HYDRODYNAMIC POSITION FORCES AND TORQUE OF THE UNDERWATER ROBOTS

Octavian TARABUTA
Vasile DOBREF
"Mircea cel Batran" Naval Academy, Constanta, Romania

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Abstract: Propulsion and control of the underwater vehicles require a careful evaluation of the hydrodynamic forces that appear while moving. Drag, lift and drift are important forces that determine the vehicle's attitude and behaviour when it operates in various missions. Estimating theoretically these dynamic parameters is a critical step in designing such submerged vehicles. In this paper, the authors intend to define the initial procedures in assessing the forces and torque that act onto an underwater robot.

1. EXPRESSIONS OF THE HYDRODYNAMIC POSITION FORCES AND TORQUE

They consist of the lift, the drag and the drift. They are due to friction with water and therefor they appear in the boundary layer. They are applied in the pressure centre.To define them in a proper way, let's consider an infinitesimal surface element of the remotely operated vehicle (ROV). It is subjected to an elementary hydrodynamic force F that can be decomposed on the velocity reference system (G, x_1, y_1, z_1) in three components:

- dR_{x1} which is an elementary drag,
- dR_{y1} which is an elementary lift,
- dR_{z1} which is an elementary drift.

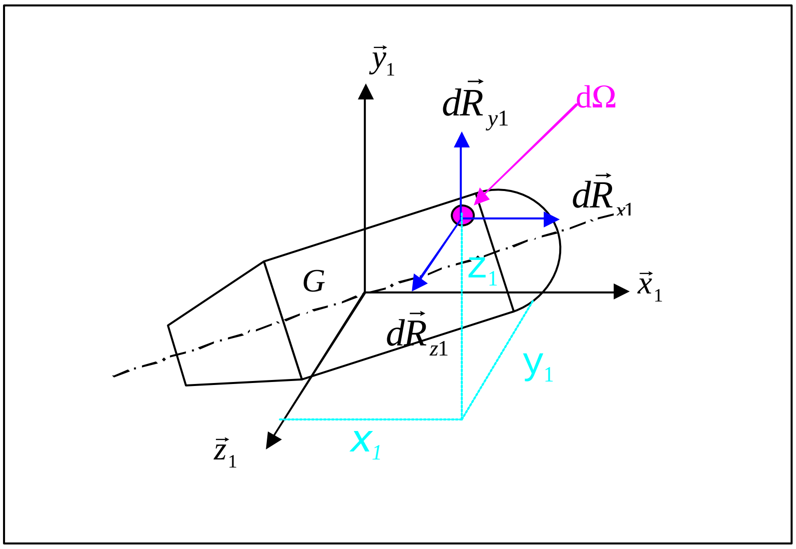

Fig 1. Elementary hydrodynamic forces

Thus, we have

$$dR_{x1} = \left[p\cos\left(\vec{n}, \vec{x}_1\right) + t\cos\left(\vec{t}\,', \vec{x}_1\right)\right] d\Omega$$
$$dR_{y1} = \left[p\cos\left(\vec{n}, \vec{y}_1\right) + t\cos\left(\vec{t}\,', \vec{y}_1\right)\right] d\Omega$$
$$dR_{z1} = \left[p\cos\left(\vec{n}, \vec{z}_1\right) + t\cos\left(\vec{t}\,', \vec{z}_1\right)\right] d\Omega$$

where $t = m\dfrac{d\vec{V}}{d\vec{n}}$ is the friction coefficient, which is linked to the dynamic pressure $\dfrac{r\,V^2}{2}$ and ? is the static pressure. In our case the position pressure ?gz is neglected.

Hence we can find global forces by integrating the elementary ones:

$$R_{x1} = \iint_\Omega dR_{x1}\, d\Omega$$
$$R_{y1} = \iint_\Omega dR_{y1}\, d\Omega$$
$$R_{z1} = \iint_\Omega dR_{z1}\, d\Omega$$

which leads to

$$R_{x1} = \iint_\Omega \left[p\cos(\vec{n},\vec{x}_1) + t\cos(\vec{t}\,',\vec{x}_1)\right] d\Omega$$
$$R_{y1} = \iint_\Omega \left[p\cos(\vec{n},\vec{y}_1) + t\cos(\vec{t}\,',\vec{y}_1)\right] d\Omega$$
$$R_{z1} = \iint_\Omega \left[p\cos(\vec{n},\vec{z}_1) + t\cos(\vec{t}\,',\vec{z}_1)\right] d\Omega$$

We can express the amount under the integral symbol as an adimensional quantity by dividing ? and t by $\dfrac{r\,V^2}{2}$ where V is the ROV's velocity, and dividing d? by a characteristic surface ?, which is the ROV's wet surface. It leads to a new expression of the hydrodynamic forces:

$$R_{x1} = \frac{r\,V^2}{2}\Omega\iint_\Omega\left[\frac{p}{\frac{r\,V^2}{2}}\cos(\vec{n},\vec{x}_1) + \frac{t}{\frac{r\,V^2}{2}}\cos(\vec{t}\,',\vec{x}_1)\right]\frac{d\Omega}{\Omega}$$

$$R_{y1} = \frac{r\,V^2}{2}\Omega\iint_\Omega\left[\frac{p}{\frac{r\,V^2}{2}}\cos(\vec{n},\vec{y}_1) + \frac{t}{\frac{r\,V^2}{2}}\cos(\vec{t}\,',\vec{y}_1)\right]\frac{d\Omega}{\Omega}$$

$$R_{z1} = \frac{r\,V^2}{2}\Omega\iint_\Omega\left[\frac{p}{\frac{r\,V^2}{2}}\cos(\vec{n},\vec{z}_1) + \frac{t}{\frac{r\,V^2}{2}}\cos(\vec{t}\,',\vec{z}_1)\right]\frac{d\Omega}{\Omega}$$

or else

$$R_{x1} = C_{x1}\frac{r\,V^2}{2}\Omega \quad \text{with } C_{x1} = \iint_\Omega\left[\frac{p}{\frac{r\,V^2}{2}}\cos(\vec{n},\vec{x}_1) + \frac{t}{\frac{r\,V^2}{2}}\cos(\vec{t}\,',\vec{x}_1)\right]\frac{d\Omega}{\Omega}$$

$$R_{y1} = C_{y1}\frac{r\,V^2}{2}\Omega \quad \text{with } C_{y1} = \iint_\Omega\left[\frac{p}{\frac{r\,V^2}{2}}\cos(\vec{n},\vec{y}_1) + \frac{t}{\frac{r\,V^2}{2}}\cos(\vec{t}\,',\vec{y}_1)\right]\frac{d\Omega}{\Omega}$$

$$R_{z1} = C_{z1} \frac{r V^2}{2} \Omega \qquad \text{with } C_{z1} = \iint_\Omega \left[\frac{p}{\frac{r V^2}{2}} \cos(\vec{n}, \vec{z}_1) + \frac{t}{\frac{r V^2}{2}} \cos(\vec{t}', \vec{z}_1) \right] \frac{d\Omega}{\Omega}$$

Those coefficients are the drag coefficient (C_{x1}), the lift coefficient (C_{y1}) and the drift lateral force coefficient (C_{z1}). They depend on the speed *V* and the Reynold's number *Re*, the ROV's shape and its roughness.

We actually have the local torque (see figure 1):

$$dM_{x1} = y_1 dR_{z1} - z_1 dR_{y1}$$
$$dM_{y1} = z_1 dR_{x1} - x_1 dR_{z1}$$
$$dM_{z1} = x_1 dR_{y1} - y_1 dR_{x1}$$

With the same method used about the forces, we can express the global torque:

$$M_{x1} = \iint_\Omega [y_1 dR_{z1} - z_1 dR_{y1}] d\Omega$$
$$M_{y1} = \iint_\Omega [z_1 dR_{x1} - x_1 dR_{z1}] d\Omega$$
$$M_{z1} = \iint_\Omega [x_1 dR_{y1} - y_1 dR_{x1}] d\Omega$$

Now to use adimensional amounts, we have to divide dR_1 by $\frac{r V^2}{2}$ and the coordinate x_1, y_1, z_1 by a characteristic length L, the total length of the ROV.

Consequently, we have:

$$M_{x1} = \frac{r V^2}{2} \Omega L \cdot \iint_\Omega \left[\frac{y_1}{L} \frac{dR_{z1}}{\frac{r V^2}{2}} - \frac{z_1}{L} \frac{dR_{y1}}{\frac{r V^2}{2}} \right] \frac{d\Omega}{\Omega}$$

$$M_{y1} = \frac{r V^2}{2} \Omega L \cdot \iint_\Omega \left[\frac{z_1}{L} \frac{dR_{x1}}{\frac{r V^2}{2}} - \frac{x_1}{L} \frac{dR_{z1}}{\frac{r V^2}{2}} \right] \frac{d\Omega}{\Omega}$$

$$M_{z1} = \frac{r V^2}{2} \Omega L \cdot \iint_\Omega \left[\frac{x_1}{L} \frac{dR_{y1}}{\frac{r V^2}{2}} - \frac{y_1}{L} \frac{dR_{x1}}{\frac{r V^2}{2}} \right] \frac{d\Omega}{\Omega}$$

or else,

$$M_{x1} = C_{mx1} \frac{r V^2}{2} \Omega L \qquad \text{with } C_{mx1} = \cdot \iint_\Omega \left[\frac{y_1}{L} \frac{dR_{z1}}{\frac{r V^2}{2}} - \frac{z_1}{L} \frac{dR_{y1}}{\frac{r V^2}{2}} \right] \frac{d\Omega}{\Omega}$$

The similar torque will have the coefficients according to their respective axes.
where the coefficient C_{x1}, C_{y1}, C_{z1}, C_{mx1}, C_{my1}, C_{mz1} are the hydrodynamic coefficients depending on the reference surface and length we chose.

2. HYDRODYNAMIC COEFFICIENT ASSESSMENT

The drag force is composed of two different part :
- the *friction drag* R_f which is the surrounding-fluid reaction to the propelling force T, it is opposed to the engine thrust.
- the *shape drag* R_s is due to the ROV's shape. When considering a symmetrical body moving through an ideal fluid flux, a low pressure zone appears and create a suction phenomenon.

$$R_{x1} = R_f + R_s$$

or in more details:

$$R_{x1} = C_{x1} \frac{r V^2}{2} \Omega \quad = \quad C_f \frac{r V^2}{2} \Omega + C_s \frac{r V^2}{2} \Omega = R_f + R_s$$

That is why we have:

$$C_{x1} = C_f + C_s$$

The hydrodynamic coefficient C_f is the drag coefficient given by the ROV's body itself. Considering the body surface as a non rough plan, we can use the expression:

$$C_f = \frac{0.455}{(\log(\mathrm{Re}))^{2.58}}.$$

It fits with the case where $a = b = d_V = d_H = 0$.

The hydrodynamic coefficient C_S is the shape drag coefficient. Using the Papnnel definition, we can express it as:

$$C_s = 0.09 \frac{S}{\Omega_w} \sqrt{\bar{l}}$$

with S, the maximum cross section of the ROV,
?$_W$, the wet surface without the stabilisers,
$\bar{l} = \frac{\sqrt{S}}{2l}$, the conicity coefficient,
l is the conical part length.

In our case, C_S can be approximated by:

$$C_s = \frac{0.047 d^{2.5}}{\Omega_w l^{0.5}}$$

Hence the drag force is:

$$R_{x1} = \left(\frac{0.455}{(\log(\mathrm{Re}))^{2.58}} + \frac{0.047 d^{2.5}}{\Omega_w l^{0.5}} \right) \frac{r V^2}{2} \Omega$$

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