12} m_2 \ddot{u}_{22} + E u_{21} + a_{02}(\alpha_{11} m_1 + \alpha_{12} m_2) = 0 \\ \alpha_{21} m_1 \ddot{u}_{21} + \alpha_{22} m_2 \ddot{u}_{22} + E u_{22} + a_{02}(\alpha_{21} m_1 + \alpha_{22} m_2) = 0 \end{aligned} \tag{1}$$

Concordant figure 2, using Hamilton's variation principle, it results the mathematical model of the precise method, based on the linear elastic continuous medium type mechanical model, in the shape of an equation with partial derivatives:

$$E \cdot I \cdot \frac{\partial^4 u_2(x,t)}{\partial x^4} - \rho \cdot I \cdot \frac{\partial^4 u_2(x,t)}{\partial x^2 \partial t^2} + \rho A \frac{\partial^2 u_2(x,t)}{\partial t^2} + \rho A a_{02} = 0 \tag{2}$$

where:

$u_2(x,t)$	is the transversal linear elastic displacement;
$a_{02} = -\omega_0^2 r \sin(\omega_o t)$	is the acceleration of the extremity O of the OA bar;
ω_0	is the angular velocity of the O_0O crank;
ρ	is the specific mass of the OA bar;
$A = b \cdot h$	is the cross section area of the OA bar;
E	is the Young's modulus;
$I_{zz} = I = \dfrac{bh^3}{12}$	is the geometric moment of inertia of the transversal section of the OA bar with respect to Oz axis;
b	is the transversal section breadth of the OA bar;
h	is the transversal section thickness of the OA bar;
$m_t = \rho bhL$	is the total mass for the OA bar;
r	is the length of the O_0O crank;
L	is the length of the OA bar;

$$\alpha_{11} = \alpha_{22} = \frac{4 \cdot L^3}{243 \cdot I}; \alpha_{12} = \alpha_{21} = \frac{7 \cdot L^3}{243 \cdot I};$$

$\vec{u}_{21}(t)$ and $\vec{u}_{22}(t)$ represent the transversal displacement of the P_1 and P_2 points.

2. THE TRANSVERSAL DISPLACEMENT FIELD

In the exact approach, by integrating equation (2) using Laplace and Fourier integral transforms finite in sinus (displacements and moments in O and A are zero), with homogenous initial conditions meaning $u_2(x,0)=f(x)=0$ and $\frac{\partial u_2(x,0)}{\partial t} = g(x) = 0$, it results the transversal displacement field in the shape of the following series:

$$u_2(x,t) = a \cdot \sum_{n=1}^{\infty} \frac{a_{1,n} \cdot \left[a_{2,n} \cdot \sin(\omega_n t) + a_{3,n} \cdot \sin(\omega_0 t)\right]}{a_{3,n}^2 - a_{2,n}^2} \cdot \sin(\alpha_n x), \tag{3}$$

where:

$$a = \frac{2A \cdot \rho \cdot r \cdot \omega_0^2}{L\sqrt{E \cdot I}}; a_{1,n} = \frac{\cos(L \cdot \alpha_n - 1)}{\alpha_n^3}; a_{2,n} = \omega_0 \sqrt{\rho\left(A + I \cdot \alpha_n^2\right)}; a_{3,n} = -\alpha_n^2 \sqrt{E \cdot I};$$

$$\omega_n = \alpha_n^2 \sqrt{\frac{E \cdot I}{\rho\left(A + I \cdot \alpha_n^2\right)}}; \alpha_n \frac{n\pi}{L}.$$

By integrating the differential equation system (1) and by applying the Laplace transform with respect to the time it results an algebraic system with the unknowns $u_{21}(s)$ and $u_{22}(s)$ which can be solved elementary. By inverting the Laplace transform, it results the transversal displacement of the points P_1 and P_2.

$$u_{21}(t) = \frac{1}{2\sqrt{E} \cdot \left[E - m\omega_0^2(\alpha_{11} + \alpha_{12})\right]} \cdot \left\{\sqrt{E^3} \cdot \left[\left(u_{21,0} - u_{22,0}\right) \cdot \cos\left(\sqrt{\frac{E}{m(\alpha_{11} - \alpha_{12})}} \cdot t\right) + \right.\right.$$

$$\left. + \left(u_{21,0} + u_{22,0}\right)\cos\left(\sqrt{\frac{E}{m(\alpha_{11} + \alpha_{12})}} t\right)\right] + \left[E - m\omega_0^2(\alpha_{11} + \alpha_{12})\right]\sqrt{m(\alpha_{11} - \alpha_{12})}\left(u'_{21,0} - \right.$$

$$\left. - u'_{22,0}\right)\sin\left(\sqrt{\frac{E}{m(\alpha_{11} - \alpha_{12})}} t\right) + \left[E\left(u'_{21,0} + u'_{22,0}\right) - m\omega_0^2(\alpha_{11} + \alpha_{12})\left(2r\omega_0 + u'_{21,0} + u'_{22,0}\right)\right] \cdot$$

$$\cdot \sqrt{m(\alpha_{11} + \alpha_{12})} \cdot \sin\left(\sqrt{\frac{E}{m(\alpha_{11} + \alpha_{12})}} \cdot t\right) + \sqrt{E} \cdot m \cdot \omega_0^2 \cdot (\alpha_{11} + \alpha_{12}) \cdot \left[2 \cdot r \cdot \sin(\omega_0 t) + \right.$$

$$\left.\left. + \left(-u_{21,0} + u_{22,0}\right) \cdot \cos\left(\sqrt{\frac{E}{m(\alpha_{11} - \alpha_{12})}} \cdot t\right) - \left(u_{21,0} + u_{22,0}\right) \cdot \cos\left(\sqrt{\frac{E}{m(\alpha_{11} + \alpha_{12})}} \cdot t\right)\right]\right\}; \tag{4}$$

$$u_{22}(t)=\frac{1}{2\sqrt{E}\cdot\left[E-m\omega_0^2(\alpha_{11}+\alpha_{12})\right]}\cdot\left\{\sqrt{E^3}\cdot\left[\left(-u_{21,0}+u_{22,0}\right)\cdot\cos\left(\sqrt{\frac{E}{m(\alpha_{11}-\alpha_{12})}}\cdot t\right)+\right.\right.$$

$$\left.+\left(u_{21,0}+u_{22,0}\right)\cos\left(\sqrt{\frac{E}{m(\alpha_{11}+\alpha_{12})}}t\right)\right]+\left[-E+m\omega_0^2(\alpha_{11}+\alpha_{12})\right]\sqrt{m(\alpha_{11}-\alpha_{12})}\left(u'_{21,0}-\right.$$

$$\left.-u'_{22,0}\right)\sin\left(\sqrt{\frac{E}{m(\alpha_{11}-\alpha_{12})}}t\right)+\left[E\left(u'_{21,0}+u'_{22,0}\right)-m\omega_0^2(\alpha_{11}+\alpha_{12})\left(2r\omega_0+u'_{21,0}+u'_{22,0}\right)\right]\cdot$$

$$\cdot\sqrt{m(\alpha_{11}+\alpha_{12})}\cdot\sin\left(\sqrt{\frac{E}{m(\alpha_{11}+\alpha_{12})}}\cdot t\right)-\sqrt{E}\cdot m\cdot\omega_0^2\cdot(\alpha_{11}+\alpha_{12})\cdot\left[-2\cdot r\cdot\sin(\omega_0 t)+\right.$$

$$\left.\left.+\left(-u_{21,0}+u_{22,0}\right)\cdot\cos\left(\sqrt{\frac{E}{m(\alpha_{11}-\alpha_{12})}}\cdot t\right)+\left(u_{21,0}+u_{22,0}\right)\cdot\cos\left(\sqrt{\frac{E}{m(\alpha_{11}+\alpha_{12})}}\cdot t\right)\right]\right\}, \quad (5)$$

where:

$u_{21,0}=u_{21}(0)$; $u_{22,0}=u_{22}(0)$; $u_{21,0}=u_{21}(0)$; $u_{22,0}=u_{22}(0)$ are the initial conditions.

3. GRAPHICAL REPRESENTATION OF THE CROSS DISPLACEMENTS

It is considered a particular case with:

L=0,445[m]; r=0,04[m]; ω_0=76,61[s^{-1}]; b=0,018[m]; h=0,005[m]; E=2,1*10^{11}[N/m^2]; ρ=7888[kg/m^3]

By using only the first four terms of the serial development (3), it results the diagrams from figure 3 and figure 4.

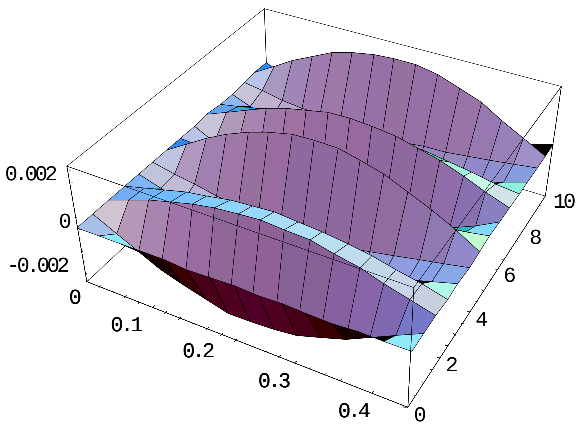

Fig.3 $u_2=u_2(x,t)$

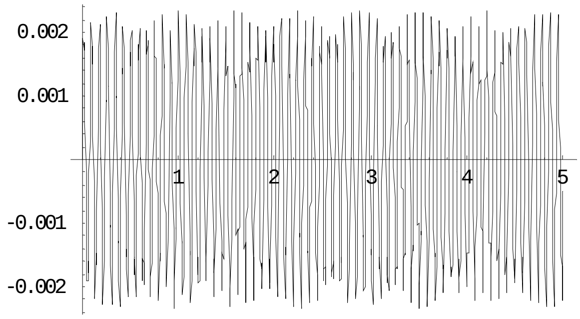

***Fig.4** $u_2=u_2(L/3,t)$*

In the initial homogenous conditions, from (4) and (5) results $u_{21}(t)=u_{22}(t)$ represented in figure 5.

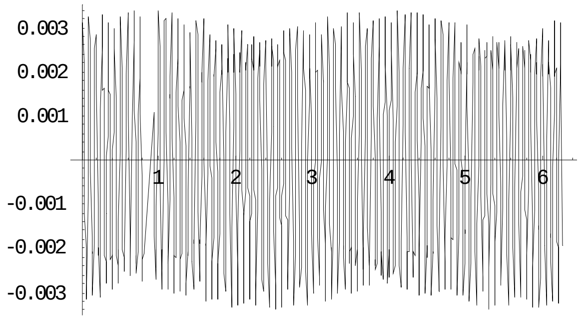

***Fig.5** $u_{21}(t)=u_{22}(t)$*

4. CONCLUSIONS

For the present time applications, in order to obtain certain calculus accuracy, one can use the approximate method, the resulting differences between the displacements from the above shown graphics being acceptable.

This method has the advantage of the integration of a differential equation system unlike the exact method that involves the integration of far more complicate partial derivatives equations.

The knowledge of the displacement field as a result of the vibrations of the kinematical elements as constitutive parts of some mechanism represents a preliminary stage of the determination of the induced stresses and deformations needed in any kind of research and design activities concerning machines in general and motor vehicles in particular.

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