SOME CONSIDERATIONS REGARDING COLLISION MODELS

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Abstract

In this paper it is proposed the presentation of some collision models of rigid bodies. Generally, the collision models may be divided in two groups. The first group is formed by the algebraic models. This group of models supposes that the input and output quantities simultaneously satisfy more algebraic or transcendental equations. The algebraic models depend only on the impulse-response rigidity of the colliding bodies. The second group of collision models is based on differential or evolution equations. Incremental rigid body collision models are based on an assumption of force-response rigidity. The models witch are treated in the paper appertain to the first category.

1. BRACH'S MODEL

Brach [1] solves numerous practical problems using the coefficient of restitution. He also introduces μ as the ratio of the tangential and the normal impulse components. This coefficient, defined as the impulse ratio, is necessary to treat the oblique impact problems, where the tangential relative velocity intervenes. If a Coulomb model is used for friction between surfaces then μ can be related to the dynamic coefficient of friction. As well, μ can be positive or negative to ensure that the tangential force is dissipative, as it is usually the case. Brach also notes that the restitution coefficient can be allowed to take negative values between 0 and -1. That means that some energy has been lost during impact but without velocity reversal (as an example, a projectile penetrating through a barrier; the penetration work reduces the velocity of the projectile without reversing it). For oblique contacts, Brach proposes to use the tangential restitution coefficient e_t to relate tangential velocities before and after impact. He also shows that μ and e_t are related. Therefore, only two independent coefficients e and μ are required to solve impact problems. An important result comes out of Brach's analysis: the final system energy cannot be zero for a perfectly inelastic and frictionless impact (e=0 and μ=0).

2. SMITH'S MODEL

An algebraic model proposed by Smith [2] uses the cinematic definition of the coefficient of normal restitution, and a frictional impulse that is defied using an intuitively appealing weighted average of the pre-collision and post-collision tangential components of the relative velocities. Smith shows that this model is guaranteed not to create kinetic energy in a collision, i.e. it satisfies assumption of non-negative energy dissipation - kinetic energy is not created in a collision (For most collisions studied in practice, the net kinetic energy of rigid body motion in the bodies after the collision will be less than or equal to the initial kinetic energy. In this paper it is assumed that the collisions cause non-negative dissipation of kinetic energy into heat or other forms of energy. For brevity the paper considers to non-negative dissipation of kinetic energy as a “conservation of energy”).

If P_N is the normal component of the transmitted impulse, P_T the second vector denoting the tangential component of the impulse, v_{iN} and v_{fN} the normal components of the pre-collision and post-collision relative velocities, respectively v_{iT} and v_{fT} the second vectors denoting the tangential components of the pre-collision and post-collision relative velocities, then for Smith’s model

$$P_T = \mu P_N \frac{\|v_{iT}\| v_{iT} + \|v_{fT}\| v_{fT}}{\|v_{iT}\|^2 + \|v_{fT}\|^2} \tag{1}$$

The cinematic (Newtonian) definition of the coefficient of restitution e is used, i.e.

$$v_{fN} = -e v_{iN} \tag{2}$$

where e is assumed to lie between zero and one. The impulse P and the relative velocities v_i and v_f still must satisfy $M(v_f - v_i) = P$. This provides three simultaneous equations which must be solved for P_N and v_{fT} (two equations in 2D):

$$P_N \left\{ \begin{array}{c} 1 \\ \mu \dfrac{\|v_{iT}\| v_{iT} + \|v_{fT}\| v_{fT}}{\|v_{iT}\|^2 + \|v_{fT}\|^2} \end{array} \right\} = M \left\{ \begin{array}{c} -(1+e) v_{iN} \\ v_{fT} - v_{iT} \end{array} \right\} \tag{3}$$

Smith himself indicates in his paper that the model ignores actual details of the frictional interaction between the bodies. It is quite likely that the predictions of the model will be inaccurate in many cases.

3. KANE AND LEVINSON’S MODEL

A commonly used collision model described briefly in Kane and Levinson’s text [3] with explicit attention to the direction of the tangential component of the contact impulse, uses cinematic restitution and a frictional impulse based on the post-collision relative velocity direction. The collision law is expressed by the equation

$$v_{fN} = -e v_{iN} \tag{5}$$

And the condition is one of these ones:

- $v_{fT} = 0$ and $\|P_T\| \le \mu P_N$
- $P_T = \mu P_N \dfrac{v_{fT}}{\|v_{fT}\|}$

These conditions, in addition to $M(v_f - v_i) = P$, are sufficient to determine the outcome of the collision. This collision law can predict large increases in system kinetic energy, as noted by Kane and Levinson.

Some overly simple models, such as Kane and Levinson’s model, violate conservation of energy for apparently reasonable values of various input parameters. This feature of collision models has attracted some attention of late, and several relatively recent papers on the topic check for energy conservation.

4. ROUTH'S MODEL

Routh's model is based on the Lagrangian equations of motion of a collision, and some results derived from it.

One of the key features of Routh's model [4], [5] is that it assumes infinite tangential stiffness in the contact region, so it cannot incorporate tangential restitution; but the normal and tangential stiffness of most bodies are of similar magnitude.

Routh's method uses the normal impulse as the integration variable. As a consequence, the normal force is unknown, and the physical condition (normal force equal to zero) cannot be used to determine the termination of the collision.

In the Routh's model the normal force relates the normal impulse true the time differential:

$$dP_N = F_N dt \tag{6}$$

and therefore the tangential impulse can be related to the normal impulse by:

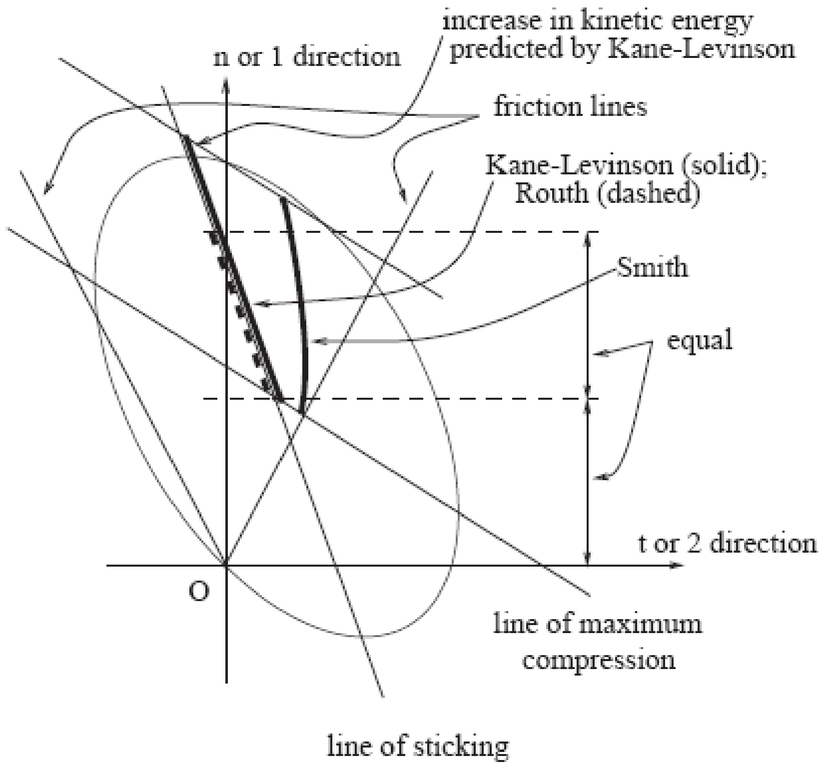
$$dP_T = F_T dt = \frac{F_T}{F_N} dP_N \tag{7}$$

The equations of motion for a collision can be written as a function of time or as a function of the normal impulse, P_N. When integrating with respect to P_N, only knowledge of the ratio between the tangential and normal forces is required, and this, together with the assumption of infinite tangential stiffness and Coulomb's law for friction, forms the basis of Routh's model. Not needing a full description of the forces throughout the collision gives Routh's model its simplicity, but it also poses some significant challenges: the difficulty of having a good criterion for the termination of the collision, and the need to assume infinite tangential stiffness (zero tangential compliance). As most materials tangential stiffness is of the same order of magnitude as their normal stiffness, the assumption of infinite tangential stiffness can be unrealistic.

5. Conclusions

The collision laws of Kane and Levinson, Smith and Routh have only one free collision parameter, called the coefficient of normal restitution *e,* in addition to the coefficient of friction. Therefore, for generic collisions with given μ, each of these collision laws can only access one-dimensional subsets of the accessible region, in both 2D and 3D collisions. The collision laws depend on two free collision parameters, in addition to the coefficient of friction. Therefore, for generic collisions with given μ these three laws can access two-dimensional subsets of the accessible region for both 2D and 3D collisions. None of these three collision laws can access the entire accessible region, however, even for 2D collisions.

In accordance with Caterjee studies [6], the graphical view of increase in energy predicted by some collision models, for the case of a sticking collision in 2D is shown in Figure. For this same collision, the regions accessible to the laws of Kane and Levinson, of Routh, and of Smith, are shown in the Figure. It is seen that for some choices of collision parameters, Kane-Levinson model can predict an increase in system kinetic energy.



Region accessible in impulse space to laws of Routh, Kane and Levinson, Smith

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