ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering

APROXIMATION OF FORMING LIMITS AND EXPERIMENTAL OBSERVATION ON SUPERPLASTIC DEFORMATION

G. Grebenisan[1], S. Muresan[1], Al. Rus[1]

[1]University of Oradea, Romania
grebe@uoradea.ro, smuresan@uoradea.ro, alrus@uoradea.ro

Abstract: *Even if certain mathematical models, which are developed for describing the atomic mechanism of superplastic deformation are validated by experiments, a mathematical model necessary to be validate, in case of gasostatic forming will be the aims of area researchers. Thereto, it is obvious necessary, since necking of superplastic gas-blow formed parts were observed, by action of uniform field of forces, considering a controlled process: such as a predicted gas pressure cycles, required to ensure that the maximum strain rate over the work piece approximately equals the pre-defined target deformation rate. In this paper, studies are carried out to investigate the simplest way to develop a behavior law, however as long as this process is a mechanical one, it cannot be neglected at all, though, with minimum of approximations, a mathematical model, by geometry changes expression, will be done.*

KEYWORDS*:* mathematical model; mechanical properties; metals and alloys; Computational analysis, superplasticity

INTRODUCTION

Let C be a full cylindrical body which have the radius R and the thickness ?l in the space with coordinates (Ouvw). An external "action" F transform this body into a hemispherical type body C' situate in the space with coordinates (Oxyz). We know that the external surface of C' is a hemisphere with radius and the thickness of C', which is ?l to the base, decrease at the value ?l-?s to the top (see Figure 1).

THE METHOD

In this paper we want to find a mathematical expression (or expressions) for the "action" F. The hypotheses are

- the parametric equations of C:

$$\begin{cases} \mathbf{u}^2 + \mathbf{v}^2 \le \mathbf{R}^2 \\ \mathbf{w} \in [\mathbf{a}, \Delta\mathbf{l}] \end{cases} \qquad (1)$$

and the parametric equation of the external surface of C':

$$\frac{x^2}{R^2}+\frac{y^2}{R^2}+\frac{z^2}{R^2}=1 \qquad (2)$$

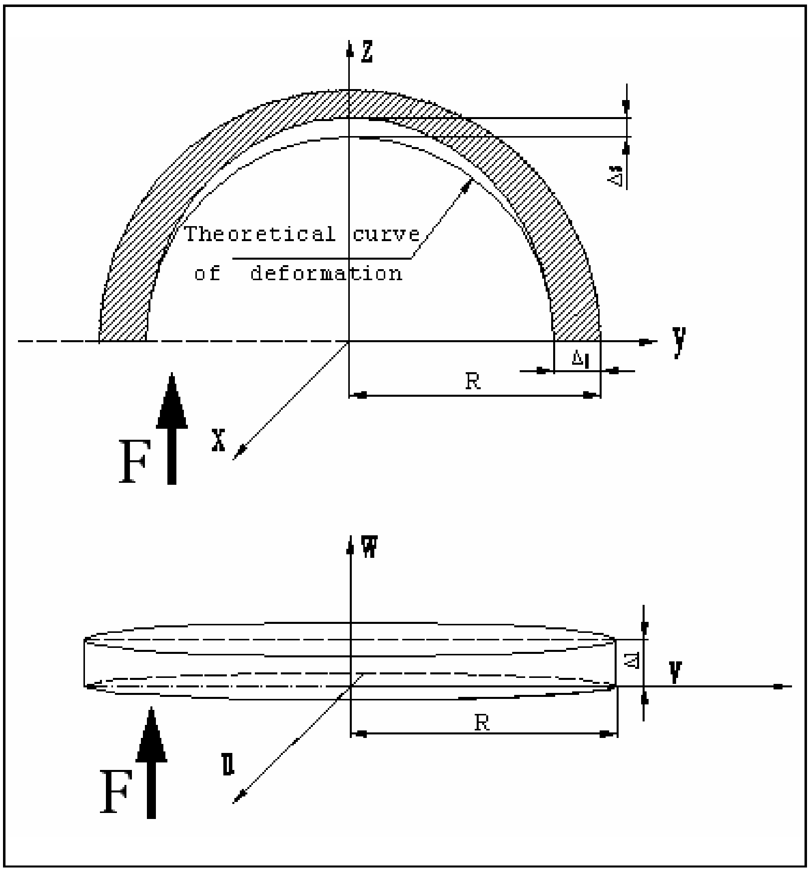

Fig. 1 – Geometrical model, before and after deformation

We suppose that the internal surface of the body C' is ellipsoid:

$$\frac{x^2}{(R-\Delta l)^2}+\frac{y^2}{(R-\Delta l)^2}+\frac{z^2}{(R-\Delta l+\Delta s)^2}=1 \qquad (3)$$

We want to find a continuous way to pass from the internal surface S_{int}, defined by, (3) to the external surface S_{ext}, defined by (2).

Since F acting especially in the vertical direction, we will give for F a mathematical equation which depend by the variable w from [0,?l]. Hence we consider (4):

$$\frac{x^2}{(R-\Delta l+w)^2}+\frac{y^2}{(R-\Delta l+w)^2}+\frac{z^2}{\left[R-\Delta l+\Delta s+\left(1-\frac{\Delta s}{\Delta l}\right)\cdot w\right]^2}=1 \qquad (4)$$

We verify equation (4) taking into account that the image under F of the bottom surface of C (which have the equation w=0) must be the internal surface , S_{int}, of C' (it's easy to see that if we take w=0 in (4) we obtain equation (3)) and, also, the image under F of the above surface of C (which have the equation w=?l) must be the external surface, S_{ext},, of C' (just like in the previous case, taking w=?l in (4), we obtain equation (2)). But to find the image under F of any point of C (not just for the point of the bottom and above surface) we must give F as a function which will be the parametrical equation of (4).

$$\mathbf{F}:\mathbf{C}\to\mathbf{C}',\qquad (\mathbf{u},\mathbf{v},\mathbf{w})\to\mathbf{F}(\mathbf{u},\mathbf{v},\mathbf{w})=\left(\mathbf{x}(\mathbf{u},\mathbf{v},\mathbf{w}),\ \mathbf{y}(\mathbf{u},\mathbf{v},\mathbf{w}),\ \mathbf{z}(\mathbf{u},\mathbf{v},\mathbf{w})\right) \tag{5}$$

FIRST TRY

Starting from (4) we consider the following parametrical equations

$$\begin{cases}\mathbf{x}=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\xi\\ \mathbf{y}=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\eta\\ \mathbf{z}=\left[\mathbf{R}-\Delta\boldsymbol{\lambda}+\Delta\boldsymbol{\sigma}+\left(\mathbf{1}-\dfrac{\Delta\boldsymbol{\sigma}}{\Delta\boldsymbol{\lambda}}\right)\cdot\mathbf{w}\right]\cdot\sqrt{\mathbf{1}-\xi^2-\eta^2}\end{cases}\quad \text{where } \mathbf{w}\in[\mathbf{0},\ \Delta\boldsymbol{\lambda}] \text{ and } \xi^2+\eta^2\le\mathbf{1} \tag{6}$$

Can we suppose ?=u and ?=v?

If we compare the hypothesis over that the variables u and v: $\mathbf{u}^2+\mathbf{v}^2\le\mathbf{R}^2$ with the condition on the variables ξ and η: $\xi^2+\eta^2\le\mathbf{1}$ we obtain the affirmative answer at the question only if $\mathbf{R}\le\mathbf{1}$. More than that, the case $\mathbf{R}<\mathbf{1}$ is not working since then z can't take the value 0, hence the body C' "floating" at the distance $\mathbf{R}\cdot\sqrt{\mathbf{1}-\mathbf{R}^2}$ over the plane (Ouv) which mean that F is not well defined. Hence, as a conclusion of this try, if **R=1** we have $\mathbf{F}:\mathbf{C}\to\mathbf{C}'$ given by

$$\mathbf{F}:\begin{cases}\mathbf{x}(\mathbf{u},\mathbf{v},\mathbf{w})=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\mathbf{u}\\ \mathbf{y}(\mathbf{u},\mathbf{v},\mathbf{w})=(\mathbf{R}-\Delta\boldsymbol{\lambda}+\mathbf{w})\cdot\mathbf{v}\\ \mathbf{z}(\mathbf{u},\mathbf{v},\mathbf{w})=\left[\mathbf{R}-\Delta\boldsymbol{\lambda}+\Delta\boldsymbol{\sigma}+\left(\mathbf{1}-\dfrac{\Delta\boldsymbol{\sigma}}{\Delta\boldsymbol{\lambda}}\right)\cdot\mathbf{w}\right]\cdot\sqrt{\mathbf{1}-\mathbf{u}^2-\mathbf{v}^2}\end{cases}$$

where $\mathbf{u}^2+\mathbf{v}^2\le\mathbf{1}$.

SECOND TRY

We will consider again the parametrical equation (6) and we study if is possible to replace $\xi=\dfrac{\mathbf{u}}{\mathbf{R}}$ and $\eta=\dfrac{\mathbf{v}}{\mathbf{R}}$. It's obvious that in this case we have the equivalence between the conditions $\xi^2+\eta^2\le\mathbf{1}\Leftrightarrow\mathbf{u}^2+\mathbf{v}^2\le\mathbf{R}^2$

Hence, without restrictions, we find a better definition of the function $\mathbf{F}:\mathbf{C}\to\mathbf{C}'$ by the following equations:

$$\mathbf{F}:\begin{cases} \mathbf{x}(\mathbf{u},\mathbf{v},\mathbf{w}) = (\mathbf{R}-\Delta\lambda+\mathbf{w})\cdot\dfrac{\mathbf{u}}{\mathbf{R}} \\ \mathbf{y}(\mathbf{u},\mathbf{v},\mathbf{w}) = (\mathbf{R}-\Delta\lambda+\mathbf{w})\cdot\dfrac{\mathbf{v}}{\mathbf{R}} \\ \mathbf{z}(\mathbf{u},\mathbf{v},\mathbf{w}) = \left[\mathbf{R}-\Delta\lambda+\Delta\sigma+\left(\mathbf{1}-\dfrac{\Delta\sigma}{\Delta\lambda}\right)\cdot\mathbf{w}\right]\cdot\dfrac{\mathbf{1}}{\mathbf{R}}\cdot\sqrt{\mathbf{R}-\mathbf{u}^2-\mathbf{v}^2} \end{cases} \tag{7}$$

where $\mathbf{u}^2+\mathbf{v}^2 \le \mathbf{R}^2$ and $\mathbf{w}\in[\mathbf{0},\ \Delta\lambda]$.

THIRD TRY

Multiplying (4) with expression $(\mathbf{R}-\Delta\lambda+\mathbf{w})^2$ we find:

$$\mathbf{x}^2+\mathbf{y}^2+\frac{\mathbf{z}^2}{\dfrac{\left[\mathbf{R}-\Delta\lambda+\Delta\sigma+\left(\mathbf{1}-\dfrac{\Delta\sigma}{\Delta\lambda}\right)\cdot\mathbf{w}\right]^2}{(\mathbf{R}-\Delta\lambda+\mathbf{w})^2}} = (\mathbf{R}-\Delta\lambda+\mathbf{w})^2 \tag{8}$$

which have the following parametrical equations

$$\begin{cases} \mathbf{x} = \xi \\ \mathbf{y} = \eta \\ \mathbf{z} = \dfrac{\mathbf{R}-\Delta\lambda+\Delta\sigma+\left(\mathbf{1}-\dfrac{\Delta\sigma}{\Delta\lambda}\right)\cdot\mathbf{w}}{\mathbf{R}-\Delta\lambda+\mathbf{w}}\cdot\sqrt{(\mathbf{R}-\Delta\lambda+\mathbf{w})^2-\xi^2-\eta^2} \end{cases} \tag{9}$$

where $\xi^2+\eta^2 \le (\mathbf{R}-\Delta\lambda+\mathbf{w})^2$ and $\mathbf{w}\in[\mathbf{0},\ \Delta\lambda]$.

Can we suppose ?=u and ?=v?

Like at the first try, we compare the conditions on the variables u and v: $\mathbf{u}^2+\mathbf{v}^2 \le \mathbf{R}^2$ and the condition on the variables ξ and η : $\xi^2+\eta^2 \le (\mathbf{R}-\Delta\lambda+\mathbf{w})^2$ for $\mathbf{w}\in[\mathbf{0},\ \Delta\lambda]$.

The answer it appear to be affirmative but, again, F is not well defined, since, in this case, F is defined only on a subset $\mathbf{C_1} \subset \mathbf{C}$, where $\mathbf{C_1} = \{(\mathbf{u},\mathbf{v},\mathbf{w}) \in \mathbf{C};\mathbf{u}^2+\mathbf{v}^2 \le (\mathbf{R}-\Delta\lambda+\mathbf{w})^2\}$. Hence we have $\mathbf{F_1}:\mathbf{C_1} \to \mathbf{C}'$:

$$F_1 : \begin{cases} x(u,v,w) = u \\ y(u,v,w) = v \\ z(u,v,w) = \left[\dfrac{R - \Delta\lambda + \Delta\sigma + \left(1 - \dfrac{\Delta\sigma}{\Delta\lambda}\right)\cdot w}{R - \Delta\lambda + w} \right] \cdot \sqrt{(R - \Delta\lambda + w)^2 - u^2 - v^2} \end{cases} \tag{10}$$

It's obvious that the restriction $F|_{C_1}$ must be F_1. What happens with the point of $C \setminus C_1$? We have at least two possible answers:

1) We define $F_2 : C \setminus C_1 \to C'$, given by:

$$F_2 : \begin{cases} x = u \\ y = v \\ z = 0 \end{cases} \tag{11}$$

and then

$$F : C \to C', \; F(u,v,w) = \begin{cases} F_1(u,v,w), & \text{if} \quad u^2 + v^2 \le (R - \Delta\lambda + w)^2 \\ F_2(u,v,w), & \text{if} \quad (R - \Delta\lambda + w)^2 < u^2 + v^2 \le R^2 \end{cases}$$

2) We can consider at the beginning that C is a full body (see Figure 2):

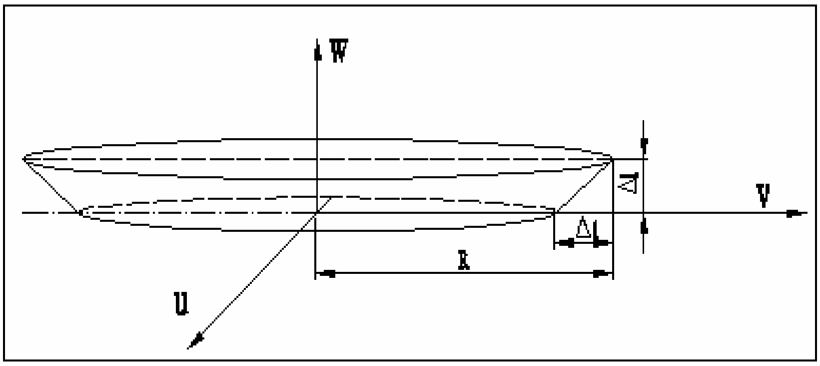

Fig. 2 The geometrical model shape modified

defined by equations

$$\begin{cases} u^2 + v^2 \le (R - \Delta\lambda + w)^2 \\ w \in [0, \Delta\lambda] \end{cases} \tag{12}$$

and not a full cylindrical body as we assumed before. In this case $F = F_1$.

FOURTH TRY

We consider the equations (4) given by (9). Here we take $\xi = \left(\frac{\mathbf{R} - \Delta\lambda + \omega}{\mathbf{R}}\right)\cdot \mathbf{u}$ and $\eta = \left(\frac{\mathbf{R} - \Delta\lambda + \omega}{\mathbf{R}}\right)\cdot \mathbf{v}$. Thereby, the equivalence between the conditions on variables u, v and ξ and η, respectively, is easy to be observed, so: $\xi^2 + \eta^2 \le (\mathbf{R} - \Delta\lambda + \omega)^2 \Leftrightarrow \mathbf{u}^2 + \mathbf{v}^2 \le \mathbf{R}^2$.

Hence we have $\mathbf{F} : \mathbf{C} \rightarrow \mathbf{C}'$, given by:

$$\mathbf{F}: \begin{cases} \mathbf{x}(\mathbf{u},\mathbf{v},\omega) = (\mathbf{R} - \Delta\lambda + \omega)\cdot\frac{\mathbf{u}}{\mathbf{R}} \\ \mathbf{y}(\mathbf{u},\mathbf{v},\omega) = (\mathbf{R} - \Delta\lambda + \omega)\cdot\frac{\mathbf{v}}{\mathbf{R}} \\ \mathbf{z} = \left[\frac{\mathbf{R} - \Delta\lambda + \Delta\sigma + \left(1 - \frac{\Delta\sigma}{\Delta\lambda}\right)\cdot\omega}{\mathbf{R} - \Delta\lambda + \omega}\right]\cdot\sqrt{(\mathbf{R} - \Delta\lambda + \omega)^2 - \left[\frac{(\mathbf{R} - \Delta\lambda + \omega)^2}{\mathbf{R}^2}\right]\cdot\mathbf{u}^2 - \left[\frac{(\mathbf{R} - \Delta\lambda + \omega)^2}{\mathbf{R}^2}\right]\cdot\mathbf{v}^2} \end{cases}$$

which is identical with F defined by equation (7).

EXPERIMENTS AND RESULTS

- There was realized experiments on a number of 11 samples of FORMALL alloy, with 51[mm], in diameter, thickness 1,2[mm], at forming temperature T_{def}=510[^{o}C] and strain rate value $\dot{\varepsilon} = 0.1x10^{-3}[s^{-1}]$;
- Using the Coordinates Measuring Machine of type MC 1200, with radius of the tool of 0.2332 [mm], was realized measurements and the results was stocked into a Delphi data file. Using this data the actual profile was realized also helped on AutoCAD facilities. On each section was realized graphical representation of the variation of transversal section;
- Deformed test pieces were sliced by an electrical method (no local melting zone admitted) for determining dimensions on transversal section. In the next images are emphasized results of measurements: pressure variation curve, strain variation and section dimensions (i.e. thickness). The analysis by MATLAB Programming Environment offered an image of material behavior during superplastic deep gasostatic forming. The interpolation and fitting procedure were interactive numerical methods used here to study the cross-section varied scene. All observation are made on pole zone and on corner radius zone, i.e. the most exposed zones of the test pieces. Following images are experiments results work.

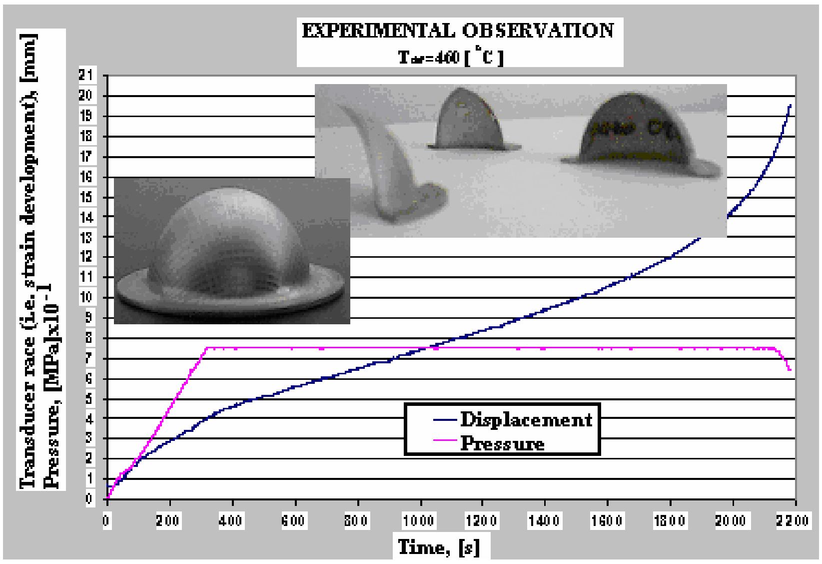

Fig. 4 – The pressure and strain variation, against the time ~starting thickness of 1.1 mm~

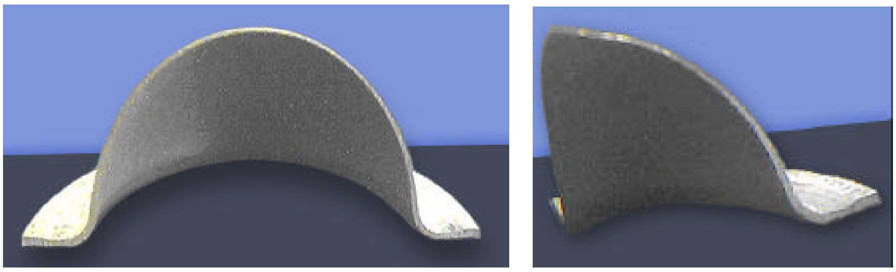

Fig. 5 – Deformed hemispherical shell

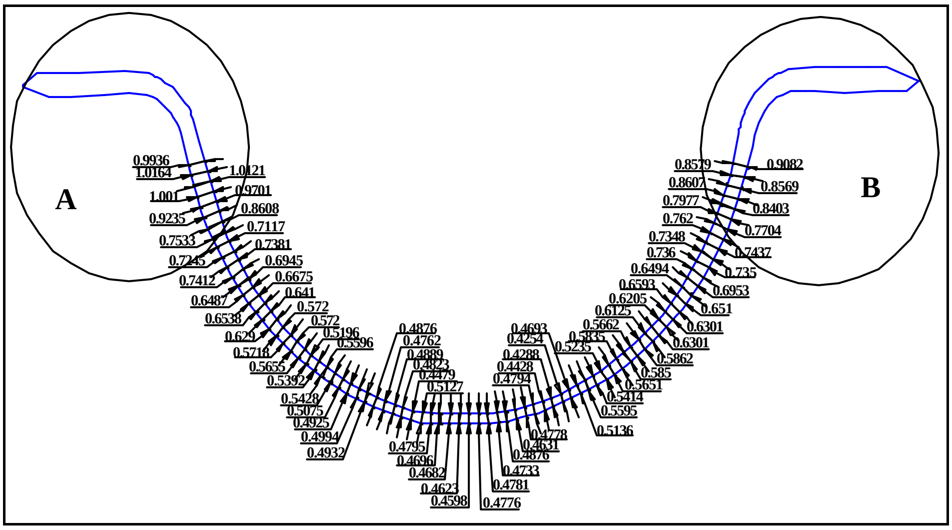

Fig. 6 – Dispersion of measurements points on axial section

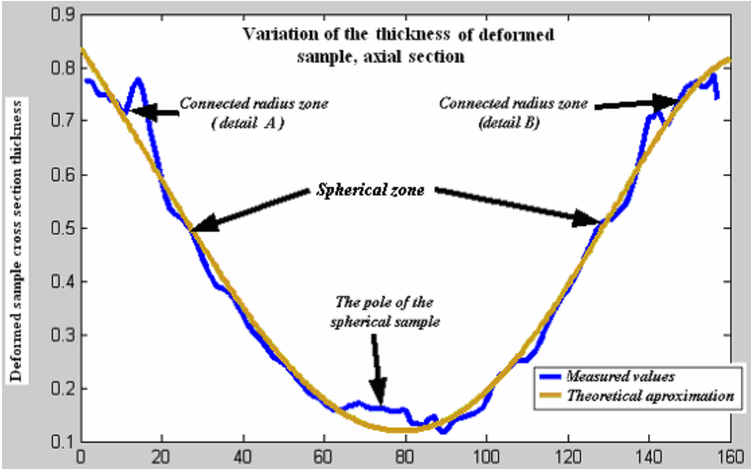

Fig. 7 – Actual and theoretical approximation of thickness variation

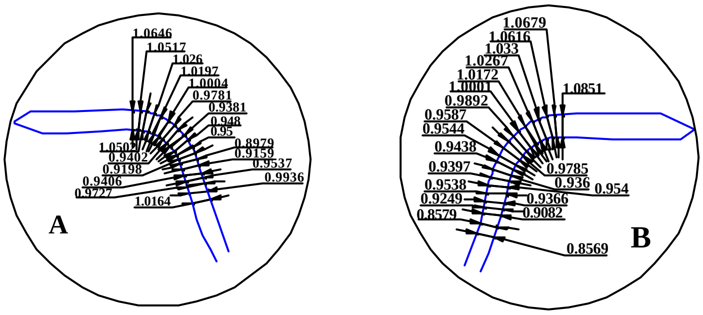

Fig. 8 – Dispersion of measurements points on the corner radius

REMARKS

As we can see it's not easy to give a mathematical model with general validity. Thereto, since the variables of mathematical models can generate discontinuities or variations when we implement this models in computational analysis it's possible to obtain unconvincing results. The models which are developed here will be include in software program which simulate the theoretical and real superplastic behavior of the materials, under gasostatic forming conditions.

DISCUSSION

The gasostatic superplastic deep forming process, such as a high complexity one, is governed by laws that are uncommon and no easy for modeling. Because the superplastic materials, commonly, at normal conditions, does not support plastic deformation (e.g. titanium alloys), any mathematical formulation have to constraint, to limits their viability.

ACKNOWLEDGEMENTS

This work is a section of a complete mathematical and computational analysis of the superplastic phenomenon, applied on gasostatic forming process, considering a simple way for geometric and evolution of shapes changes.

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