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AN OPTIMAL CONSUMPTION POLICY MODEL FOR ECONOMIC GROWTH

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Abstract. The paper reconsiders the Solow-Swan neoclassical growth model, where, beside capital stocks and labor, the impact of a third production factor is studied. The exhaustible natural resources having its own dynamics will lead to a greater influence of the substitution in production. The problem which arrises is if the conventional evaluation of the national income can be modified as to consider the depletion of the natural resources and the depreciation of the quality of enviroment.

1. INTRODUCTION

In the resources theory there are two intertemporal allocation rules: Hotteling rule and Hartwick rule. The first appears in Dasgupta-Heal-Solow model as a local efficiency condition and stipulates that along a path of efficient employment of resources, the price of exhaustible resources grows by a rate equal to the interest rate. It follows that a null value of the agregate net investments leads to the constancy in time of the consumption. The Hartwick rule was stated for a production economy where consumption at any moment t depends not only on the natural capital extraction, but also on the stock of manufactured capital available at moment t.

Solow and Hartwick have established a sustainability condition of a maximal constant consumption flow in a closed economy which disposes of an exhaustible resource as input, a reproductible capital and uses a constant technology in order to produce consumption goods.

2. THE APPLICATION OF HARTWICK RULE IN A MULTISECTORIAL ECONOMY

Let us consider a two sectors economy. One of them uses as input an exhaustible resource denoted by $s(t)$, $s(0) = s_0 > 0$. The row material $m(t)$ and the capital stock $k(t)$, $k(0) = k_0(t) > 0$ are used as inputs in the other sector. The production function is $y(t) = f(k(t), m(t))$ and the output $y(t)$ is divided between consumption $c(t)$ and accumulation or investments $i(t) = k'(t)$. Let us suppose that there is no depreciation of the capital, all technological progress is endogenous and the level of population is maintained constant. The competitive intertemporal equilibrium provides market prices for the capital goods.

The properties of the consumption function are:

(i) function f is continuous

(ii) function f has positive partial derivatives of first order

$$\frac{\partial f(k,m)}{\partial k} > 0 \text{ and } \frac{\partial f(k,m)}{\partial m} > 0, \ \forall \ k, m > 0$$

(iii) function f has negative partial derivatives of second order

$$\frac{\partial f^2(k,m)}{\partial k^2} < 0 \text{ şi } \frac{\partial f^2(k,m)}{\partial m^2} < 0, \ \forall \ k, m > 0$$

(iv) $\lim_{k\to\infty} f_k(k, m) = 0, m > 0$ and $\lim_{k\to\infty} f_m(k, m) = 0, m > 0$.

Definition 2.1. *A triplet of functions of the form (c(t), k(t), i(t)), where c(t) ≥ 0, m(t) ≥ 0, k(t) ≥ 0, s(t) ≥ 0 and f(k(t), m(t)) ≥ c(t) + i(t), t ≥ 0 is called admisible path for the economic growth.*

Let *Ω = {(c(t), k(t), i(t))| t ≥ 0}* denote the set of all admisible paths. We will presume that following properties hold:

(i) *Ω* is convex and closed

(ii) if *(c(t), k(t), i(t))* $\in \Omega$ then *k ≥ 0*

(iii) if $j \le i$ and *(c, k, i)* $\in \Omega$ then *(c, k, j)* $\in \Omega$.

The vector of consumption goods generates utility, described by relation *u(c) = u(c(t))*, where the properties of function *u* which indicates the utility provided by a level of consumption per capita *c* are:

(i) function *u* is differentiable and $\lim_{c\to\infty} u'(c) = 0$

(ii) function *u* is no decreasing

(iii) function *u* is concave.

Let *p(t)* and *q(t)* denote respectively the present value prices of the consumption goods and capital stocks at moment *t*. Let *λ(t)* denote the utility discount factor at moment *t*. If we consider $\lambda(t) = \lambda(0)e^{-\delta t}$ then the discount rate is constant, given by

$$\delta = -\frac{\lambda'(t)}{\lambda(t)} = \frac{\lambda(t)}{\int_t^\infty \lambda(s)ds}.$$

Definition 2.2. *An admisible growth path (c(t), k(t), i(t)), t ≥ 0 is called feasible if k'(t) = i(t),* $k(0) = k_0 > 0$*, s'(t) = - m(t),* $s(0) = s_0 > 0$*.*

Definition 2.3. *The feasible path (c(t), k(t), i(t)), t ≥ 0 is said to be competitive relative to the consumption positive discount factors {λ(t)}, t ≥ 0 and to the nonnegative discount prices {p(t), q(t)}, t ≥ 0 if for all t following relations hold:*

(i) *the instant utility λ(t)u(c) – p(t)c is maximized by* $c^*(t)$

(ii) *the instant income is maximized, namely* $(c^*(t), k^*(t), i^*(t))$ *maximizes the expresion p(t)c + q(t)i + q'(t)k for all triplets (c, k, i)* $\in \Omega$*.*

Remark. The competitive conditions (i) and (ii) can also be expressed as

$$\lambda(t)c^*(t) + p(t)i^*(t) - q(t)m^*(t) + p'(t)k^*(t) + q'(t)s^*(t) \ge$$
$$\ge \lambda(t)c + p(t)i - q(t)m + p'(t)k + q'(t)s$$

Remark. To sustain the fact that *p(t)c + q(t)i + q'(t)k* denotes the instant profit we will express the prices of the consumption goods and of the production means in current utility terms $P(t) = \frac{p(t)}{\lambda(t)}$ respectively $Q(t) = \frac{q(t)}{\lambda(t)}$. We obtain a nonarbitrarily condition

$$Q'(t) = \frac{d\frac{q(t)}{\lambda(t)}}{dt} = \frac{q'(t)}{\lambda(t)} - q(t)\frac{\lambda'(t)}{\lambda^2(t)} = \frac{q'(t)}{\lambda(t)} + \delta_0(t)Q(t),$$

which proves the applicability of Hotteling rule: along a path of efficient employment of resources, the price of exhaustible resources grows by a rate equal to the interest rate. Further it follows that

$$\frac{p(t)}{\lambda(t)}c + \frac{q(t)}{\lambda(t)}k' + \frac{q'(t)}{\lambda(t)}k = P(t)c + Q(t)k' - \left[\delta_0(t)Q(t) - Q'(t)\right]k,$$

where $P(t)c + Q(t)k'$ denotes the current value of production and $[\delta_0(t)Q(t) - Q'(t)]k$ the current cost of capital. Therefore, any competitive path is efficient due to the finite value of the discounted sum of instant utilities. Meanwhile, the transversality condition for the level of capital $\lim_{t\to\infty} q(t)k^*(t) = 0$ holds.

Definition 2.4. *The competitive path ($c^*(t)$, $k^*(t)$, $i^*(t)$), $t \geq 0$ is said to be regular relative to the consumption positive discount factors $\{\lambda(t)\}$, $t \geq 0$ and to the nonnegative discount prices $\{p(t), q(t)\}$, $t \geq 0$ if for all t following relations hold:*

(i) *there exists* $\int_0^\infty \lambda(t)u(c^*(t))dt < \infty$

(ii) $\lim_{t\to\infty} q(t)k^*(t) = 0.$

Definition 2.5. *The feasible path ($c(t)$, $k(t)$, $i(t)$), $t \geq 0$ is said to be of type maximin if following relation*

$$\inf_{t\in[0,\infty)} c(t) \geq \inf_{t\in[0,\infty)} c_*(t)$$

holds for all feasible paths ($c_*(t)$, $k_*(t)$, $i_*(t)$) $\in \Omega$.

Let us consider that in the studied model there exists a path of type maximin ($c(t)$, $k(t)$, $i(t)$), $t \geq 0$ so that

$$\inf_{t\in[0,\infty)} c(t) = c^* > 0.$$

Proposition 2.6. *If the path ($c^*(t)$, $k^*(t)$, $i^*(t)$), $t \geq 0$ is regular relative to the consumption positive discount factors $\{\lambda(t)\}$, $t \geq 0$ and to the nonnegative discount prices $\{p(t), q(t)\}$, $t \geq 0$ then ($c^*(t)$, $k^*(t)$, $i^*(t)$), $t \geq 0$ maximizes*

$$\int_0^\infty \lambda(t)u(c^*(t))dt$$

on the set of all admisible consumption plans $k(t)$, $t \geq 0$ and $k(0) = k_0$.

Proof. Let ($c(t)$, $k(t)$, $i(t)$) $\in \Omega$ so that $k(0) = k_0$. According to condition (i) of Definition 2.3 and further condition (ii) it follows that

$$\int_0^T \lambda(t)\left[u(c(t)) - u(c^*(t))\right]dt \leq \int_0^T p(t)\left[c(t) - c^*(t)\right]dt \leq$$

$$\leq \int_0^T \left\{q(t)\left[k'^*(t) - k'(t)\right] + q'(t)\left[k^*(t) - k(t)\right]\right\}dt =$$

$$= q(T)\left[k^*(T) - k(T)\right] - q(0)\left[k^*(0) - k(0)\right] \leq q(T)k^*(T).$$

According to Definition 2.4 the conclusion holds. □

Proposition 2.7. *Following relations hold*

(i) *If the path ($c^*(t)$, $k^*(t)$, $i^*(t)$), $t \geq 0$ is competitive and $c^*(t) > 0$, $\forall t \geq 0$ then the relation*

$$\lambda(t)\frac{\partial u(c(t))}{\partial c}(c^*(t)) = p(t)$$

holds for all consumption goods

(ii) *If the set Ω is smooth and the path ($c^*(t)$, $k^*(t)$, $i^*(t)$) is competitive then*

$$p(t)c'^*(t) + \frac{d(q(t)k'(t))}{dt}(k^*(t)) = 0.$$

Proof. (i) follows from condition (i) of Definition 2.3, value c^* being the maximum point of expression $\lambda(t)u(c) - p(t)c$.