KINEMATIC AND STATIC MODELLING OF THE SAFETY CLUTCHES WITH BALLS

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Abstract: The existent information in the technical literature concerning the safety clutches with balls are reduced, as a rule, to descriptive information, difficult to use in the design and optimisation processes of the transmissions that include such clutches. The papers that tackle these clutches are mainly referred to monographs destined to cars and tractors transmissions as well to monographs and papers destined to clutches in general.

1. INTRODUCTION

At present the safety clutches are designed on the basis of some models that estimate in a very small extent the real progress of these clutches working. Up to the present the achieved researches in the area of automatic intermittent clutches, had in view, among other things, the solution of the following problems:

1. The establishment of some calculus schemes and of some design relations of these clutches, corresponding with the complete connected working situation and with the uncoupling process.
2. The clutches study by means of some equivalent plane mechanisms valid for the entire uncoupling process.
3. Defining of some appreciation criteria for the functional performances. These criteria are used for the selection and the comparative appreciation of the safety clutches.

This paper proposes, for the problem 2 enunciated above, to emphasize some aspects concerning the study of he safety clutches with balls (aspects also necessary for the dynamic analysis [2]).

The analysis of the uncoupling process for the safety clutches with balls is achieved, at present, on the basis of some equivalent plane mechanisms. This paper proposes the study of the clutches on the basis of space equivalent mechanisms, that to allow solving of the following problems:

- The equivalent space mechanisms – used for the study of the uncoupling process – have as an input motion in the mechanism a rotational motion; this leads to the possibility of tacking into consideration, in the later on strength calculations, both of the inertia moment of the driving clutch and, even more, of the inertia moments corresponding to the transmissions assembled before the clutch and of the inertia moment of the motor.
- The kinematic analysis of he space mechanisms consists of:
  - * the determination of the position functions of the ball (balls), in terms of the relative rotation angle between the two semi-clutches (angles that represents the independent motion of the equivalent space mechanism);
  - * the determination of the velocity and acceleration functions of the ball (balls); the expressions of these velocities and accelerations are necessary for the later on calculations, necessary for the designing of these clutches (for instance to the inertia forces calculus).
- The relative rotation angle between the semi-clutches is a function depending on time,

the determination of this dependence making possible the plotting of the diagrams for the torque transmitted by the clutch in the uncoupling process, in terms of time. The determination of the dependence angle-time will be achieved by solving of the motion equations, corresponding to the equivalent space mechanisms, for the two stages of uncoupling process (the solving of these equations is possible only on the computer, using numerical methods). There will be taken into considerations the following time functions:

$$\varphi(t)=\varphi_1(t)-\varphi_3(t);\ \omega_1(t);\ \omega_3(t);\ \varepsilon_1(t);\ \varepsilon_3(t); \tag{1}$$

$$M_1=M_1(F_a,\varphi_1,\varphi_3);\ M_3=M_3(F_a,\varphi_1,\varphi_3). \tag{2}$$

Taking into consideration, the algorithm of dynamic analysis supposes:

- the wording of the dynamic modelling problem;
- the static and kinematic modelling of the clutch;
- the modelling of the correlation induced by the mechanical characteristics of the motors and end-users;
- the modelling of the semi-clutches motions by means of the Lagrange equations of the second species,

the paper proposes the static and kinematic modelling of a safety clutch with balls [2].

2. KINEMATIC AND STATIC MODELLING OF THE SAFETY CLUTCHES WITH BALLS

The safety clutches with balls [2] can be modelled as bimobile mechanisms (M=2), with one input and one output, where the semi-clutches motions, φ_1 and φ_3, are considered

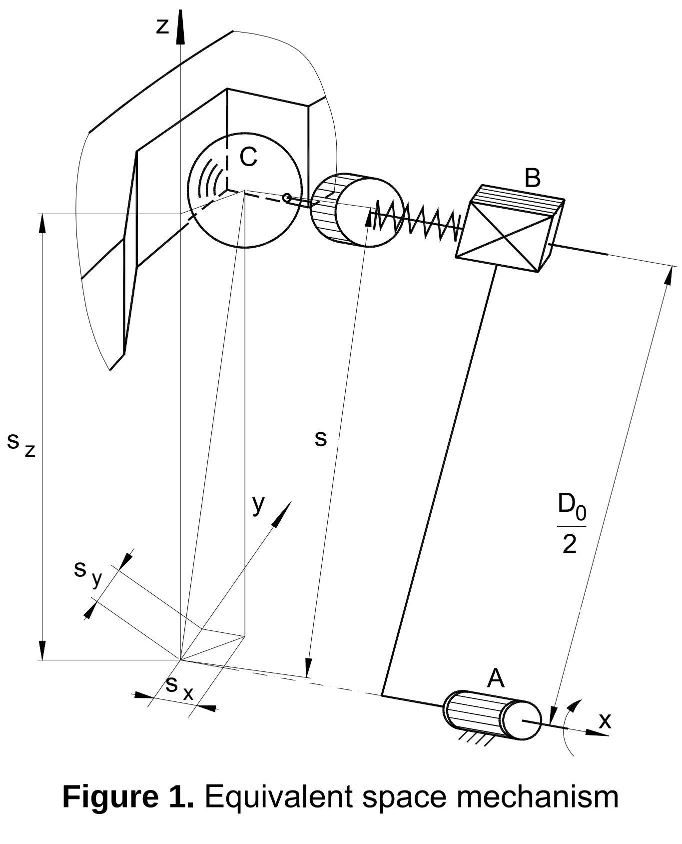
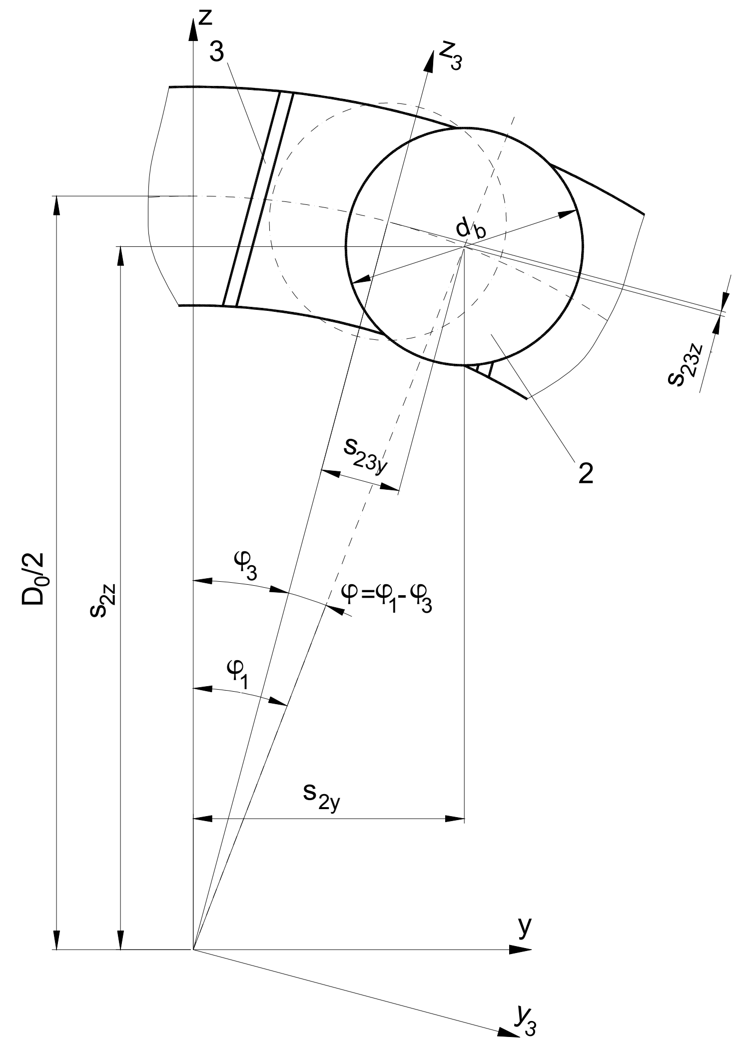


Figure 1. Equivalent space mechanism



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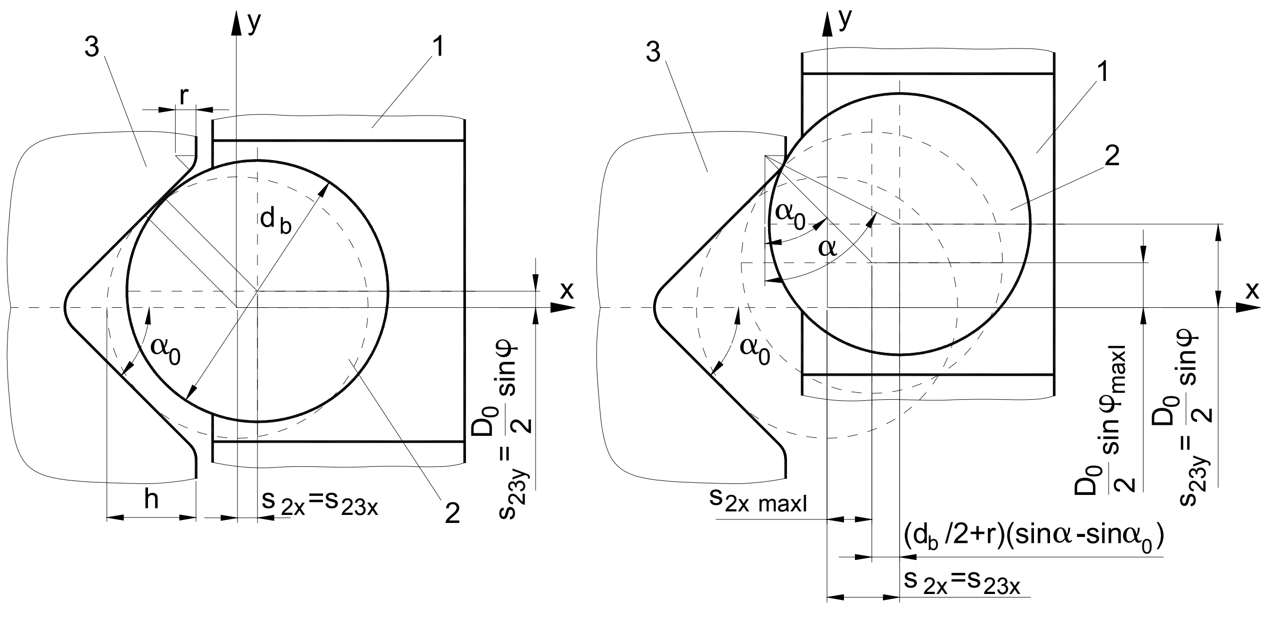

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Figure 2. Geometry of the active rabbets and the calculation scheme of the transmission functions of the ball 2

Table 1

Expressions of the position, velocity and acceleration functions for the ball 2

The first uncoupling stage	The second uncoupling stage	Complete uncoupling
❑ Variation area of the relative rotation angle between semi-clutches $\varphi_1-\varphi_3$		
$\varphi_1-\varphi_3\in[0,\varphi_{maxI}]$	$\varphi_1-\varphi_3\in(\varphi_{maxI},\varphi_{maxII}]$	$(\varphi_{maxII},\varphi_{max}]$
$\varphi_{maxI}=\arcsin\left[\frac{2h-(d_b+2r)(1-\sin\alpha_0)}{D_0}tg\alpha_0\right]$; $\varphi_{maxII}=\arcsin\left[\frac{2h\sin\alpha_0+(d_b+2r)(1-\sin\alpha_0)}{D_0\cos\alpha_0}\right]$ $\varphi_{max}=\frac{2\pi}{nz}-\varphi_{maxII}$		
❑ Contact angle between the ball 2 and the driven semi-clutch 3		
$\alpha=\alpha_0$	$\alpha=\arccos\left(\frac{2htg\alpha_0-D_0\sin(\varphi_1-\varphi_3)}{d_b+2r}+\frac{1-\sin\alpha_0}{\cos\alpha_0}\right)$	$\alpha=90^o$
❑ Relative position function of the ball 2 depending on the driving semi-clutch 1: s_{21}		
$[s_{21}]=\begin{bmatrix}\frac{D_0}{2}\frac{\sin(\varphi_1-\varphi_3)}{tg\alpha_0}\\0\\0\end{bmatrix}$	$[s_{21}]=\begin{bmatrix}\frac{D_0}{2}\frac{\sin\varphi_{maxI}}{tg\alpha_0}+\left(\frac{d_b}{2}+r\right)(\sin\alpha-\sin\alpha_0)\\0\\0\end{bmatrix}$	$[s_{21}]=\begin{bmatrix}0\\0\\0\end{bmatrix}$
❑ Relative velocity function of the ball 2 depending on the driving semi-clutch 1: v_{21}		
$[v_{21}]=\begin{bmatrix}v_{21x}\\v_{21y}\\v_{21z}\end{bmatrix}=\begin{bmatrix}\frac{D_0}{2}\frac{\cos(\varphi_1-\varphi_3)}{tg\alpha}(\dot{\varphi}_1-\dot{\varphi}_3)\\0\\0\end{bmatrix}$		$[v_{21}]=\begin{bmatrix}v_{21x}\\v_{21y}\\v_{21z}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$
❑ Relative position function of the ball 2 depending on the driven semi-clutch 3: s_{23}		
$\begin{bmatrix}\frac{D_0}{2}\frac{\sin(\varphi_1-\varphi_3)}{tg\alpha_0}\\\frac{D_0}{2}\sin(\varphi_1-\varphi_3)\\\frac{D_0}{2}(1-\cos(\varphi_1-\varphi_3))\end{bmatrix}$	$\begin{bmatrix}\frac{D_0}{2}\frac{\sin\varphi_{maxI}}{tg\alpha_0}+\left(\frac{d_b}{2}+r\right)(\sin\alpha-\sin\alpha_0)\\\frac{D_0}{2}\sin(\varphi_1-\varphi_3)\\\frac{D_0}{2}-\frac{D_0}{2}\cos(\varphi_1-\varphi_3)\end{bmatrix}$	$\begin{bmatrix}0\\\frac{D_0}{2}\sin(\varphi_1-\varphi_3)\\\frac{D_0}{2}(1-\cos(\varphi_1-\varphi_3))\end{bmatrix}$
❑ Relative velocity function of the ball 2 depending on the driven semi-clutch 3: v_{23}		
$[v_{23}]=\begin{bmatrix}v_{23x}\\v_{23y}\\v_{23z}\end{bmatrix}=\begin{bmatrix}\frac{D_0}{2}\frac{\cos(\varphi_1-\varphi_3)}{tg\alpha}(\dot{\varphi}_1-\dot{\varphi}_3)\\\frac{D_0\cos(\varphi_1-\varphi_3)}{2}(\dot{\varphi}_1-\dot{\varphi}_3)\\\frac{D_0\sin(\varphi_1-\varphi_3)}{2}(\dot{\varphi}_1-\dot{\varphi}_3)\end{bmatrix}$		$\begin{bmatrix}0\\\frac{D_0\cos(\varphi_1-\varphi_3)}{2}(\dot{\varphi}_1-\dot{\varphi}_3)\\\frac{D_0\sin(\varphi_1-\varphi_3)}{2}(\dot{\varphi}_1-\dot{\varphi}_3)\end{bmatrix}$

Table 1 (continuation)

❑ Position function of the ball 2: s_2		
$[s_2]=\begin{bmatrix}\frac{D_0}{2}\frac{\sin(\varphi_1-\varphi_3)}{tg\alpha_0}\\ \frac{D_0}{2}\sin\varphi_1\\ \frac{D_0}{2}\cos\varphi_1\end{bmatrix}$	$\begin{bmatrix}\frac{D_0}{2}\frac{\sin\varphi_{maxl}}{tg\alpha_0}+\left(\frac{d_b}{2}+r\right)(\sin\alpha-\sin\alpha_0)\\ \frac{D_0}{2}\sin\varphi_1\\ \frac{D_0}{2}\cos\varphi_1\end{bmatrix}$	$[s_2]=\begin{bmatrix}s_{2x}\\ s_{2y}\\ s_{2z}\end{bmatrix}=\begin{bmatrix}0\\ \frac{D_0}{2}\sin\varphi_1\\ \frac{D_0}{2}\cos\varphi_1\end{bmatrix}$
❑ Velocity function of the ball 2: v_2		
$[v_2]=\begin{bmatrix}v_{2x}\\ v_{2y}\\ v_{2z}\end{bmatrix}=\begin{bmatrix}\frac{D_0}{2}\frac{\cos(\varphi_1-\varphi_3)}{tg\alpha}(\dot\varphi_1-\dot\varphi_3)\\ \frac{D_0}{2}\cos\varphi_1\cdot\dot\varphi_1\\ -\frac{D_0}{2}\sin\varphi_1\cdot\dot\varphi_1\end{bmatrix}$		$[v_2]=\begin{bmatrix}0\\ \frac{D_0}{2}\cos\varphi_1\cdot\dot\varphi_1\\ -\frac{D_0}{2}\sin\varphi_1\cdot\dot\varphi_1\end{bmatrix}$
❑ Acceleration function of the ball 2: a_2		
$a_{2y}=\frac{D_0}{2}\left[-\sin\varphi_1\cdot\dot\varphi_1^2+\cos\varphi_1\cdot\ddot\varphi_1\right]$ $a_{2z}=\frac{D_0}{2}\left[-\cos\varphi_1\cdot\dot\varphi_1^2-\sin\varphi_1\cdot\ddot\varphi_1\right]$ The first stage: $a_{2x}=\frac{D_0}{2}\left[-\frac{\sin(\varphi_1-\varphi_3)}{tg\alpha_0}(\dot\varphi_1-\dot\varphi_3)^2+\frac{\cos(\varphi_1-\varphi_3)}{tg\alpha_0}(\ddot\varphi_1-\ddot\varphi_3)\right]$ The second stage: $a_{2x}=\frac{D_0}{2}\left[-\left(\frac{\sin(\varphi_1-\varphi_3)}{tg\alpha}+\frac{\cos^2(\varphi_1-\varphi_3)}{\sin^3\alpha}\frac{D_0}{d_b+2r}\right)(\dot\varphi_1-\dot\varphi_3)^2+\frac{\cos(\varphi_1-\varphi_3)}{tg\alpha}(\ddot\varphi_1-\ddot\varphi_3)\right]$ The complete uncoupling stage: $a_{2x}=0$		

independent parameters (Figure 1). The contact of the balls 2 with the trapezium active rabbets, radial disposed on the driving semi-clutch 1, is materialised by the translation pairs B; the balls contact with the trapezium rabbets, disposed frontally on the driven semi-clutch 3, is represented by the pairs C; the connection between the balls 2 and the pressure washer 4 will be considered an external one [2]. In this way, it is assured the observance of the structural condition of the mechanism existence [1]: L=3>M=2>0, where L represents the external connections number (power inputs and outputs), and M the degree of freedom.

The adopted mechanism model is defined considering the following assumptions:

- the kinematic elements are rigid;
- the kinematic pairs are permanent, geometrical, and stationary.

For the safety clutches with trapezium rabbets, the uncoupling process takes places in two stages. In the first stage the ball 2 moves on the plane surface of the active rabbets; this stage ends when the contact point between the ball and the active rabbet reaches the extreme point of the rabbet (the intersection point between the active plane surface and the connected zone of radius r). In the second stage, the ball 2 turns around the extreme point of the rabbet; this stage ends when the ball 2 becomes tangent to the exterior surface of the semi-clutch 3. Figure 2 represents the ball positions for the first and the second uncoupling stages, considering both semi-clutches moving; thus the angular relative displacement between the two semi-clutches is $\varphi = \varphi_1 - \varphi_3$.

Knowing the geometry of the active rabbets, there can be determined the transmission function $s_a = s_a(\varphi_1, \varphi_3) = s_{23x}$, as well as the relative and absolute position functions of the balls 2: s_{21} and s_2. By successive derivation of these position functions, the velocities and accelerations expressions are obtained; these expressions are depending on the independent motions of the mechanism. Table 1 systematizes the expressions of the position, velocities and accelerations functions, the expressions being determined for the two uncoupling stages and also for the uncoupling stage.

3. CONCLUSIONS

For the bimobile equivalent mechanism of the clutch, the determination of the transmission functions for the forces (relations (2)) will be achieved using the Lagrange equations of the second species. In this context, the modelling of the semi-clutches motions makes necessary the static and kinematic modelling of the clutch proposed.

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