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THE INFLUENCE OF THE KINEMATICAL PARAMETERS CONCERNING THE VIBRATIONS OF THE VISCOELASTIC CONNECTINGROD FOR A SLIDER-CRANK MECHANISM

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Abstract. In this paper is presented, first, the mathematical model of the vibrations for the viscoelastic connecting rod from a slider-crank mechanism. Afterwards, the cross displacements field is determined using an iterative method. Finally, for a concrete case, the cross displacements variation diagrams depending on time and the point distribution are presented.

1. THE MOVEMENT MATHEMATICAL MODEL

By using Hamilton's principle [1], the mathematical model for the vibrations of a linear elastic bar having a plan parallel motion, (fig. 1 and fig.2) was determined under the form:

$$[L]\{u\}+[M_4]\{a_0\}+\{V\}_1+[M_7]\{f\}+\{V_2\}=\{0\} \quad , \tag{1}$$

where $[L]\bullet$ is a differential operator, having the expression:

$$[L]\bullet=[M_1]\frac{\partial^4\bullet}{\partial x^4}+[M_2]\frac{\partial^4\bullet}{\partial x^2\partial t^2}+[M_3]\frac{\partial^2\bullet}{\partial x^2}+[M_4]\frac{\partial^2\bullet}{\partial t^2}+[M_5]\frac{\partial\bullet}{\partial t}+[M_6]\bullet \tag{2}$$

The matrices and the vectors from the expressions (1) and (2) are:

$$[M_1]=E\cdot I\cdot\begin{bmatrix}0&0\\0&1\end{bmatrix};[M_2]=\rho\cdot I\cdot\begin{bmatrix}0&0\\0&-1\end{bmatrix};[M_3]=E\cdot A\cdot\begin{bmatrix}1&0\\0&1\end{bmatrix};[M_4]=\rho\cdot A\cdot\begin{bmatrix}-1&0\\0&1\end{bmatrix};[M_7]=\begin{bmatrix}1&0\\0&-1\end{bmatrix};$$

$$\{V_1\}=\rho\cdot A\cdot x\cdot\{\ \omega^2;\varepsilon\}^T;\{V_2\}=\left\{o;\frac{\partial m}{\partial x}\right\}^T;\{u\}=\{u_1;u_2\}^T;\{a_0\}=\{a_{01};a_{02}\}^T;\{f\}=\{f_1;f_2\}^T;\{0\}=\{0;0\}^T,$$

$$[M_5]=[0];[M_6]=\rho A\omega^2[M_7],$$

where: $\vec{u}$ - the linear elastic displacement;
$\vec{\omega}$ - the angular velocity of the OA bar;
$\vec{\varepsilon}$ - the angular acceleration of the OA bar;
$\vec{a}_0$ - the acceleration of the extremity point O of the OA bar;
$\vec{f}(x,t)$ - the exterior force on the length unity;
$\vec{m}(x,t)$ - the exterior moment on the unity length unity;
ρ - the specific mass ;
$A(x)$ - the plan area of the bar;
E - the Young modulus;
$I_{zz}=I$ - the geometrical moment of inertia of the plan area about the neutral axis of the bar.

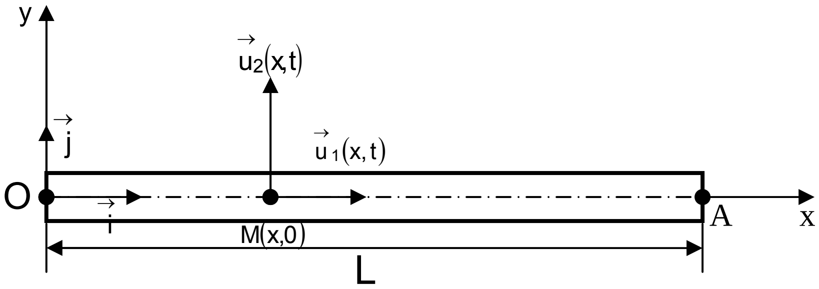

Fig.1 The bar in plan-parallel motion

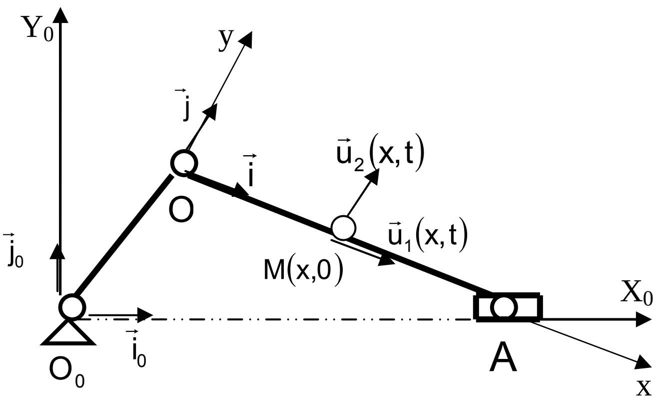

Fig. 2 The slider-crank mechanism R(RRT)

Grouping those terms which grant to the mathematical model the invariant in time model quality and considering $\{V_2\} = \{0\}$ and $\{f\} = \{0\}$, that is the case of free vibrations, it results the mathematical model under the form:

$$[L_0]\{u\} + [M_4]\{a_0\} + \{V_1\} + \{F\} = \{0\}, \tag{1'}$$

where the differential operator $[L_0]\bullet$ is:

$$[L_0]\bullet = [M_1]\frac{\partial^4 \bullet}{\partial x^4} + [M_2]\frac{\partial^4 \bullet}{\partial x^2 \partial t^2} + [M_3]\frac{\partial^2 \bullet}{\partial x^2} + [M_4]\frac{\partial^2 \bullet}{\partial t^2}, \tag{2'}$$

and:

$$\{F\} = \{F_1, F_2\} = [M_6]\{u\} \quad , \tag{3}$$

contains the terms above mentioned.

Neglecting in (1') the vector $\{F\}$, it results the matrix equation:

$$[L_0]\{u\} + [M_4]\{a_0\} + \{V_1\} = \{0\} \ , \tag{1''}$$

that is an uncoupled model, linear and with constant coefficients in a first approximation.

By applying to the equation (1") the unilateral Laplace transform in relation with time and replacing the E modulus with $\tilde{E}(s)$, it results the matrix equation of the first approximation, in Laplace's images, of the vibrations for the viscoelastic connecting rod OA of the slider-crank mechanism R(RRT) (fig.2) under the form:

$$[L_0(s)]\{\tilde{u}^{(1)}\} + [M_4]\{\tilde{a}_0\} + \{\tilde{V}_1\} = \{0\} \tag{3'}$$

where:

$$[L_0(s)] = [M_1(s)]\frac{\partial \bullet}{\partial x^4} + [M_8(s)]\frac{\partial \bullet}{\partial x^2} + [M_4(s)]\bullet; [M_1(s)] = \tilde{E}(s)\cdot I \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix};$$

$$[M_8(s)] = \begin{bmatrix} \widetilde{E}(s)A & 0 \\ 0 & -\rho \cdot I \cdot s^2 \end{bmatrix}; [M_4(s)] = s^2[M_4]; \{\widetilde{a}_0\} = \{\widetilde{a}_{01}(s); \widetilde{a}_{02}(s)\}^T;$$

$$(\widetilde{V}_1) = \rho A x \{\widetilde{g}(s); \widetilde{\varepsilon}(s)\}^T; \omega^2(t) = g(t); \{\widetilde{u}^{(1)}\} = \{\widetilde{u}_1^{(1)}(x,s); \widetilde{u}_2^{(1)}(x,s)\}^T;$$

$$\widetilde{E}(s) = \frac{a_0 s}{b_0 s + b_1}; a_0 = 9GK; b_0 = 3K + G; b_1 = \frac{3KG}{\eta};$$

$$K = \frac{\nu E}{(1+\nu)(1-2\nu)} + \frac{2}{3}G;$$

$\widetilde{E}(s)$ has the expression proper to the Maxwell mathematical model specific to the polymers in a solid state (plastics);

G - rigidity modulus;
K - compressibility modulus;
ν - Poisson cross contraction coefficient;
η - the afferent constant to the Newton type component of the Maxwell's model.

2. THE DYNAMIC RESPONSE

By applying, in (3'), the finite in sine and cosine Laplace transforms, to the first and, respectively, to the second equation, result uncoupled algebraic systems with the displacements $u_{1,c}^{(1)}(n,s)$ and $u_{2,s}^{(1)}(n,s)$ in Laplace and Fourier images in sine, respectively, in cosine unknowns. Afterwards, inverting the Laplace and Fourier transforms, it results the solution in a first approximation $\{u^{(1)}(x,t)\}$. With $\{u^{(1)}(x,t)\}$, the vectors $\{F^{(1)}\}$ are calculated, in a first approximation. This one is introduced in the equation (1'), resulting the mathematical model in the second approximation. Solving this one, with the help of the integral transforms, is obtained the solution $\{u^{(2)}(x,t)\}$ in the second approximation. Resuming the repetitive process, it results the mathematical model in the "j" approximation:

$$[L_0]\{u^{(j)}(x,t)\} + [M_4]\{a_0\} + \{V\}_1 + \{F^{(j-1)}\} = \{0\} \quad (4)$$

where:

$$\{F^{(j-1)}\} = [M_6]\{u^{(j-1)}(x,t)\},\ j=1,2,...,n;\ \{F^{(0)}\} = \{0\};$$

$$\{u^{(j)}\} = \{u_1^{(j)}(x,t), u_2^{(j)}(x,t)\}^T;$$

$$\{u^{(j-1)}\} = \{u_1^{(j-1)}(x,t), u_2^{(j-1)}(x,t)\}^T$$

The solution in the "j" approximation should be:

$$u_1^{(j)}(x,t) = \frac{1}{L} \cdot u_{1,c}^{(j)}(o,t) + \frac{2}{L}\sum_{n=1}^{n=\infty} u_{1,c}^{(j)}(n,t) \cdot \cos(\alpha_n \cdot x) \quad (5)$$

$$u_2^{(j)}(x,t) = \frac{2}{L}\sum_{n=1}^{n=\infty} u_{2,s}^{(j)}(n,t) \cdot \sin(\alpha_n \cdot x), \quad (6)$$

where $u_{1,c}^{(j)}(n,t)$ and $u_{2,s}^{(j)}(n,t)$ are the finite transform Fourier in cosine, respectively in sine, of the longitudinal, respectively of the cross elastic displacements.

The successive approximation process is finished when $\forall n, \left\| \{u^{(j)}\} - \{u^{(j-1)}\} \right\| \le \varepsilon$, where $\varepsilon > 0$ and little enough according to the demanded calculus precision.

The connecting rod OA being double jointed, the boundary conditions which have permitted the application of the two Fourier transform of the original functions and, respectively, for their Laplace images, were:

$$\frac{\partial u_1(0,t)}{\partial x}=\frac{\partial u_1(L,t)}{\partial x}=0;u_2(0,t)=u_2(L,t)=0;\frac{\partial^2 u_2(0,t)}{\partial x^2}=\frac{\partial^2 u_2(L,t)}{\partial x^2}=0,$$

$$\frac{\partial u_1(0,s)}{\partial x}=\frac{\partial u_1(L,s)}{\partial x}=0;u_2(0,s)=u_2(L,s)=0;\frac{\partial^2 u_2(0,s)}{\partial x^2}=\frac{\partial^2 u_2(L,s)}{\partial x^2}=0.$$

The boundary conditions are:

$$u_1(x,o)=0;\frac{\partial u_1(x,0)}{\partial t}=0;u_2(x,0)=0;\frac{\partial u_2(x,0)}{\partial t}=0\,.$$

The cross displacement field, in a first approximation, in case of the free vibrations of the connecting rod OA of the R(RRT) mechanism from the figure 2, represented by the equation (1”), where:

$$\{a_o\}=\left\{\omega_0^2 r\left[\frac{r}{L}\sin^2(\omega_0 t)-\cos(\omega_0 t)\right];-\omega_0^2 r\left[\sin(\omega_0 t)+\frac{r}{2L}\sin(2\omega_0 t)\right]\right\}^T;$$

$$\omega=-\frac{r}{L}\omega_0\cos(\omega_0 t);\varepsilon=\frac{\omega_0^2 r}{L}\sin(\omega_0 t),$$

should be given, as result of the application of the preceding integration algorithm, by a type (6) function with j=1, that is the function:

$$u_2^{(1)}(x,t)=\frac{2}{L}\sum_{n=1}^{n=\infty}u_{2,s}^{(1)}(n,t)\cdot\sin(\alpha_n\cdot x),\qquad (6')$$

$$u_{2,s}^{(1)}(n,t)=\frac{a_{n,4}}{a_{n,1}}\{a_{n,3}\omega_0[\cosh(\Omega_{n,1}t)-\sinh(\Omega_{n,1}t)][a_1+a_{n,5}-2a_2a_{n,1}\sinh(\Omega_{n,1}t)-2a_2a_{n,1}\cosh(\Omega_{n,1}t)$$
$$+a_2a_{n,1}+a_{n,6}\sinh(\omega_n t)+a_{n,6}\cosh(\omega_n t)]\}+\frac{a_{n,7}\omega_0}{a_{n,1}a_{n,2}}\left[\cosh\left(\frac{b_1}{b_0}t\right)-\sinh\left(\frac{b_1}{b_0}t\right)\right]\{a_{n,8}[-1+\cosh(\omega_n t)$$
$$+\sinh(\omega_n t)][\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)]+a_3a_{n,1}\omega_0\sin(\omega_0 t)\left[\cosh\left(\frac{b_1}{b_0}t\right)+\sinh\left(\frac{b_1}{b_0}t\right)\right]-a_{n,9}\cdot$$
$$\cdot[\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)][-b_1a_2+b_1a_{n,1}+a_{n,10}\cosh(\omega_n t)+a_{n,10}\sinh(\omega_n t)+a_4+2(-b_1a_{n,1}\cdot$$
$$\cdot\cos\omega_0 t+b_0a_{n,1}\omega_0\sin\omega_0 t)\cosh(\Omega_{n,1}t)+2a_{n,1}(-b_1\cos\omega_0 t+b_0\omega_0\sin\omega_0 t)\sinh(\Omega_{n,1}t)]\}+\frac{a_{n,11}\omega_0}{a_{n,1}a_{n,2}}\cdot$$
$$\cdot\left[\cosh\left(\frac{b_1}{b_0}t\right)-\sinh\left(\frac{b_1}{b_0}t\right)\right]\{(a_{n,8}-a_{n,9}a_2b_1+4b_0^2\omega_0^2a_{n,9}a_6)[-1+\cosh(\omega_n t)+\sinh(\omega_n t)]\cdot$$
$$\cdot[\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)]+a_{n,1}[a_5\omega_0\sin(2\omega_0 t)-2a_{n,9}b_1\cos(2\omega_0 t)+4b_0^2\omega_0^2a_{n,9}\sin(2\omega_0 t)]\cdot$$
$$\cdot\left[\cosh\left(\frac{b_1}{b_0}t\right)+\sinh\left(\frac{b_1}{b_0}t\right)\right]+a_{n,9}b_1a_{n,1}[\cosh(\Omega_{n,2}t)+\sinh(\Omega_{n,2}t)]+a_{n,9}b_1a_{n,1}[\cosh(\Omega_{n,1}t)+$$
$$+\sinh(\Omega_{n,1}t)]\}\qquad (7)$$

where:

$$a_1=-A\rho b_1^2;a_2=b_1\sqrt{A\rho};a_3=2A\rho\sqrt{A\rho}(b_1^2+b_0^2\omega_0^2);a_4=-2\sqrt{A\rho}\cdot b_0^2\omega_0^2;a_5=4A\rho\sqrt{A\rho}(b_1^2+4b_0^2\omega_0^2);$$

$$a_6 = 2\sqrt{A\rho}; a_{n,1} = \sqrt{A\rho b_1^2 - 4 \cdot I \cdot a_0 b_0 \alpha_n^4}; a_{n,2} = A^2\rho^2\omega_0^2\left(b_1^2 + b_0^2\omega_0^2\right) + I \cdot a_0\alpha_n^4\left(I \cdot a_0\alpha_n^4 - 2A\rho b_0\omega_0^2\right);$$

$$a_{n,3} == -4\sin(L\alpha_n) - r \cdot \alpha_n \cos(L\alpha_n) + \alpha_n(r + 4L); a_{n,4} = -\frac{r \cdot \sqrt{A\rho}}{8L \cdot I \cdot a_0\alpha_n^6}; a_{n,5} = 2 \cdot I \cdot a_0 b_0 \alpha_n^4; a_{n,6} =$$

$$= a_2 a_{n,1} + a_2^2 - a_{n,5}; a_{n,7} = \frac{r\sqrt{A\rho}\left[\sin(L\alpha_n) - L\alpha_n\right]}{2L\alpha_n^2}; a_{n,8} = 2 \cdot I^2 \cdot a_0^2 b_0 \alpha_n^8; a_{n,9} = I \cdot \sqrt{A\rho} \cdot a_0\alpha_n^4; a_{n,10} =$$

$$= a_2 b_1 + b_1 a_{n,1} - a_4; a_{n,11} = \frac{r^2\sqrt{A\rho}\left[\cos(L\alpha_n) - 1\right]}{8L\alpha_n}; \Omega_{n,1} = \frac{b_1}{2b_0} + \frac{\omega_n}{2}; \Omega_{n,1} = \frac{b_1}{2b_0} + \frac{\omega_n}{2}; \Omega_{n,2} = \frac{b_1}{2b_0} - \frac{\omega_n}{2};$$

$$\omega_n = \frac{a_{n,1}}{b_0 \cdot \sqrt{A\rho}}.$$

3. THE CROSS DISPLACEMENT FUNCTION DIAGRAMS

Let's consider a concrete case with:
L=0,445[m];r=0,04[m]; ω_0 =76,61[s^{-1}];b=0,018[m];h=0,005[m]; ρ=1213,3 [Kg/m^3];
G=0,0118979$\cdot 10^{11}\left[\frac{N}{m^2}\right]$; $\eta = 4{,}5 \cdot 3600 \cdot 10^{13}\left[\frac{Ns}{m^2}\right]$; K=0,02424455$\cdot 10^{11}\left[\frac{N}{m^2}\right]$;

With the definition law (6') in which $u_{2,s}^{(1)}(n,t)$ is given by the expression (7) and with the concrete data above, result the graphical representations from the figure 3 and 4 of the functions $u_2^{(1)}(x,t)$, respectively $u_2^{(1)}\left(\frac{L}{3},t\right)$, i=1,2.

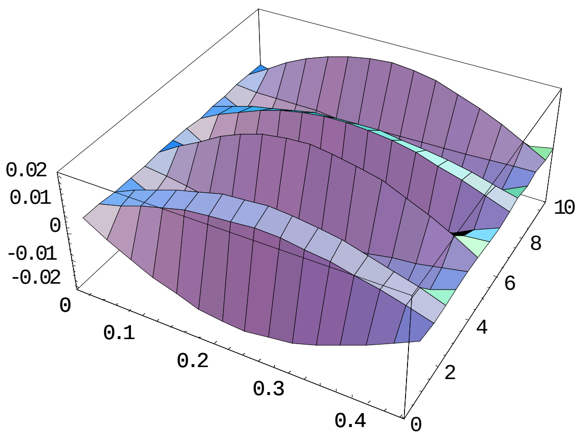

Fig.3 $u_2 = u_2^{(1)}(x,t)$

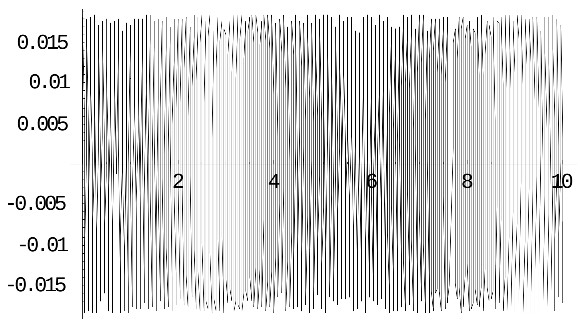

Fig.4 $u_2 = u_2^{(1)}\left(\frac{L}{3},t\right)$

4. CONCLUSIONS

The present paper insists on the cross displacement field because the influence of the longitudinal displacements on the stress and strain state is little enough. The definition law (6') proves the influence of the kinematical parameters on the cross displacement and, implicitly, on the stress and strain state, that intervene in the strength calculations necessary to the designing activity.

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