GEARS AND BARS MECHANISM SYNTHESIS WITH IMPOSED ASSOCIATED POSITIONS

Adina CATANEANU, Dan Bagnaru
University of Craiova, Faculty of Mechanics, a.cataneanu@rdslink.ro

Keywords: synthesis, imposed associated position

Abstract. The purpose of this paper is to present a modern method for the geometrical - kinematical synthesis of a complex plane mechanism which has in its structure, besides the type bar elements, also a gear. The lengths of the elements can be determined based on the associated positions at which the transmission functions are known.

1. INTRODUCTION

The geometrical-kinematical synthesis is part of the mechanism synthesis for which is demanded a correlation between the positions of the guided elements and the positions of the guiding elements. It is necessary to determine the lengths of the elements from the mechanism structure. The number of the relative-associated positions, where the zero order transmission functions are given, depends on the number of functional sides (all the segments of finite length located between two kinematical pairs), on the number of driving elements for which the transmission functions are known and on the number of the independent cycles.

2. MECHANISM SYNTESIS WITH IMPOSED RELATIVELY-ASSOCIATED POSITIONS

The mechanism presented in the figure 1 has, in its structure, besides the bar elements, the gear that consists in the toothed-wheels linked in B, respectively, in C, the first one being solid to the guiding element AB and the second one to the guided element CD.

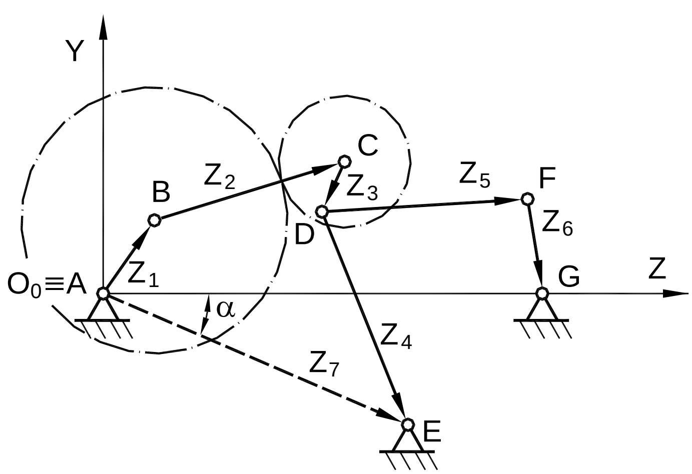

Fig. 1 The gears and bars mechanism

To each element is attached one complex number noted, in the initial position z_i, $i = \overline{1,6}$, while the segment AE is noted z_7. From the closing condition for the independent cycle ABCDFG, to the 10 position, can be written:

$$z_1 + z_2 + z_3 + z_5 + z_6 = 1;$$
$$z_1 e^{i\varphi_j^1} + z_2 e^{i\varphi_j^2} + z_3 e^{i\varphi_j^3} + z_5 e^{i\varphi_j^5} + z_6 e^{i\varphi_j^6} = 1 \quad j = \overline{1,10} \tag{1}$$

and for the independent cycle ABCDE:

$$z_1 + z_2 + z_3 + z_4 = z_7;$$
$$z_1 e^{i\varphi_j^1} + z_2 e^{i\varphi_j^2} + z_3 e^{i\varphi_j^3} + z_4 e^{i\varphi_j^4} = z_7, \quad j = \overline{1,10}. \tag{2}$$

In these relations $\varphi_j^k,\ k = \overline{1,6}$ represent the position angles for the driving element AB, respectively, for the intermediary elements BC, CD, DE, DF and for the guided element FG, face to the initial position, which are unknown. From the first five equations of the system (1) results:

$$z_k = \frac{\Delta_k}{\Delta}, \quad k = 1,2,3,5,6, \tag{3}$$

where:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ e^{i\varphi_1^1} & e^{i\varphi_1^2} & e^{i\varphi_1^3} & e^{i\varphi_1^5} & e^{i\varphi_1^6} \\ e^{i\varphi_2^1} & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^5} & e^{i\varphi_2^6} \\ e^{i\varphi_3^1} & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^5} & e^{i\varphi_3^6} \\ e^{i\varphi_4^1} & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^5} & e^{i\varphi_4^6} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{i\varphi_1^2} & e^{i\varphi_1^3} & e^{i\varphi_1^5} & e^{i\varphi_1^6} \\ 1 & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^5} & e^{i\varphi_2^6} \\ 1 & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^5} & e^{i\varphi_3^6} \\ 1 & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^5} & e^{i\varphi_4^6} \end{vmatrix}, \ldots.. \tag{4}$$

In the system (3), the unknowns which interfere in calculus of $z_k,\ k = 1,2,3,5,6$ are $\varphi_j^2,\ \varphi_j^3,\ \varphi_j^5,\ j = 1,2,3,4$. By substituting (3) into the last six equations of the system (1) it results the system:

$$\Delta_1 e^{i\varphi_j^1} + \Delta_2 e^{i\varphi_j^2} + \Delta_3 e^{i\varphi_j^3} + \Delta_5 e^{i\varphi_j^5} + \Delta_6 e^{i\varphi_j^6} = \Delta, \quad j = \overline{5,10}. \tag{5}$$

With the expressions of the determinants (4), the system (5) becomes:

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{i\varphi_1^1} & e^{i\varphi_1^2} & e^{i\varphi_1^3} & e^{i\varphi_1^5} & e^{i\varphi_1^6} \\ 1 & e^{i\varphi_2^1} & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^5} & e^{i\varphi_2^6} \\ 1 & e^{i\varphi_3^1} & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^5} & e^{i\varphi_3^6} \\ 1 & e^{i\varphi_4^1} & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^5} & e^{i\varphi_4^6} \\ 1 & e^{i\varphi_j^1} & e^{i\varphi_j^2} & e^{i\varphi_j^3} & e^{i\varphi_j^5} & e^{i\varphi_j^6} \end{vmatrix} = 0, \quad j = \overline{5,10} \tag{6}$$

with the unknowns $\varphi_j^5,\ j = \overline{5,10}$. The determinants (6) are developed after the last but one column and, in concentrate writing, results:

$$D_{j+} D_j^1 e^{i\varphi_1^5} + D_j^2 e^{i\varphi_2^5} + D_j^3 e^{i\varphi_3^5} + D_j^4 e^{i\varphi_4^5} + D e^{i\varphi_j^5} = 0, \quad j = \overline{5,10}, \tag{7}$$

where:

$$D = a + ib = -\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{i\varphi_1^1} & e^{i\varphi_1^2} & e^{i\varphi_1^3} & e^{i\varphi_1^6} \\ 1 & e^{i\varphi_2^1} & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^6} \\ 1 & e^{i\varphi_3^1} & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^6} \\ 1 & e^{i\varphi_4^1} & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^6} \end{vmatrix}, \quad D_j = a_j + ib_j = \begin{vmatrix} 1 & e^{i\varphi_1^1} & e^{i\varphi_1^2} & e^{i\varphi_1^3} & e^{i\varphi_1^6} \\ 1 & e^{i\varphi_2^1} & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^6} \\ 1 & e^{i\varphi_3^1} & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^6} \\ 1 & e^{i\varphi_4^1} & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^6} \\ 1 & e^{i\varphi_j^1} & e^{i\varphi_j^2} & e^{i\varphi_j^3} & e^{i\varphi_j^6} \end{vmatrix},$$

$$D_j^1 = a_j^1 + ib_j^1 = -\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{i\varphi_2^1} & e^{i\varphi_2^2} & e^{i\varphi_2^3} & e^{i\varphi_2^6} \\ 1 & e^{i\varphi_3^1} & e^{i\varphi_3^2} & e^{i\varphi_3^3} & e^{i\varphi_3^6} \\ 1 & e^{i\varphi_4^1} & e^{i\varphi_4^2} & e^{i\varphi_4^3} & e^{i\varphi_4^6} \\ 1 & e^{i\varphi_j^1} & e^{i\varphi_j^2} & e^{i\varphi_j^3} & e^{i\varphi_j^6} \end{vmatrix}, \ldots\ldots\ldots\ldots .$$

(8)

Further on, it is introduced the complex conjugated system of the system (7):

$$\overline{D_j} + \overline{D_j^1} e^{-i\varphi_1^5} + \overline{D_j^2} e^{-i\varphi_2^5} + \overline{D_j^3} e^{-i\varphi_3^5} + \overline{D_j^4} e^{-i\varphi_4^5} + \overline{D} e^{-i\varphi_j^5} = 0, \quad j = \overline{5,10},$$

(9)

where $\overline{D}, \overline{D_j}, \overline{D_j^1}, \overline{D_j^2}, \overline{D_j^3}, \overline{D_j^4}$ are the conjugates of the complex numbers $D, D_j, D_j^1, D_j^2, D_j^3, D_j^4$. At last, by eliminating the unknowns $\varphi_j^5, \ j = \overline{5,10}$, by multiplying (9) with (7), it results:

$$D\overline{D} = D_j\overline{D_j} + \sum_{k=1}^{4} D_j^k \overline{D_j^k} + \sum_{k=1}^{4} D_j \overline{D_j^k} e^{-i\varphi_k^5} + \sum_{k=1}^{4} D_j^k \overline{D_j} e^{i\varphi_k^5} + \sum_{k=2,3,4} D_j^1 \overline{D_j^k} e^{i\left(\varphi_1^5 - \varphi_k^5\right)} +$$
$$+ \sum_{k=1,3,4} D_j^2 \overline{D_j^k} e^{i\left(\varphi_2^5 - \varphi_k^5\right)} + \sum_{k=1,2,4} D_j^3 \overline{D_j^k} e^{i\left(\varphi_3^5 - \varphi_k^5\right)} + \sum_{k=1,2,3} D_j^4 \overline{D_j^k} e^{i\left(\varphi_4^5 - \varphi_k^5\right)}, \quad j = \overline{5,10}.$$

(10)

From the first equation of the system (2), it results:

$$z_4 = z_7 - z_1 - z_2 - z_3,$$

(11)

which substituted in the remained equations of this system leads to the obtaining of the following system:

$$z_1 e^{i\varphi_j^1} + z_2 e^{i\varphi_j^2} + z_3 e^{i\varphi_j^3} + \left(z_7 - z_1 - z_2 - z_3\right) e^{i\varphi_j^4} = z_7, \ j = \overline{1,10}.$$

(12)

With (3) in (12) it results the system:

$$\Delta_1 e^{i\varphi_j^1} + \Delta_2 e^{i\varphi_j^2} + \Delta_3 e^{i\varphi_j^3} + \left(\Delta |z_7| e^{i\alpha} - \Delta_1 - \Delta_2 - \Delta_3\right) e^{i\varphi_j^4} = \Delta |z_7| e^{i\alpha}, \ j = \overline{1,10},$$

(13)

where:

$$z_7 = |z_7| e^{i\alpha}.$$

(14)

To eliminate the unknowns $\varphi_j^4, j = \overline{1,10}$, it is introduced the complex-conjugated system of the system (13):

$$\overline{\Delta_1} e^{i\varphi_j^1} + \overline{\Delta_2} e^{i\varphi_j^2} + \overline{\Delta_3} e^{i\varphi_j^3} + \left(\overline{\Delta} |z_7| e^{i\alpha} - \overline{\Delta_1} - \overline{\Delta_2} - \overline{\Delta_3}\right) e^{i\varphi_j^4} = \overline{\Delta} |z_7| e^{i\alpha}, \ j = \overline{1,10}$$

(15)

and, this way, from (13) and (15) are eliminated the unknowns $\varphi_j^4, j=\overline{1,10}$. It results:

$$\left|z_7\right|e^{i\alpha}\left(\sum_{k=1}^{3}\Delta\overline{\Delta_k}-\sum_{k=1}^{3}\Delta\overline{\Delta_k}e^{-i\varphi_j^k}\right)+\left|z_7\right|e^{-i\alpha}\left(\sum_{k=1}^{3}\Delta_k\overline{\Delta}-\sum_{k=1}^{3}\Delta_k\overline{\Delta}e^{i\varphi_j^k}\right)-$$
$$-\Delta_1\overline{\Delta}_2\left[1-e^{i\left(\varphi_j^1-\varphi_j^2\right)}\right]--\Delta_2\overline{\Delta}_1\left[1-e^{-i\left(\varphi_j^1-\varphi_j^2\right)}\right]-\Delta_1\overline{\Delta}_3\left[1-e^{i\left(\varphi_j^1-\varphi_j^3\right)}\right]-$$
$$-\Delta_3\overline{\Delta}_1\left[1-e^{-i\left(\varphi_j^1-\varphi_j^3\right)}\right]-\Delta_2\overline{\Delta}_3\left[1-e^{-i\left(\varphi_j^2-\varphi_j^3\right)}\right]-\Delta_3\overline{\Delta}_2\left[1-e^{-i\left(\varphi_j^2-\varphi_j^3\right)}\right]=0,\ j=\overline{1,10}. \quad (16)$$

Considering:

$$R_j+iI_j=\sum_{k=1}^{3}\Delta\overline{\Delta}_k-\sum_{k=1}^{3}\Delta\overline{\Delta}_k e^{-i\varphi_j^k};$$
$$R_j^{'}+iI_j^{'}=\Delta_1\overline{\Delta}_2\left[1-e^{-i\left(\varphi_j^1-\varphi_j^2\right)}\right];$$
$$R_j^{''}+iI_j^{''}=\Delta_1\overline{\Delta}_3\left[1-e^{i\left(\varphi_j^1-\varphi_j^3\right)}\right];$$
$$R_j^{'''}+iI_j^{'''}=\Delta_2\overline{\Delta}_3\left[1-e^{i\left(\varphi_j^2-\varphi_j^3\right)}\right];$$
$$\rho_j=\sqrt{R_j^2+I_j^2},\quad tg\alpha_j=\frac{I_j}{R_j},\quad j=\overline{1,10}, \quad (17)$$

the relations (16) conduct to the system:

$$\left|z_7\right|\rho_j\cos\left(\alpha+\alpha_j\right)-\Re_j=0,\ \ j=\overline{1,10}, \quad (18)$$

where:

$$\Re_j=R_j^{'}+R_j^{''}+R_j^{'''}. \quad (19)$$

By eliminating $\left|z_7\right|$ from the system (18), it results:

$$\frac{\Re_1}{\Re_j}=\frac{\rho_1\cos\left(\alpha+\alpha_1\right)}{\rho_j\cos\left(\alpha+\alpha_j\right)},\quad j=\overline{2,10}, \quad (20)$$

which can be written:

$$tg\alpha=\frac{\rho_1\Re_j\cos\alpha_1-\Re_1\rho_j\cos\alpha_j}{\rho_1\Re_j\sin\alpha_1-\Re_1\rho_j\sin\alpha_j},\quad j=\overline{2,10}. \quad (21)$$

By eliminating α from (21) the following system is obtained:

$$\frac{\rho_1\Re_2\cos\alpha_1-\Re_1\rho_2\cos\alpha_2}{\rho_1\Re_2\sin\alpha_1-\Re_1\rho_2\sin\alpha_2}=\frac{\rho_1\Re_j\cos\alpha_1-\Re_1\rho_j\cos\alpha_j}{\rho_1\Re_j\sin\alpha_1-\Re_1\rho_j\sin\alpha_j},\quad j=\overline{3,10}. \quad (22)$$

Further on, by grouping together (10) and (22) it results a nonlinear system which can be solved by numerical methods. With the obtained solution, from (3), are calculated z_1, z_2, z_3, z_5, z_6. From the system (18) z_7 is determined. Afterwards, from (11), results

z_4 and, this way, is achieved the geometrical-kinematical synthesis based on the relative-associated positions.

For example, the complex system (13) can be changed in an algebraically system with the real unknowns $\varphi_k^5,\ k=\overline{1,4}$, on the form:

$$f_j\left(\varphi_1^5,\varphi_2^5,\varphi_3^5,\varphi_4^5\right)=0,\quad j=\overline{5,8}, \tag{23}$$

where:

$$\begin{aligned}
&f_j\left(\varphi_1^5,\varphi_2^5,\varphi_3^5,\varphi_4^5\right)=(a_j)^2+(b_j)^2-(a)^2-(b)^2+\sum_{k=1}^{4}\left[\left(a_j^k\right)^2+\left(b_j^k\right)^2\right]+\\
&+2\sum_{k=2}^{4}\left[\left(a_ja_j^k+b_jb_j^k\right)\cos\varphi_k^5-\left(a_jb_j^k+b_ja_j^k\right)\sin\varphi_k^5\right]+2\sum_{k=2}^{4}\left[\left(a_j^1a_j^k+b_j^1b_j^k\right)\right.\\
&\cdot\cos\left(\varphi_1^5-\varphi_k^5\right)-\left.\left(b_j^1a_j^k+a_j^1b_j^k\right)\sin\left(\varphi_1^5-\varphi_k^5\right)\right]+2\sum_{k=3}^{4}\left[\left(a_j^2a_j^k+b_j^2b_j^k\right)\right.\\
&\cdot\cos\left(\varphi_2^5-\varphi_k^5\right)-\left.\left(b_j^2a_j^k+a_j^2b_j^k\right)\sin\left(\varphi_2^5-\varphi_k^5\right)\right]+2\left[\left(a_j^3a_j^4+b_j^3b_j^4\right)\cos\left(\varphi_3^5-\varphi_4^5\right)-\right.\\
&-\left.\left(b_j^3a_j^4+a_j^3b_j^{4k}\right)\sin\left(\varphi_3^5-\varphi_4^5\right)\right]\quad j=\overline{5,8}.
\end{aligned} \tag{24}$$

Let's consider the column matrices:

$$\varphi=\begin{bmatrix}\varphi_1^5\\ \varphi_2^5\\ \varphi_3^5\\ \varphi_4^5\end{bmatrix},\quad f=\begin{bmatrix}f_5\\ f_6\\ f_7\\ f_8\end{bmatrix}. \tag{25}$$

The system (23), under matric form, becomes:

$$f(\varphi)=0. \tag{23'}$$

The Jacobi matrix joined to the system (23') is:

$$\begin{bmatrix}
\frac{\partial f_5}{\partial\varphi_1^5} & \frac{\partial f_5}{\partial\varphi_2^5} & \frac{\partial f_5}{\partial\varphi_3^5} & \frac{\partial f_5}{\partial\varphi_4^5}\\
\frac{\partial f_6}{\partial\varphi_1^5} & \frac{\partial f_6}{\partial\varphi_2^5} & \frac{\partial f_6}{\partial\varphi_3^5} & \frac{\partial f_6}{\partial\varphi_4^5}\\
\frac{\partial f_7}{\partial\varphi_1^5} & \frac{\partial f_7}{\partial\varphi_2^5} & \frac{\partial f_7}{\partial\varphi_3^5} & \frac{\partial f_7}{\partial\varphi_4^5}\\
\frac{\partial f_8}{\partial\varphi_1^5} & \frac{\partial f_8}{\partial\varphi_2^5} & \frac{\partial f_8}{\partial\varphi_3^5} & \frac{\partial f_8}{\partial\varphi_4^5}
\end{bmatrix}. \tag{26}$$

For the unknowns there are selected the initial values $\varphi_1^{5,0},\ \varphi_2^{5,0},\varphi_3^{5,0},\varphi_4^{5,0}$, which represent the initial approximate solution. According to the Newton method, the solution of the system (23') is given by the matric relation:

$$\varphi^{p+1}=\varphi^p-J^{-1}\left(\varphi^p\right)f\left(\varphi^p\right)\quad p=1,2,3,\ldots\ldots, \tag{27}$$

where $J^{-1}\left(\varphi^p\right)$ is the reciprocal of the matrix $J\left(\varphi^p\right)$.

The initial approximate solution φ^0 must be chosen thus the process shall be convergent. With the solution φ^1 it is passing to the second approximation etc., the iterative process going on until the desired precision is obtained thus:

$$\left\|\varphi - \varphi^p\right\| \le \varepsilon,$$

where $\left\|\varphi - \varphi^p\right\|$ is the norm "m" of the column matrix $\left(\varphi - \varphi^p\right)$.

The complex system (22) can be transformed into an algebraically system with the real unknowns $\varphi_j^2,\ \ j = \overline{3,10}$ under the form:

$$f_j\left(\varphi_j^2\right) = 0, \quad j = \overline{3,10}, \tag{28}$$

that can be solved in the same way.

3. CONCLUSIONS

The presented method offers the possibility to determine the lengths of the component elements for a complex gears and bars mechanism. This method is based on the associated positions, at which the transmission functions are known. If the positions of the guiding element associated with the position of the guided elements are given in relation to a fixed frame, this position is named absolute associated positions. Because the frame is fixed to the chassis, the absolute associated positions are even the zero order transmission functions.

BIBLIOGRAPHY

1. Cataneanu, A., Apostol, M., Asupra cinematicii mecanismelor cu roti dintate si parghii, Proceedings of the Scientific Communications Meeting of "Aurel Vlaicu" University, Third Edition, Arad, 16th-17th Mau 1996, vol 3, pag. 61-66.
2. Cataneanu, A., Apostol, M., Kinematic Analysis of a Complex Mechanism, The Seventh IFToMM International Symposium on Linkages and Computer Aided Design Methods-Theory and Practice of Mechanisms, Bucharest 26-30 August 1997, vol3, pp.135-140.
3. Handra-Luca, V., Stoica, I.A., Introducere in teoria mecanismelor, Editura Dacia Cluj-Napoca, vol. I, 1982.
4. Handra-Luca, V., Functiile de transmitere in studiul mecanismelor, Editura Academoei, Bucuresti, 1983.