PARTICULARITIES OF THE UNDERWATER ROBOTS' KINEMATIC EQUATIONS

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Abstract: Underwater vehicles are widely employed in various aquatic environments due to their operational advantages over the divers. High pressure, low temperature and currents put a strict limit to the human operators capabilities. In maritime missions, either industrial or military ones, the underwater robots can cope with the tough environment they are supposed to work in. The study of their maneuverability and stability along the trajectory, much needed for a proper functioning, starts with the statement of the movement equations.

1. REFERENCE SYSTEMS

The study of the underwater robots motion requires three reference systems as there are different aspects that need to be discussed according to their best definition. These coordinate systems are all Cartesian and are the following:

The velocity reference system is related to the remotely operated vehicle's (ROV) trajectory. All the quantities expressed in this reference system are indexed by a subscript. The velocity reference system is defined by:

- the structure's gravity centre, G stands for the reference system origin, O_1
- the x_1 axis is parallel to the velocity vector V (which is tangent to the trajectory)
- the y_1 axis belongs to the cinematic plan and is perpendicular to the (O_1, x_1) axis
- the z_1 axis is perpendicular to both (O_1, x_1) and (O_1, y_1) axes.

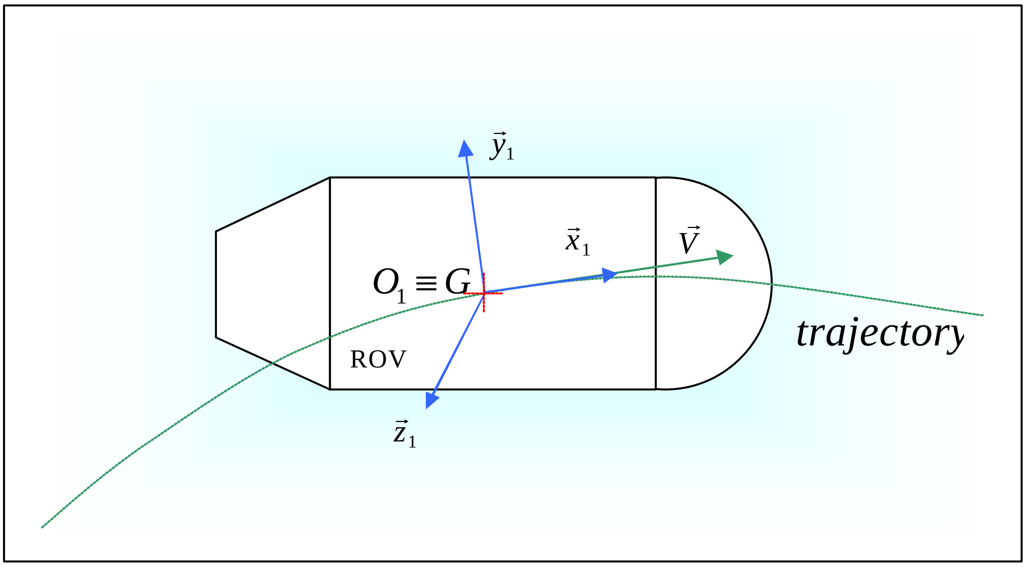

Fig. 1. The velocity reference system

This reference system allows the determination of the hydrodynamic coefficients and the added masses. The explanations about added masses will be given later.

The relative reference system is attached to the structure. It is chosen using the solid geometry and properties. The ROV is a symmetrical revolution system. Therefore the reference system is defined as:

- the reference system origin, O is also the gravity centre, G
- the x-axis is parallel to the longitudinal axis of the structure
- the y-axis belongs to the cinematic plan and is perpendicular to the (O, x) axis
- the z-axis is perpendicular to both (O, x) and (O, y) axes.

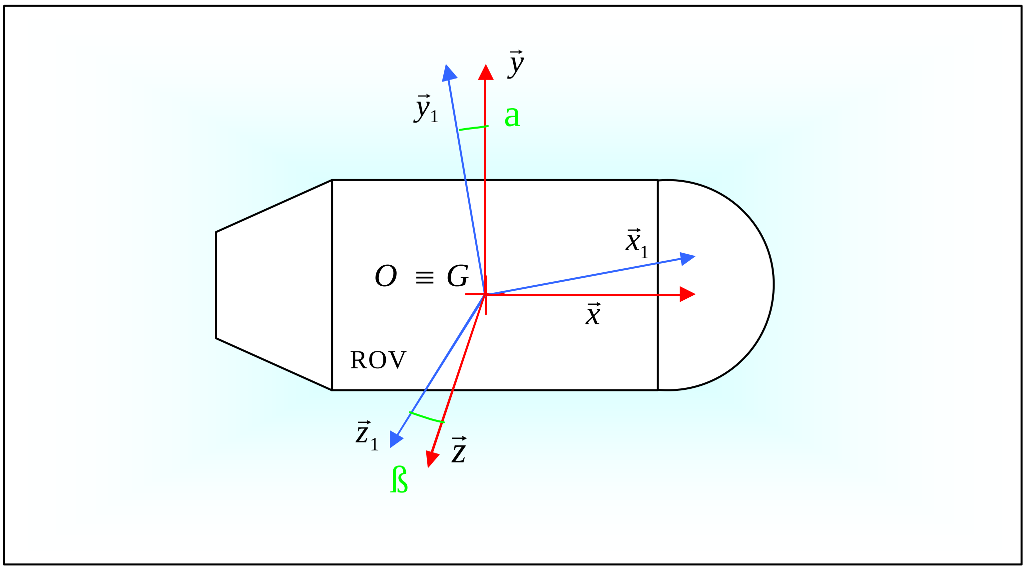

Fig. 2. The relative reference system

This reference system leads to the determination of two angular kinematic parameters:

- a, which is the attack angle
- ß, which is the drift angle.

This reference system is quite important. It is the reference system in which essential forces are expressed, for example the upward thrust. Indeed sonar's indications are linked to the relative reference system of the ROV.

Therefore we need to express the linear and angular velocities in this reference system. Let's introduce some cinematic information:

- the linear velocity along x-axis, v_x is called the surge,
- the linear velocity along y-axis, v_y is called the heave,
- the linear velocity along z-axis, v_z is called the sway,
- the angular velocity around x-axis, ? $_x$ is called the roll rate,
- the angular velocity around y-axis, ? $_y$ is called the yaw rate,
- the angular velocity around z-axis, ? $_z$ is called the pitch rate,

We can define a linear velocity vector:

$$\vec{V} = \left\{ \begin{matrix} v_x \\ v_y \\ v_z \end{matrix} \right\}_R ,$$

but also a angular velocity vector:

$$\vec{w} = \begin{Bmatrix} w_x \\ w_y \\ w_z \end{Bmatrix}_R .$$

Hence we obtain a global velocity vector:

$$\vec{U} = \begin{Bmatrix} \{\vec{V}\} \\ \{\vec{w}\} \end{Bmatrix}_R = \begin{Bmatrix} v_x \\ v_y \\ v_z \\ w_x \\ w_y \\ w_z \end{Bmatrix}_R$$

This last vector contains all the motion parameters of the vehicle. Its components can be expressed in function of the supplied velocity is to say $\vec{V}_1 = v \cdot \vec{x}_1$.

The absolute reference system is fixed to the Earth. All the quantities expressed in this reference system are indexed by a subscript. The absolute reference system is composed of:

- a fixed point that stands for the reference system origin, O_t
- the x_t axis belongs to the horizontal plan and is pointing the ROV's wanted direction
- the z_t axis belongs to the horizontal plan and is perpendicular to the (O_t, x_t) axis
- the y_t axis is perpendicular to both (O_t, x_t) and (O_t, y_t) axes.

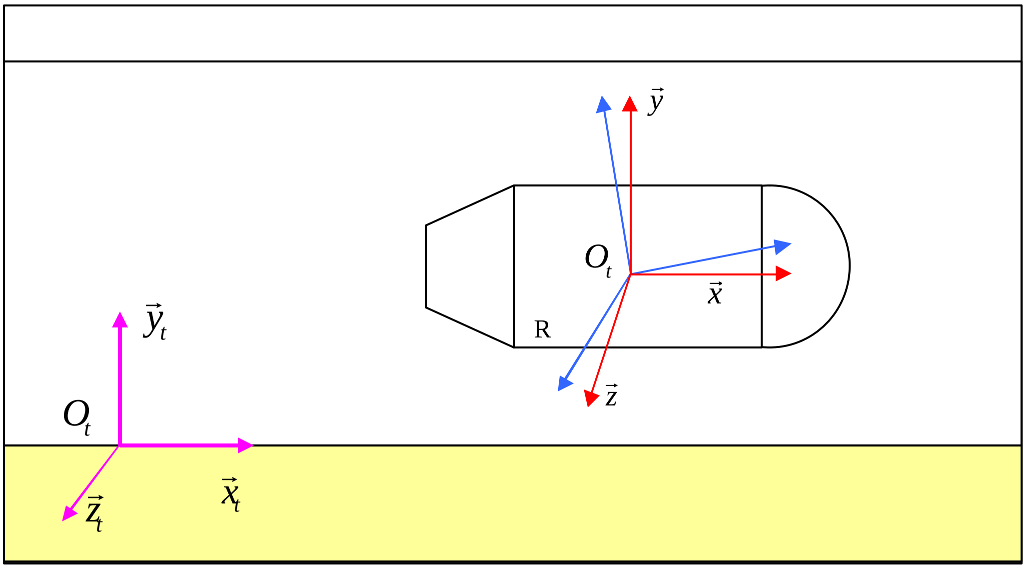

Fig. 3. The absolute reference system

This reference system is used in order to study the ROV's motion. The relations between this reference system and the previous one bring to the fore the three Euler's angles:

- the pitch, ? which is the angle between (O_t, xX) and (O_t, x_t) where (O_t, xX) is the x axis projection on the (O_t, x_t, y_t) plan linked to O_t,
- the roll, f which is the angle between (O_t, xX) and (O_t, x_t) where (O_t, xX) is the x axis projection on the (O_t, x_t, z_t) plan linked to O_t,
- ? which is the rotation around the (O, y) axis.

2. CHANGING EQUATIONS FROM A REFERENCE SYSTEM TO ANOTHER

In order to compare all the quantities we are dealing with, we have to find a way to express them in the same reference system. That is why we use transfer matrixes to change from a reference system to another one.

The transfer between those two reference systems needs two angular parameters. Actually we get to a transfer matrix:

$$A_{ab} = \begin{bmatrix} \cos a \cos b & -\sin a \cos b & \sin b \\ \sin a & \cos a & 0 \\ -\sin b \cos a & \sin a \sin b & \cos b \end{bmatrix}$$

(from the velocity reference system to the relative reference system)

The transfer between the relative and the absolute, fixed reference system needs three angular parameters: ?

Actually we get to a transfer matrix:

$$A_{yjq} = \begin{bmatrix} \cos y \cos q & \sin y \sin j - \sin q \cos y \cos j & \sin y \cos j + \sin q \sin j \cos y \\ \sin q & \cos q \cos j & -\sin j \cos q \\ -\sin y \cos q & \sin j \cos y + \sin y \sin q \cos j & \cos y \cos j - \sin y \sin q \sin j \end{bmatrix}$$

(from the relative reference system to the absolute one)

3. THE KINEMATIC EQUATIONS

The cinematic equations are the relation that expresses the global velocity, $\vec{U}$ in function of the cinematic parameters, a, b, y, q, j . We can express linear velocities in function of data as the supplied speed and the attack or drift angles.

$$v_x = v \cdot \cos(a)\cos(b)$$

$$v_y = v \cdot \sin(a)$$

$$v_z = -v \cdot \cos(a)\sin(b)$$

$$w_x = j + \dot{y}\sin(q)$$

$$w_y = \dot{q}\sin(j) + \dot{y}\cos(q)\cos(j)$$

$$w_z = \dot{q}\cos(j) - \dot{y}\cos(q)\sin(j)$$

We may note that $\dot{y}, \dot{q}, \dot{j}$ are the angular velocities around respectively, x-axis, y-axis, z-axis in the absolute reference system.

Therefore, we can write:

$$\vec{U}=\begin{bmatrix}\cos(a)\cos(b) & \sin(a) & -\cos(a)\sin(b) & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \sin(q) & 0 & 1\\ 0 & 0 & 0 & \cos(q)\cos(j) & \sin(j) & 0\\ 0 & 0 & 0 & -\cos(q)\sin(j) & \cos(j) & 0\end{bmatrix}\begin{Bmatrix}v\\ 0\\ 0\\ \dot{y}\\ \dot{q}\\ \dot{j}\end{Bmatrix}$$

4. LEFT HAND PART OF EQUATIONS DEVELOPMENT

We're all familiar with the momentum theorem and the similar one involved in rotations:

$$\left|\frac{d(\vec{H}+\vec{H}_1)}{dt}\right|_t=\vec{F}\quad \text{and}\quad \left|\frac{d(\vec{K}+\vec{K}_1)}{dt}\right|_t=\vec{M}\quad ,$$

where we define: H as the ROV's momentum
H_1 as the surrounding fluid momentum
K as the ROV's kinetic momentum
K_1 as the surrounding fluid kinetic momentum.
Using the derivation rules we get:

$$\left|\frac{d(\vec{H}+\vec{H}_1)}{dt}\right|_t=\frac{d(\vec{H}+\vec{H}_1)}{dt}+\vec{w}\times(\vec{H}+\vec{H}_1)=\vec{F}$$

$$\left|\frac{d(\vec{K}+\vec{K}_1)}{dt}\right|_t=\frac{d(\vec{K}+\vec{K}_1)}{dt}+\vec{w}\times(\vec{K}+\vec{K}_1)+\vec{v}\times(\vec{H}+\vec{H}_1)=\vec{M}$$

Therefore we actually have six equations:

$$\frac{d(H_x+H_{1x})}{dt}+w_y(H_z+H_{1z})+w_z(H_y+H_{1y})=F_x$$
$$\frac{d(H_y+H_{1y})}{dt}+w_z(H_x+H_{1x})+w_x(H_z+H_{1z})=F_y$$
$$\frac{d(H_z+H_{1z})}{dt}+w_x(H_y+H_{1y})+w_y(H_x+H_{1x})=F_z$$

and

$$\frac{d(K_x+K_{1x})}{dt}+w_y(K_z+K_{1z})-w_z(K_y+K_{1y})+v_y(H_z+H_{1z})-v_z(H_y+H_{1y})=M_x$$
$$\frac{d(K_y+K_{1y})}{dt}+w_z(K_x+K_{1x})-w_x(K_z+K_{1z})+v_z(H_x+H_{1x})-v_x(H_z+H_{1z})=M_y$$
$$\frac{d(K_z+K_{1z})}{dt}+w_x(K_y+K_{1y})-w_y(K_x+K_{1x})+v_x(H_y+H_{1y})-v_y(H_x+H_{1x})=M_z$$

On the other hand, we know that the derivatives of energy with respect to velocity and angular velocity are, respectively:

$$H_i = \frac{\partial T_c}{\partial v_i} \text{ and } K_i = \frac{\partial T_c}{\partial w_i} \text{, with i=\{x, y, z\}.}$$

Introducing those new relations in the formulae above we obtain:

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial v_v}\right) + w_y \frac{\partial T_c}{\partial v_z} - w_z \frac{\partial T_c}{\partial v_y} = F_x$$

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial v_y}\right) + w_z \frac{\partial T_c}{\partial v_x} - w_x \frac{\partial T_c}{\partial v_z} = F_y$$

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial v_z}\right) + w_x \frac{\partial T_c}{\partial v_y} - w_y \frac{\partial T_c}{\partial v_x} = F_z$$

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial w_x}\right) + w_y \frac{\partial T_c}{\partial w_z} - w_z \frac{\partial T_c}{\partial w_y} + v_y \frac{\partial T_c}{\partial v_z} - v_z \frac{\partial T_c}{\partial v_y} = M_x$$

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial w_y}\right) + w_z \frac{\partial T_c}{\partial w_x} - w_x \frac{\partial T_c}{\partial w_z} + v_z \frac{\partial T_c}{\partial v_x} - v_z \frac{\partial T_c}{\partial v_z} = M_y$$

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial w_z}\right) + w_x \frac{\partial T_c}{\partial w_y} - w_y \frac{\partial T_c}{\partial w_x} + v_x \frac{\partial T_c}{\partial v_y} - v_y \frac{\partial T_c}{\partial v_x} = M_z$$

But in our case, T_c is composed of two kinetic energies: that of the ROV as a structure (T_1) and that of the fluid swept along by the ROV's motion (T_2).

$$T_c = T_1 + T_2$$

This last kinetic energy deals with a mathematical model called in general "added masses". The study of the added masses represents a next step in defining the complete equations of the ROV's motion.

As a conclusion, we may say that the ROV's motion equations present two notable characteristics: First, they require three refernce systems in order to express all parameters involved – and eventually their alignment through Euler's rotations – and secondly, the underwater robot's movement also should take in account the surrounding fluid which interacts energetically with the vehicle.

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