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NUMERICAL SIMULATION OF THE SUPERPLASTIC DEFORMATION

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Abstract:*Based on recently authors' works, [1], this paper, consists on an extension of volumes generation mathematical model, and are focused on a step by step procedure of volume generation. This work are mainly studied the behavior of finite elements of gas-blow formed parts (such as hemispherical work piece). The aims of the work are to investigate, by common calculus ideas, the final expression of numerical model, that contains, nor redundant unknown, nor inexpressive equations.*

KEYWORDS*:* mathematical model; mechanical properties; metals and alloys; Computational analysis, superplasticity

THE METHOD

In this paper we will study some properties of the application $\mathbf{F}:\mathbf{C}\rightarrow\mathbf{C}'$, $\mathbf{F}(\mathbf{u},\mathbf{v},\mathbf{w})=(\mathbf{x},\mathbf{y},\mathbf{z})$ given by

$$\mathbf{F}:\begin{cases}\mathbf{x}(\mathbf{u},\mathbf{v},\mathbf{w})=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\dfrac{\mathbf{u}}{\mathbf{R}}\\[2mm] \mathbf{y}(\mathbf{u},\mathbf{v},\mathbf{w})=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\dfrac{\mathbf{v}}{\mathbf{R}}\\[2mm] \mathbf{z}(\mathbf{u},\mathbf{v},\mathbf{w})=\left[\mathbf{R}-\Delta\boldsymbol{\lambda}+\Delta\boldsymbol{\sigma}+\left(\mathbf{1}-\dfrac{\Delta\boldsymbol{\sigma}}{\Delta\boldsymbol{\lambda}}\right)\cdot\mathbf{w}\right]\cdot\dfrac{\mathbf{1}}{\mathbf{R}}\cdot\sqrt{\mathbf{R}-\mathbf{u}^2-\mathbf{v}^2}\end{cases}\qquad(1')$$

where $\mathbf{u}^2+\mathbf{v}^2\le\mathbf{R}^2$ and $\mathbf{w}\in[\mathbf{0},\ \Delta\boldsymbol{\lambda}]$, are the parametric equations of volume **C**, [1].

First of all, if exists (u_1,v_1,w_1) and (u_2,v_2,w_2) in C such that $\mathbf{F}(\mathbf{u}_1,\mathbf{v}_1,\mathbf{w}_1)=\mathbf{F}(\mathbf{u}_2,\mathbf{v}_2,\mathbf{w}_2)$ then, it's obviously from definition of F, that $(\mathbf{u}_1=\mathbf{u}_2,\ \mathbf{v}_1=\mathbf{v}_2,\ \mathbf{w}_1=\mathbf{w}_2)$ which verify injectivity of F.

Now we consider $\mathbf{P}(\mathbf{X},\mathbf{Y},\mathbf{Z})$ a point in C', which means that will exist an unique $\mathbf{W}_0\in[\mathbf{0},\Delta\boldsymbol{\lambda}]$ with

$$\frac{\mathbf{X}^2}{(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})^2}+\frac{\mathbf{Y}^2}{(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})^2}+\frac{\mathbf{Z}^2}{\left[\mathbf{R}-\Delta\mathbf{l}+\Delta\mathbf{s}+\left(\mathbf{1}-\frac{\Delta\mathbf{s}}{\Delta\mathbf{l}}\right)\mathbf{W_0}\right]^2}=\mathbf{1} \qquad (1)$$

Choosing $\mathbf{U_0}=\frac{\mathbf{RX}}{(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})}$, $\mathbf{V_0}=\frac{\mathbf{RY}}{(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})}$ we verify that $\mathbf{F}(\mathbf{U_0},\mathbf{V_0},\mathbf{W_0})\equiv\mathbf{P}$.

(i) <u>Conditions on parameters</u> U_0, V_0, W_0

We know that $W_0\in[0,\Delta l]$ and using (1) we have

$$\mathbf{U_0^2}+\mathbf{V_0^2}=\frac{\mathbf{R}^2(\mathbf{X}^2+\mathbf{Y}^2)}{(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})^2}=\mathbf{R}^2\left(\mathbf{1}-\frac{\mathbf{Z}^2}{[\ldots]^2}\right)\le\mathbf{R}^2$$

(ii) <u>Equation which define the application F</u>

For U_0, V_0 *and* W_0 considered bove we obtain by (1') and (1)

$$\mathbf{x}=(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})*\frac{\mathbf{U_0}}{\mathbf{R}}=\mathbf{X},$$

$$\mathbf{y}=(\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0})*\frac{\mathbf{V_0}}{\mathbf{R}}=\mathbf{Y},$$

$$\mathbf{z}=\left[\mathbf{R}-\Delta\mathbf{l}+\Delta\mathbf{s}+\left(\mathbf{1}-\frac{\Delta\mathbf{s}}{\Delta\mathbf{l}}\right)\mathbf{W_0}\right]\cdot\frac{\mathbf{1}}{\mathbf{R}}\sqrt{\mathbf{R}^2-\mathbf{U_0^2}-\mathbf{V_0^2}}=$$

$$\frac{\left[\mathbf{R}-\Delta\mathbf{l}+\Delta\mathbf{s}+\left(\mathbf{1}-\frac{\Delta\mathbf{s}}{\Delta\mathbf{l}}\right)\mathbf{W_0}\right]}{\mathbf{R}-\Delta\mathbf{l}+\mathbf{W_0}}\cdot\sqrt{\mathbf{1}-\mathbf{X}^2-\mathbf{Y}^2}=\mathbf{Z}$$

wich consists on a verification rule.

The application F is a bijection, should be demonstrate.

Considering the cylindrical coordinates we have

$$F:\mathbf{K}\to\mathbf{C}$$

$$(\rho,\vartheta,\boldsymbol{\eta})\to F(\rho,\vartheta,\boldsymbol{\eta})=(\mathbf{u},\mathbf{v},\mathbf{w})$$

given by

$$\begin{cases}\mathbf{u}(\rho,\vartheta,\boldsymbol{\eta})=\rho\,\mathbf{cos}\,\vartheta\\ \mathbf{v}(\rho,\vartheta,\boldsymbol{\eta})=\rho\,\mathbf{sin}\,\vartheta\\ \mathbf{w}(\rho,\vartheta,\boldsymbol{\eta})=\boldsymbol{\eta}\end{cases}$$

for $(\rho,\vartheta,\boldsymbol{\eta})\in\mathbf{K}=[\mathbf{0},\mathbf{R}]\times[\mathbf{0},\mathbf{2}\pi]\times[\mathbf{0},\Delta\mathbf{l}]$

It's not dificult to see that $\mathbf{\Phi}$ is also bijective. We will have finally $\mathbf{\Phi_0} \,\Phi : \mathbf{K} \to \mathbf{C}'$

$(\rho, \boldsymbol{\vartheta}, \mathbf{h}) \mapsto \mathbf{\Phi_0}\,\Phi(\rho, \boldsymbol{\vartheta}, \mathbf{h}) = (\mathbf{x}, \mathbf{y}, \mathbf{z})$ defined by the following equations

$$\begin{cases} \mathbf{x} = (\mathbf{R} - \Delta\mathbf{l} + \mathbf{h}) \cdot \dfrac{\rho \cos \vartheta}{\mathbf{R}} & \rho \in [\mathbf{0}, \mathbf{R}] \\ \mathbf{y} = (\mathbf{R} - \Delta\mathbf{l} + \mathbf{h}) \cdot \dfrac{\rho \sin \vartheta}{\mathbf{R}} & \vartheta \in [\mathbf{0}, \mathbf{2}\pi] \\ \mathbf{z} = \left[\mathbf{R} - \Delta\mathbf{l} + \Delta\mathbf{s} + \left(\mathbf{1} - \dfrac{\Delta\mathbf{l}}{\Delta\mathbf{s}}\right)\mathbf{h}\right] \cdot \dfrac{\mathbf{1}}{\mathbf{R}} \sqrt{\mathbf{R}^2 - \rho^2} & \mathbf{h} \in [\mathbf{0}, \Delta\mathbf{l}] \end{cases} \tag{2}$$

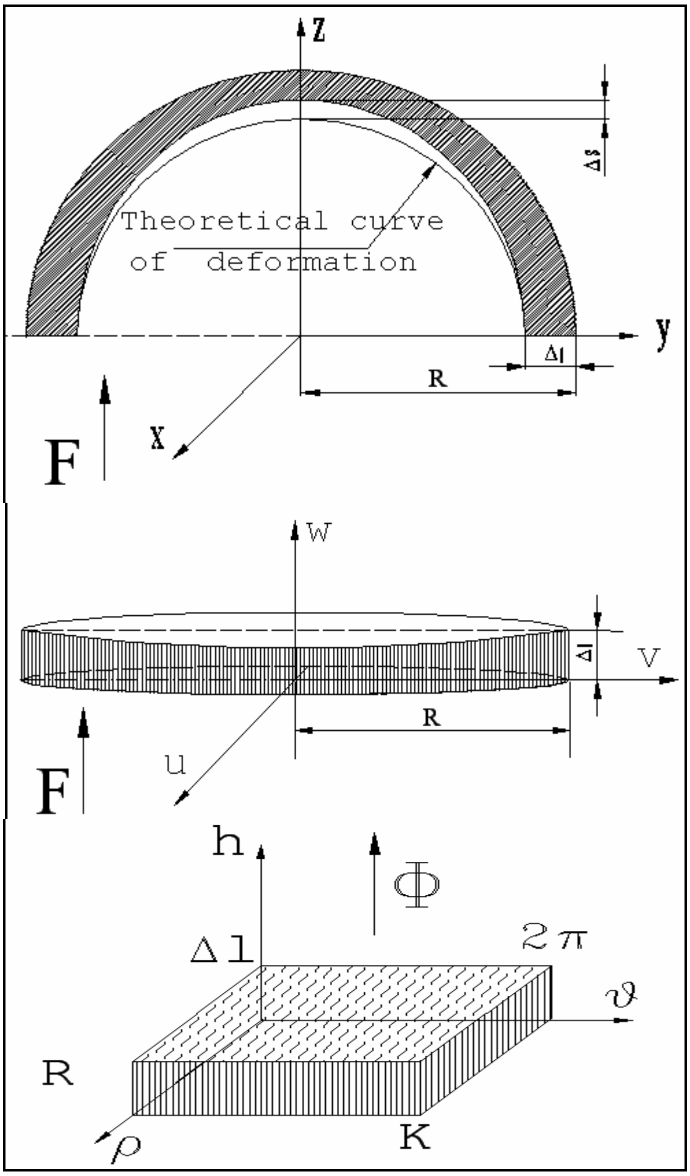


Fig. 1 – Geometrical model

We will study now the description of C' obtained from some particulars decompositions of C and K under the action F and $\Phi_0 \mathbf{\Phi}$, respectively.

Let Δ_1 be a decomposition of C in 3-dimensional closed intervals (cubes ?, parallepipedes ?) which have the faces parallel with the coordinates planes:

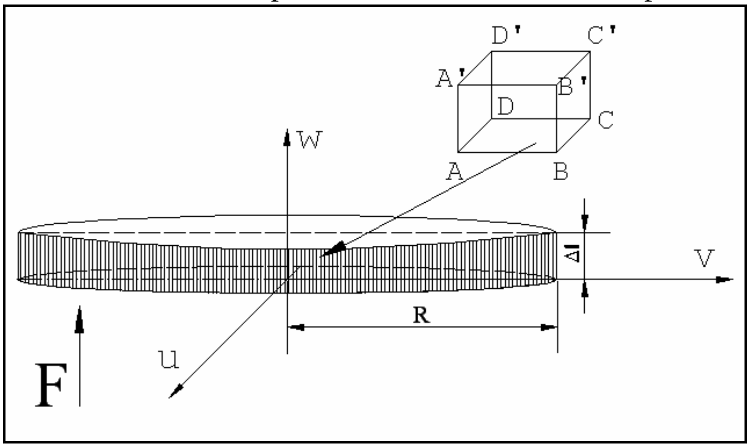


Fig. 2- Decompozition on 3D elements

- The frontal faces $(A'ABB')$ and the back face $(D'DCC')$ are parallels with the coordinates plane u=0, hence we have the equations
  of $(A'ABB'): u = U_0$
  of $(D'DCC'): u = U_1$
- The left face $(A'ADD')$ and the right face $(B'BCC')$ are parallels with the coordinates plane v=0, hence we have the equations
  of $(A'ADD'): v = V_0$
  of $(B'BCC'): v = V_1$
- The bottom ? face $(ABCD)$and the top face $(A'B'C'D')$ are parallels with the coordinates plane w=0, hence we have the equations
  of $(ABCD): w = W_0$
  of $(A'B'C'D'): w = W_1$

From the above considerations we have the coordinates of vertices

$\mathbf{A(U_0, V_0, W_0)}$ $\mathbf{B(U_0, V_1, W_0)}$ $\mathbf{C(U_1, V_1, W_0)}$ $\mathbf{D(U_1, V_0, W_0)}$

$\mathbf{A'(U_0, V_0, W_1)}$ $\mathbf{B'(U_0, V_1, W_1)}$ $\mathbf{C'(U_0, V_0, W_0)}$ $\mathbf{D'(U_1, V_0, W_1)}$

The image of the line $[\mathbf{AA'}]: \begin{cases} \mathbf{u = U_0} \\ \mathbf{v = V_0} \\ \mathbf{w \in [W_0, W_1]} \end{cases}$ under F action:

$$\begin{cases} \mathbf{x} = (\mathbf{R} - \Delta\mathbf{l} + \mathbf{w}) \cdot \dfrac{\mathbf{U_0}}{\mathbf{R}} \\ \mathbf{y} = (\mathbf{R} - \Delta\mathbf{l} + \mathbf{w}) \cdot \dfrac{\mathbf{V_0}}{\mathbf{R}} \\ \mathbf{z} = \left[\mathbf{R} - \Delta\mathbf{l} + \Delta\mathbf{s} + \left(\mathbf{1} - \dfrac{\Delta\mathbf{s}}{\Delta\mathbf{l}}\right)\mathbf{w}\right] \cdot \dfrac{\mathbf{1}}{\mathbf{R}} \sqrt{\mathbf{R}^2 - \mathbf{U}_0^2 - \mathbf{V}_0^2} \end{cases} \tag{3}$$

$$\Leftrightarrow \frac{x-(R-\Delta l)\cdot\frac{U_0}{R}}{\frac{U_0}{R}} = \frac{y-(R-\Delta l)\cdot\frac{V_0}{R}}{\frac{V_0}{R}} = \frac{z-(R-\Delta l+\Delta s)\cdot\frac{1}{R}\sqrt{R^2-U_0^2-V_0^2}}{\left(1-\frac{\Delta s}{\Delta l}\right)\cdot\frac{1}{R}\sqrt{R^2-U_0^2-V_0^2}} \quad (\forall w)$$

which is a line passing throught the point on coordinates:

$$\left((R-\Delta l)\frac{U_0}{R},\ (R-\Delta l)\frac{V_0}{R},\ (R-\Delta l+\Delta s)\frac{\sqrt{R^2-U_0^2-V_0^2}}{R}\right) \tag{4}$$

and have the directions $\left(\frac{U_0}{R}, \frac{V_0}{R}, \left(1-\frac{\Delta s}{\Delta l}\cdot\frac{1}{R}\sqrt{R^2-U_0^2-V_0^2}\right)\right)$.

Similary, for the line $[BB']: \begin{cases} u=U_0 \\ v=V_1 \\ w\in[W_0,W_1] \end{cases}$ the image under F is the line which passing through the point $\left((R-\Delta l)\frac{U_0}{R},\ (R-\Delta l)\frac{V_1}{R},\ (R-\Delta l+\Delta s)\frac{\sqrt{R^2-U_0^2-V_1^2}}{R}\right)$ and have the directions $\left(\frac{U_0}{R}, \frac{V_1}{R}, \left(1-\frac{\Delta s}{\Delta l}\cdot\frac{1}{R}\sqrt{R^2-U_0^2-V_1^2}\right)\right)$.

- For the line $[CC']: \begin{cases} u=U_1 \\ v=V_1 \\ w\in[W_0,W_1] \end{cases}$ the image under F is the line which passing through the point $\left((R-\Delta l)\frac{U_1}{R},\ (R-\Delta l)\frac{V_1}{R},\ (R-\Delta l+\Delta s)\frac{\sqrt{R^2-U_1^2-V_1^2}}{R}\right)$

  and have the directions $\left(\frac{U_1}{R}, \frac{V_1}{R}, \left(1-\frac{\Delta s}{\Delta l}\right)*\frac{1}{R}\sqrt{R^2-U_1^2-V_1^2}\right)$.

- For the line $[DD']: \begin{cases} u=U_1 \\ v=V_0 \\ w\in[W_0,W_1] \end{cases}$ the image is the line

$$\frac{x-(R-\Delta l)\frac{U_1}{R}}{\frac{U_1}{R}} = \frac{y-(R-\Delta l)\frac{V_0}{R}}{\frac{V_0}{R}} = \frac{z-(R-\Delta l+\Delta s)\sqrt{\frac{R^2-U_1^2-V_0^2}{R}}}{\left(1-\frac{\Delta s}{\Delta l}\right)*\frac{1}{R}\sqrt{R^2-U_1^2-V_0^2}} \tag{5}$$

<u>None of the images lines are parallels and have not same point passing through.</u>

The image of the line $[AB]: \begin{cases} u=U_0 \\ w=W_0 \\ v\in[V_0,V_1] \end{cases}$ under F is

$$\begin{cases} x = (R - \Delta\lambda + W_0)\cdot\dfrac{U_0}{R} \\ y = (R - \Delta\lambda + W_0)\cdot\dfrac{v}{R} \\ z = \left[R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)W_0\right]\cdot\dfrac{1}{R}\sqrt{R^2 - U_0^2 - v^2} \end{cases} \tag{6}$$

$$\Leftrightarrow \begin{cases} x = X_0 \ (\text{const}) \\ \dfrac{Ry}{(R - \Delta\lambda + \Delta\sigma)} = v \\ \dfrac{Rz}{\left[R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)W_0\right]^2} = \sqrt{R^2 - U_0^2 - v^2} \end{cases} \tag{7}$$

$$\Leftrightarrow \begin{cases} x = X_0 \ (\text{const}) \\ \dfrac{R^2y^2}{(R - \Delta\lambda + W_0)^2} + \dfrac{R^2z^2}{\left[R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)W_0\right]^2} = R^2 - U_0^2 \end{cases} \tag{8}$$

$$\Leftrightarrow \begin{cases} x = X_0 \\ \dfrac{y^2}{\dfrac{(R^2 - U_0^2)(R - \Delta\lambda + W_0)^2}{R^2}} + \dfrac{z^2}{\dfrac{\left[R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)W_0\right]^2(R^2 - U_0^2)}{R^2}} = 1 \end{cases} \tag{9}$$

which is an elipsis in plane $x = (R - \Delta\lambda + W_0)\dfrac{U_0}{R}$ parallel with the coordinate plane x=0.

The image of the line $[A'B']: \begin{cases} u = U_0 \\ w = W_1 \\ v \in [V_0, V_1] \end{cases}$ under F is an ellipsis in the plane $x = (R - \Delta\lambda + W_1)\dfrac{U_0}{R}$ (which is parallel with the previous one):

$$\frac{y^2}{\dfrac{(R^2 - U_0^2)(R - \Delta\lambda + W_1)^2}{R^2}} + \frac{z^2}{\dfrac{\left[R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)W_1\right]^2(R^2 - U_0^2)}{R^2}} = 1 \tag{10}$$

- For the line $[CD]:\begin{cases} u = U_1 \\ w = W_0 \\ v \in [V_0, V_1] \end{cases}$ the image under F is an elipse in the plane

$$x = (R - \Delta l + W_0)\frac{U_1}{R};$$

$$\frac{y^2}{\dfrac{(R^2 - U_1^2)(R - \Delta l + W_0)^2}{R^2}} + \frac{z^2}{\dfrac{\left[R - \Delta l + \Delta s + \left(1 - \dfrac{\Delta s}{\Delta l}\right)W_0\right]^2 (R^2 - U_1^2)}{R^2}} = 1 \qquad (11)$$

and the image of the line $[C'D']:\begin{cases} u = U_1 \\ w = W_1 \\ v \in [V_0, V_1] \end{cases}$ under F is an ellipsis in the plane

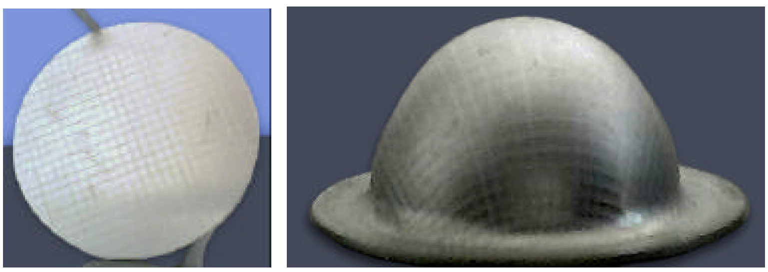
$$x = (R - \Delta l + W_1)\frac{U_1}{R};$$

$$\frac{y^2}{\dfrac{(R^2 - U_1^2)(R - \Delta l + W_1)^2}{R^2}} + \frac{z^2}{\dfrac{\left[R - \Delta l + \Delta s + \left(1 - \dfrac{\Delta s}{\Delta l}\right)W_1\right]^2 (R^2 - U_1^2)}{R^2}} = 1 \qquad (12)$$

The image under F of the lines which are parallels with O_v coordinate axe are four ellipsises each of them in some different plane parallel with coordinate plane x=0.

EXPERIMENTS AND RESULTS

- Two samples was prepared especially, by drawing a rectangular grid spaced at one millimeter distance;



a)-the sample *b)-deformed part*

Fig. 3 – A sample and the formed part

- There are observed, on *Fig. 1*, elliptical curves of deformed sample;

Deformed tested pieces were sliced by an electrical method (no local melting zone admitted) for determining dimensions on transversal section. In the next images are emphasized results of measurements: pressure variation curve, strain variation and section dimensions (i.e. thickness). The analysis by MATLAB Programming Environment offered an image of material behavior during superplastic deep gasostatic forming. The interpolation and fitting procedure with an interactive numerical method were used here to study the cross-section varied scene. All observation are made on pole zone and on corner radius zone, i.e. the most exposed zones of the test pieces. Following images are experiments results work.

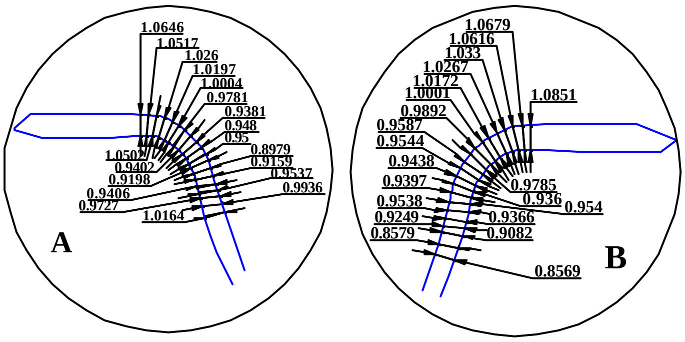


Fig. 4 – Dispersion of measurements points on the corner connected radius

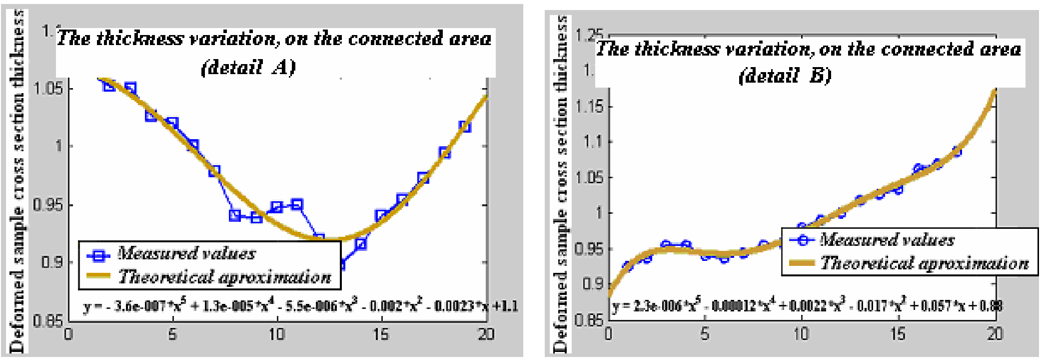


Fig. 5 – Actual and theoretical approximation of thickness variation, on connected area

CONCLUSION

Finaly, one can conclude:

- At the corner radius was observed variation of very low level, because this is the region unformed on classical laws of deformation;
- The grids are clearly deformed on the entire region. The original squares of the grid are finally rounded squares but equals deformed on the same meridian of the hemispherical shell and at the same ratio aspect on the opposite region.

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