EIGEN MODE FOR MICROWAVE TRANSMISSION

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Abstract: The electromagnetic properties of microwave transmission lines can be described using Maxwell's equations in the frequency domain. Applying a finite-volume scheme this results in an algebraic eigenmode problem. In this paper, an improved numerical computation of the eigenmodes is presented.

1. INTRODUCTION

The design of microwave integrated circuits and packages requires eficient CAD tools for a three-dimensional electromagnetic simulation because the coupling effects become critical with growing packaging density and increasing frequency. Usually, a circuit is described in terms of its scattering matrix [1], [2], [3], [4], [5]. In this approach, the structure under investigation is embedded in a set of longitudinally homogeneous transmission lines (see Figure 1), which are assumed to be infinitely long. As a first step one has to calculate the field distribution in these transmission lines.

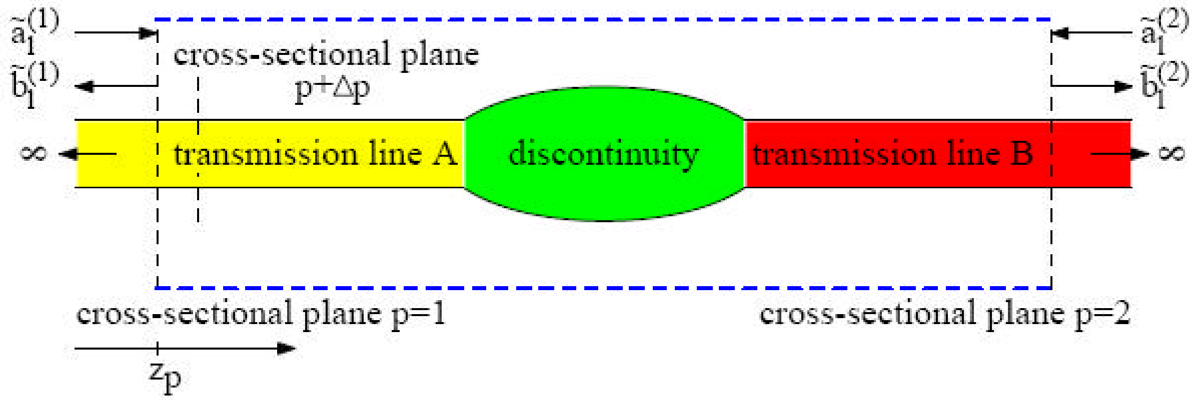

Figure 1: Structure under investigation.

Because the transmission lines are longitudinally homogeneous, the electromagnetic field can be expanded into a sum of wave modes with each of them varying exponentially in the longitudinal direction. The corresponding ansatz results in an eigenvalue problem. In a second step, the eigenfunctions determine the boundary values for the calculation of the fields in the three-dimensional structure. In order to speed up the design process, the calculations have to be performed as effectively as possible. In this paper, we treat the numerical solution of the eigenmode problem. Applying the finite-volume method to discretize the Maxwellian equations results in a standard eigenvalue problem with a large sparse matrix. We describe a suitable method to calculate the set of interesting eigenvalues and eigenvectors.

2. MATRIX REPRESENTATION OF MAXWELL'S EQUATIONS

In the frequency domain, we consider fields that vary with time according to the complex exponential function $e^{j\omega t}$. Thus, the integral form of the Maxwellian equations reads:

$$\begin{cases} \oint\limits_{\partial\Omega} \frac{1}{\tilde{\mu}\mu_0} \vec{B} \cdot d\vec{s} = \int\limits_{\Omega} \left(j\omega\tilde{\varepsilon}\varepsilon_0 \vec{E} \right) \cdot d\vec{\Omega} \\ \oint\limits_{\partial\Omega} \vec{E} \cdot d\vec{s} = \int\limits_{\Omega} \left(-j\omega\vec{B} \right) \cdot d\vec{\Omega} \\ \oint\limits_{\cup\Omega} \left(\tilde{\varepsilon}\varepsilon_0 \vec{E} \right) \cdot d\vec{\Omega} = 0 \\ \oint\limits_{\cup\Omega} \vec{B} \cdot d\vec{\Omega} = 0 \end{cases} \tag{1}$$

taking into account the constitutive relations:

$$\vec{B} = \mu\vec{H},\ \vec{D} = \varepsilon\vec{E}\text{, with } \underline{\varepsilon} = \varepsilon + \frac{k}{j\omega},\ \mu = \tilde{\mu}\mu_0,\ \underline{\varepsilon} = \tilde{\varepsilon}\varepsilon_0 \tag{2}$$

The field vectors E, H, D, B, (electric and magnetic field intensity, electric and magnetic ux density, respectively) are complex functions of the spatial coordinates only. ω is the circular frequency and $j^2 = -1$. The permeability μ, the permittivity ε, and the conductivity k are assumed to be scalar functions of the spatial coordinates. $\underline{\varepsilon}$ is the complex permittivity.

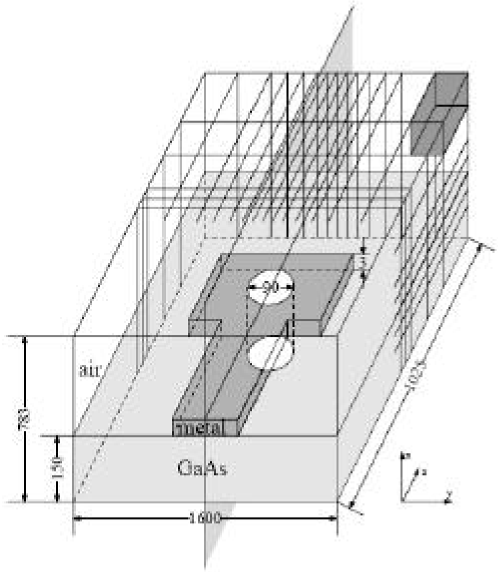

Figure 2: Via hole.

Introducing the finite-volume scheme, the region is divided into rectangular parallelepipeds (see Figure 2) using a three-dimensional nonequidistant Cartesian grid. The edges of the cells are parallel to the coordinate axes. The grid nodes (i; j; k), the lower left front corners of the parallelepipeds, are numbered by

$$l = (k-1)n_x n_y + (j-1)n_x + i, \quad i = 1(1)n_x, \quad j = 1(1)n_y, \quad k = 1(1)n_z \tag{3}$$

n_x, n_y, and n_z denote the numbers of rectangular parallelepipeds in the x-, y- and z-direction, respectively. The field vectors are expressed as

$$\vec{M} = M_x\vec{i}_x + M_y\vec{i}_y + M_z\vec{i}_z, \quad \vec{M} \in \{\vec{E}, \vec{D}, \vec{H}, \vec{B}\} \tag{4}$$

The components E_x, E_y, and E_z of the electric field E are located in the centers of the edges of the elementary cells. The components B_x, B_y, and B_z, on the other hand, are normal to the face centers [6], [7]. Thus, the electric field components form a primary grid, and the magnetic ux density components a dual grid (see Figure 3). The Maxwellian equations are now applied to each cell. We use the lowest-order integration formulae:

$$\oint\limits_{\partial\Omega} \vec{f} \cdot d\vec{s} \approx \sum (\pm f_i \cdot s_i), \quad \int\limits_{\Omega} \vec{f} \cdot d\vec{\Omega} \approx f\Omega \tag{5}$$

in order to approximate the left-hand and the right-hand sides of the first and the second Maxwellian equation. The closed path $\partial\Omega$ of the integration in the primary grid consists of 4 straight lines of length si and is the path around the periphery of a unit cell face in the grid. f_i denotes the function value in the center of the side si. In the dual grid the closed path $\partial\Omega$ of the integration consists of 8 straight lines, and fi denotes the function value in the center of the corresponding face in the primary grid (see Figure 3). i_S the area of any cell face. f denotes the function value in the center of this face. Let be:

$$\vec{e} = \begin{pmatrix} \vec{e}_x \\ \vec{e}_y \\ \vec{e}_z \end{pmatrix}, \quad \begin{matrix} \vec{e}_x = \left(e_{x1}, e_{x2}, ..., e_{xn_{xyz}}\right)^T, & e_{xl} = E_{x_{i,j,k}} \\ \vec{e}_y = \left(e_{y1}, e_{y2}, ..., e_{yn_{xyz}}\right)^T, & e_{yl} = E_{y_{i,j,k}} \\ \vec{e}_z = \left(e_{z1}, e_{z2}, ..., e_{zn_{xyz}}\right)^T, & e_{zl} = E_{z_{i,j,k}} \end{matrix} \tag{6}$$

$$\vec{b} = \begin{pmatrix} \vec{b}_x \\ \vec{b}_y \\ \vec{b}_z \end{pmatrix}, \quad \begin{matrix} \vec{b}_x = \left(b_{x1}, b_{x2}, ..., b_{xn_{xyz}}\right)^T, & b_{xl} = B_{x_{i,j,k}} \\ \vec{b}_y = \left(b_{y1}, b_{y2}, ..., b_{yn_{xyz}}\right)^T, & b_{yl} = B_{y_{i,j,k}} \\ \vec{b}_z = \left(b_{z1}, b_{z2}, ..., b_{zn_{xyz}}\right)^T, & b_{zl} = B_{z_{i,j,k}} \end{matrix} \tag{7}$$

$$l = (k-1)n_{xy} + (j-1)n_z + i, \quad n_{xy} = n_x n_y, \quad n_{xyz} = n_x n_y n_z \tag{8}$$

the vectors containing the electric and the magnetic field of the elementary cells, respectively. Let be D_S and D_A diagonal matrices and A a matrix which represents the operator of the line integral in the second Maxwellian equation, using the primary grid [4] the following matrix representation of the second Maxwellian

$$\oint_{\partial\Omega} \vec{E} \cdot d\vec{s} = \int_{\Omega} \left(-j\omega\vec{B}\right) \cdot d\vec{\Omega} \quad \Rightarrow \quad AD_x\vec{e} = -j\omega D_A\vec{b} \tag{9}$$

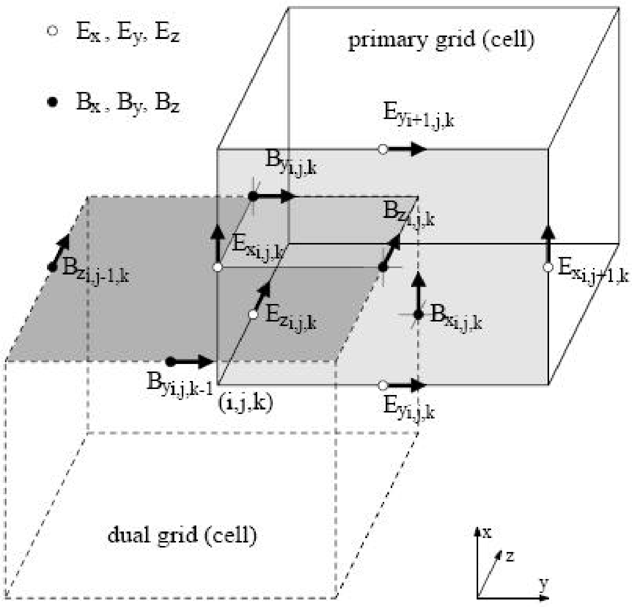

Figure 3: Primary and dual grid.

The diagonal matrices D_S and D_A contain the information on all dimensions for the specified structure and the corresponding mesh. Let be $D_{A\varepsilon}$ and $D_{s/\mu}$ diagonal matrices and A_T the transposed matrix of A, using the dual grid the following matrix representation of the modified first Maxwellian equation results:

$$\oint_{\partial\Omega} \frac{\vec{B}}{\tilde{\mu}} \cdot d\vec{s} = \int_{\Omega} \left(j\omega\tilde{\varepsilon}\varepsilon_0\mu_0\vec{E}\right) \cdot d\vec{\Omega} \quad \Rightarrow \quad A^T D_{s/\mu}\vec{b} = j\omega\varepsilon_0\mu_0 D_{A\varepsilon}\vec{e} \tag{10}$$

The diagonal matrices $D_{A\varepsilon}$ and $D_{s/\mu}$ contain the information on dimension and material for the structure and the corresponding mesh. Let be B the matrix that represents the operator of the surface integral we get the matrix representation of the electric-field divergence equation:

$$\oint_{\cup\Omega} \tilde{\varepsilon}\varepsilon_0 \vec{E}\cdot d\vec{\Omega} = 0 \quad \Rightarrow \quad BD_{A\tilde{\varepsilon}}\vec{e} = 0 \qquad (11)$$

The matrices A and B are sparse and consist only of the elements -1, 0 and 1. Combining we get the matrix representation of the system of linear algebraic equations:

$$\left(A^T D_{s/\tilde{\mu}} D_A^{-1} A D_s - k_0^2 D_{A\tilde{\varepsilon}}\right)\vec{e} = 0, \quad k_0 = \omega\sqrt{\varepsilon_0\mu_0} \qquad (12)$$

3. THE EIGENVALUE PROBLEM

In the following, we consider a longitudinally homogeneous transmission line, which means that ε and μ are functions of transverse position but are independent of the longitudinal direction z. With these assumptions, any field can be expanded into a sum of so-called modal fields:

$$\vec{E}_{k_x}(x,y,z) = \vec{E}_{k_x}(x,y)e^{-jk_x z} \qquad (13)$$

which vary exponentially in the longitudinal direction.
The $E_{kz}(x,y)$ are eigenfunctions of a partial differential equation of second order and the propagation constants kz are related to the eigenvalues. To find the corresponding solutions of the discretized Maxwellian equations we consider the field components in three consecutive elementary cells (see Figure 4).

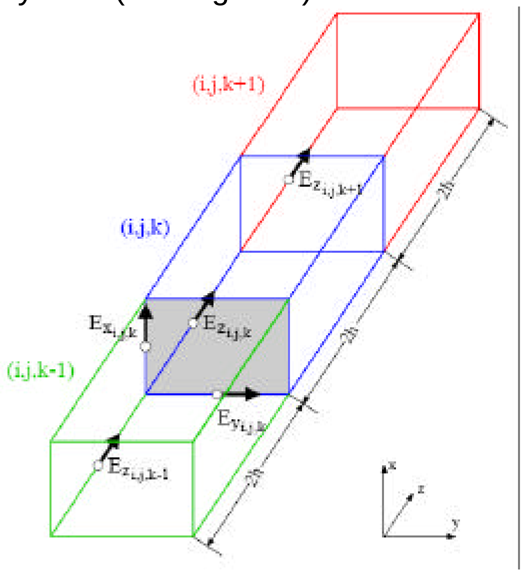

Figure 4: Reduction of the dimension.

Each cell is of lenght 2h in the z-direction where h must be chosen small enough so that $2h|k_z| << 1$. So we get an eigenvalue problem for the transverse electric field on the transmission line region. $E_{xi;j;k}$, $E_{yi;j;k}$ = const, are the $2n_{xy}$ components of the eigenfunctions with the eigenvalues

$$\gamma(h) = e^{-jk_z 2h} + e^{-jk_z 2h} - 2 = -4\sin^2(k_z h) \qquad (14)$$

Let be

$$\underline{\vec{e}} = \begin{pmatrix} \underline{\vec{e}_x} \\ \underline{\vec{e}_y} \end{pmatrix}, \quad \begin{matrix} \underline{\vec{e}_x} = \left(\underline{e_{x1}}, \underline{e_{x2}}, \ldots, \underline{e_{xn_{xyz}}}\right)^T, & \underline{e_{xl}} = E_{x_{i,j,k}} \\ \underline{\vec{e}_y} = \left(\underline{e_{y1}}, \underline{e_{y2}}, \ldots, \underline{e_{yn_{xyz}}}\right)^T, & \underline{e_{yl}} = E_{y_{i,j,k}} \end{matrix} \qquad (15)$$

with

$$l = (j-1)n_x + 1, \quad i = 1(1)n_x, \quad j = 1(1)n_n$$
$$n_{xy} = n_x n_y \quad \text{and} \quad k = 1 \quad \text{or} \quad k = n_x \tag{16}$$

The assumption k = 1 corresponds to the case in which the cross-sectional plane (see Figure 1) is located on the left-handed (x; y)-plane of the enclosure. Let be A, dim(A) = $2n_{xy}$, the matrix of the eigenvalue problem. Because of boundary conditions on the left-hand side and the bottom of the port, the dimension of the eigenvalue problem is reduced to $n = 2n_{xy}-n_b$, $nb = n_x+n_y$. In most cases we have to take into account also boundary conditions at interior boundaries, which further reduce the dimension.

4. THE PROPAGATION CONSTANTS

The propagation constants k_z can be computed from after the solution of the eigenvalue problem. We get from

$$k_{z'} = \frac{1}{h}\arcsin\left(\frac{j}{2}\sqrt{\gamma_i}\right) = \frac{j}{2h}\ln\left(\frac{\gamma_i}{2}+1+\sqrt{\frac{\gamma_i}{2}\left(\frac{\gamma_i}{2}+2\right)}\right), \quad i = 1(1)n \tag{17}$$

Using the principal values of the functions, we have always β > 0 and α > 0. A propagation constant k_z and its corresponding eigenfunction are called a mode. In technical applications only a small number m of modes are of interest. For real values of k_z, = (k_z) = 0, the modal field describes a nonattenuated wave, which can be used to transmit signals. On the other hand, the amplitude of waves with $\alpha = -\Im(k_z) > 0$ decreases with increasing z. The larger the magnitude of =(k_z) the stronger is the decay. Such waves are called evanescent waves, and can be neglected after a given length d of the transmission line. Therefore, we use to select the interesting modes the:

Criterion 1: In the discussion to follow we distinguish between sets A of all eigenvalues and A of all corresponding propagation constants, and sets ε and ε of computed eigenvalues and propagation constants, respectively. The number of elements of a set B is denoted by $|B|$. Let be

$$\overline{A}^{(r)} \cup \overline{A}^{(i)} \cup \overline{A}^{(i)} = \overline{A}, \quad \overline{A}^{(r)} \cap \overline{A}^{(i)} = \varnothing, \quad \overline{A}^{(r)} \cap \overline{A}^{(c)} = \varnothing, \quad \overline{A}^{(i)} \cap \overline{A}^{(c)} = \varnothing \tag{18}$$

The real propagation constants of the set $\overline{A}^{(r)}$ are the propagation constants with the smallest magnitude of imaginary part. In most applications with lossless materials (that means with real ε and μ) one has at least one real propagation constant. These propagation constants have to be taken into account anyway. It is important, however, to know also at least one propagation constant with the smallest nonvanishing $|\alpha|$, in order to decide whether the eigenfunctions decrease strongly enough in a given distance d.

5. ESTIMATION OF THE MAXIMUM PROPAGATION CONSTANT

We can give an upper bound $k^{(max)}$ for the real part of the interesting propagation constants in the following way. In a homogeneous lossless material (with real ε and μ) the wave number k_f of an electromagnetic wave equals

$$k_f = \omega\sqrt{\varepsilon\mu} = k_0\sqrt{\tilde{\varepsilon}\tilde{\mu}} \tag{19}$$

This value is also an upper bound for the (real) propagation constants of undamped modes in a waveguide that is completely filled with the same material. In case of an inhomogeneously filled waveguide the quantities ε and μ can be different from cell to cell. We select the maxima of these quantities:

$$\tilde{\varepsilon}^{(max)} = \max_{i,j,k}\{\tilde{\varepsilon}_{i,j,k}\}, \quad \tilde{\mu}^{(max)} = \max_{i,j,k}\{\tilde{\mu}_{i,j,k}\}, \quad k^{(max)} = k_0\sqrt{\tilde{\varepsilon}^{(max)}\tilde{\mu}^{(max)}} \qquad (20)$$

as an upper bound for the propagation constants of propagating modes. If we change to materials with small losses (that means with small imaginary parts of ε and μ), we expect that the former real propagation constants become complex with nearly the same real part as in the lossless case and with small imaginary part. Hence in this case:

$$k^{(max)} = k_0\sqrt{\left|\tilde{\varepsilon}^{(max)}\right| \cdot \left|\tilde{\mu}^{(max)}\right|} \qquad (21)$$

approximates the maximum possible real part of the interesting propagation constants.

6. CONCLUSIONS

The application of the finite-volume scheme to the boundary value problem of the Maxwellian equations that describe the electromagnetic properties of microwave transmission lines results in an eigenmode problem of high dimension. We avoid the time-consuming computation of all eigenvalues in order to calculate a selected set of propagation constants using an iterative method, which is carried out twice. The numerical effort and the storage requirements can be reduced considerably.

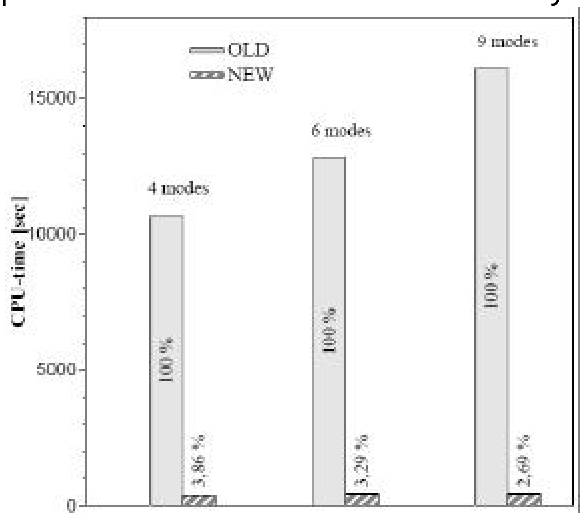

Figure 5: Computing times

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