ANALYTICAL DESCRIPTION OF AN EXPANDED METAL PLANE STRUCTURE

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Abstract. The analytical description of a structure consisting in an expanded metal plaque is given in the case of an expanding process by triangular knives.

1. Introduction

Expanded Metal is made from sheets of solid metal that are slit and stretched with each stroke of the upper die, forming a raised diamond pattern. This pattern is called regular or standard. The pattern varies by the gauge and type of material and the size of the diamond. The sheets can also be flattened by rollers for an optional style by passing them through a cold, roll reducing mill. In this paper we deal with expanded metal plane standard structures, as in figure 1. They consist in a series of neighbouring cells, having their frontier made out of two types of surfaces: plane surfaces and conic surfaces of parabolic directory curve. We use the example illustrated in figure 1, which is obtained by an expanding process using triangular knives.

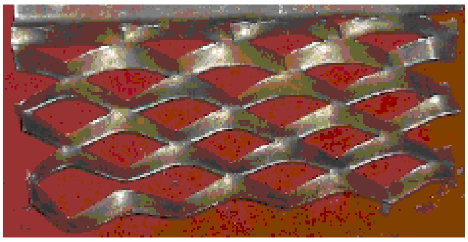

Fig. 1. Plane structure of expanded metal

The other types of expanded metal are easy to study in an analogous manner.

2. The plane surfaces

The plane plaque, which is submitted to the expanding process is described by the equation $z = 0$.

In what follows, we use the following notations: h_x = the step on Ox axis, h_y = the step on Oy, h_z = the step on Oz and q = the semi-distance between the cells, along the directions parallel to the x - axis.

The elements h_x, h_y and q are positive real numbers, while h_z is a negative real number, with respect to the three dimensional coordinates system, as in figure 2. The physical significance of these numbers is:

h_x = the length of a cut plus the length of a bridge between two neighbouring cuts;
h_y = the distance between two rows of successive cuts;
h_z = the vertical distance of penetration of the knife during two successive cuts.
q = half of the distance between two successive cuts, located on the same cuts row.
The graphical representation of the plane plaque submitted to expanding is in figure 2.

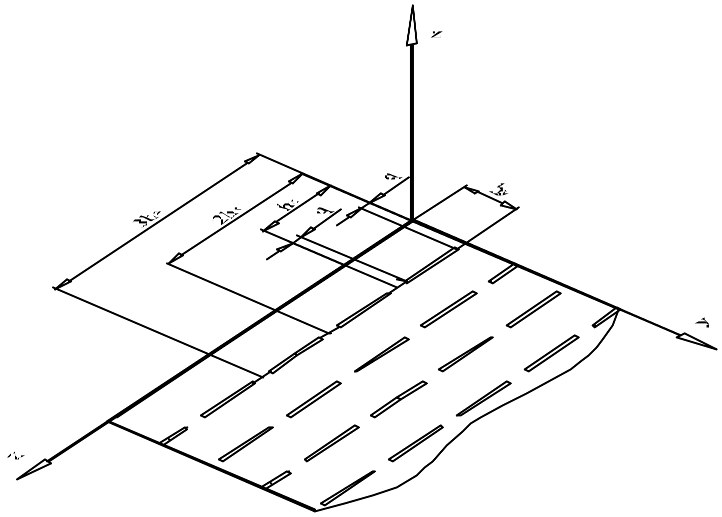

Fig. 2 The plane plaque submitted to the expanding process

In the following, we take into account only one cell resulted after expanding the plaque in figure 2, which results after cutting the straight line segment having the coordinates $((j\text{-}1)h_x, (k+1)h_y)$ and $(jh_x, (k+1)h_y)$ in the plane $z = 0$. Figure 3 depicts this cell.

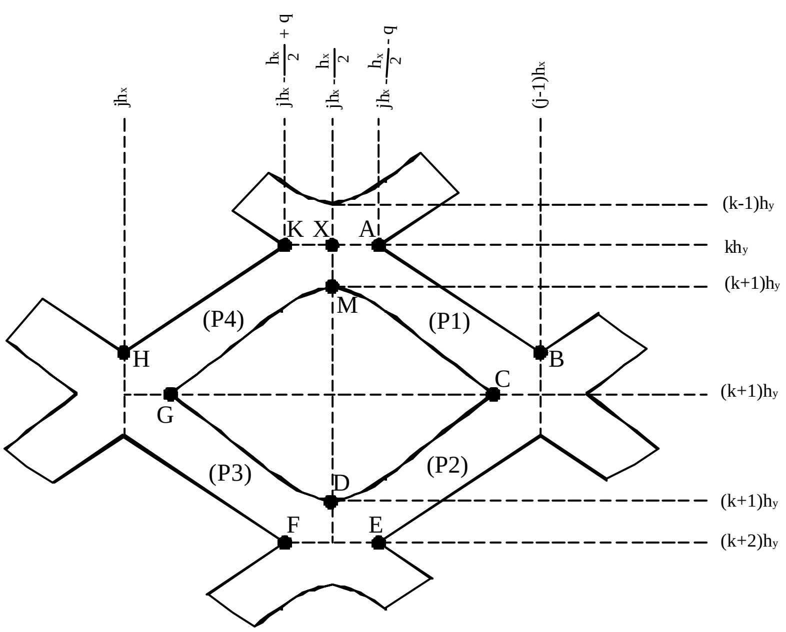

Fig. 3. The expanded metal cell

Consider that, after cutting, the point of x - coordinate $x = jh_x - \frac{h_x}{2}$ from the row having the ordinate $y = kh_y$, has vertically moved with $u = \frac{h_z}{2}$ units, such as the cut on the line y = $(k+1)h_y$ will result in the descent of the point $x = jh_x - \frac{h_x}{2}$ with $h_z - u = \frac{h_z}{2}$ more units.

The cell, which results after expanding, depicted in figure 3, is limited by four planes, denoted by P_1 = (ABC), P_2 = (CDE), P_3 = (DFG) and P_4 = (GHK). Their equations and their directions are derived in the following property.

Property 2.1. *An expanded metal plane structure contains three types of plane surfaces: one is parallel to plane xOy and the other two have the normal vectors* $\overline{N_1} = \overline{N_3}$ *and* $\overline{N_2} = \overline{N_4}$ *where:*

$$\overline{N_1}\left(-\frac{h_z h_y}{2},\ \frac{h_z q}{2},\ h_y\left(q - \frac{h_x}{2}\right)\right),$$

$$\overline{N_2}\left(-\frac{h_z h_y}{2},\ -\frac{q h_z}{2},\ h_y\left(\frac{h_x}{2} - q\right)\right).$$

Proof. *The equation of plane* P_1 is derived noticing that it contains points A, B and C, where:

$$A\left(j \cdot h_x - \frac{h_x}{2} - q,\ k \cdot h_{y},\ \frac{k}{2} \cdot h_z\right),$$

$$B\left((j-1) \cdot h_x,\ k \cdot h_{y},\ \frac{k+1}{2} \cdot h_z\right),$$

$$C\left((j-1) \cdot h_x + q,\ (k+1) \cdot h_{y},\ \frac{k+1}{2} \cdot h_z\right).$$

Then, the determinant form of the equation of this plane is:

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_B - x_A & y_B - y_A & z_B - z_A \\ x_C - x_A & y_C - y_A & z_C - z_A \end{vmatrix} = 0,$$

which becomes, replacing the coordinates and subtracting the second row from the third one:

$$\begin{vmatrix} x - j\cdot h_x + \frac{h_x}{2} + q & y - k\cdot h_y & z - \frac{k}{2}\cdot h_z \\ q - \frac{h_x}{2} & 0 & \frac{h_z}{2} \\ q & h_y & 0 \end{vmatrix} = 0 .$$

After computing the determinant, one gets the equation of plane P_1 as:

$$-\frac{h_z h_y}{2}\cdot x + \frac{h_z q}{2}\cdot y + h_y\left(q - \frac{h_x}{2}\right)\cdot z + \frac{-1-2k}{2} h_y h_z q + \frac{2j+k-1}{4} h_x h_y h_z = 0 \quad (2.1)$$

The normal direction to plane P_1, as it results from (2.1) is:

$$\overline{N_1}\left(-\frac{h_z h_y}{2}, \frac{h_z q}{2}, h_y\left(q - \frac{h_x}{2}\right)\right). \quad (2.2)$$

The equation of plane P_2 is derived observing that it contains points C, D and E, where:

$$C\left((j-1)\cdot h_x + q,\ (k+1)\cdot h_y, \frac{k+1}{2}\cdot h_z\right),$$

$$D\left(j\cdot h_x - \frac{h_x}{2},\ (k+1)\cdot h_{y,}\ \frac{k+2}{2}\cdot h_z\right),$$

$$E\left(j\cdot h_x - \frac{h_x}{2} - q,\ (k+2)\cdot h_{y,}\ \frac{k+2}{2}\cdot h_z\right).$$

As above, the equation of plane P_2 results in:

$$-\frac{h_y h_z}{2}\cdot x - \frac{h_z q}{2}\cdot y + h_y\left(\frac{h_x}{2} - q\right)\cdot z + \frac{2j-k-2}{4} h_x h_y h_z + \frac{2k+3}{2} q h_y h_z = 0. \quad (2.3)$$

The normal direction to plane P_2, as it follows from its equation, is:

$$\overline{N_2}\left(-\frac{h_z h_y}{2}, -\frac{q h_z}{2}, h_y\left(\frac{h_x}{2} - q\right)\right). \quad (2.4)$$

The equation of plane P_3 is determined by the points D, F and G, where:

$$D\left(j\cdot h_x - \frac{h_x}{2},\ (k+1)\cdot h_y,\ \frac{k+2}{2}\cdot h_z\right),$$

$$F\left(j\cdot h_x - \frac{h_x}{2} + q,\ (k+2)\cdot h_{y,}\ \frac{k+2}{2}\cdot h_z\right),$$

$$G\left(j\cdot h_x - q,\ (k+1)\cdot h_y, \frac{k+1}{2}\cdot h_z\right).$$

As above, the equation of P_3 results in:

$$-\frac{h_z h_y}{2}\cdot x+\frac{h_z q}{2}\cdot y-h_y\left(\frac{h_x}{2}-q\right)\cdot z+\frac{2j+k+1}{4}h_x h_y h_z-\frac{2k+3}{2}q h_y h_z=0. \quad (2.5)$$

The normal direction to P_3, is obviously:

$$\overline{N_3}\left(-\frac{h_z h_y}{2},\ \frac{h_z q}{2},\ -h_y\left(\frac{h_x}{2}-q\right)\right). \quad (2.6)$$

The equation of plane P_4 follows if we realise that it contains points G, H and K, where:

$$G\left(j\cdot h_x-q,\ (k+1)\cdot h_y,\frac{k+1}{2}\cdot h_z\right),$$

$$H\left(j\cdot h_x,\ k\cdot h_y,\frac{k+1}{2}\cdot h_z\right),$$

$$K\left(j\cdot h_x-\frac{h_x}{2}+q,\ k\cdot h_y,\ \frac{k}{2}\cdot h_z\right).$$

Then, this plane has the equation:

$$-\frac{h_y h_z}{2}\cdot x-\frac{q h_z}{2}\cdot y+h_y\left(\frac{h_x}{2}-q\right)\cdot z+\frac{2k+1}{2}q h_y h_z+\frac{2j-k-1}{4}h_x h_y h_z=0. \quad (2.7)$$

The normal direction to plane P_4, as it results out of its equation, is:

$$\overline{N_4}\left(-\frac{h_z h_y}{2},-\frac{h_z q}{2},\ h_y\left(\frac{h_x}{2}-q\right)\right). \quad (2.8)$$

Comparing (2.2) and (2.6), one can see that they describe the same direction. Also, the comparison of (2.4) and (2.8) shows that they represent the same direction.

The equation of plane P_5 = (AXM) is:

$$z=\frac{k}{2}h_z. \quad (2.9)$$

The property is now completely proved.

Corollary 2.2. *The equations of the plane surfaces containing the frontier of the expanded metal cell in figure 3 are:*

$$(P_1):\ -\frac{h_z h_y}{2}\cdot x+\frac{h_z q}{2}\cdot y+h_y\left(q-\frac{h_x}{2}\right)\cdot z+\frac{-1-2k}{2}h_y h_z q+\frac{2j+k-1}{4}h_x h_y h_z=0,$$

$$(P_2):\ -\frac{h_y h_z}{2}\cdot x-\frac{h_z q}{2}\cdot y+h_y\left(\frac{h_x}{2}-q\right)\cdot z+\frac{2j-k-2}{4}h_x h_y h_z+\frac{2k+3}{2}q h_y h_z=0,$$

(P3): $-\frac{h_z h_y}{2}\cdot x+\frac{h_z q}{2}\cdot y-h_y\left(\frac{h_x}{2}-q\right)\cdot z+\frac{2j+k+1}{4}h_x h_y h_z-\frac{2k+3}{2}q h_y h_z=0$,

(P4): $-\frac{h_y h_z}{2}\cdot x-\frac{q h_z}{2}\cdot y+h_y\left(\frac{h_x}{2}-q\right)\cdot z+\frac{2k+1}{2}q h_y h_z+\frac{2j-k-1}{4}h_x h_y h_z=0$,

(P5): $z=\frac{k}{2}h_z$.

3. Conic surfaces

Consider the point $M\left(jh_x-\frac{h_x}{2},\ (k+1)h_y,\ \frac{k}{2}h_z\right)$ from figure 3. The plane (P1) does not contain the point M, due to the behaviour of the material during cutting. The result is a conic surface, having the vertex in A and tangent to the horizontal plane (KAM) of equation $z=\frac{k}{2}h_z$ and to plane (P1). The conic surface has the generator AM as the intersection with the tangent plane (KAM). The equation of A is:

$$\frac{x-x_A}{x_M-x_A}=\frac{y-y_A}{y_M-y_A}=\frac{z-z_A}{z_M-z_A},$$

meaning that:

$$\frac{x-jh_x+\frac{h_x}{2}+q}{q}=\frac{y-kh_y}{h_y}=\frac{z-\frac{k}{2}h_z}{0}.$$

AM can be represented as an intersection of two planes, as it follows:

$$\begin{cases} xh_y-qy+\frac{1-2j}{2}h_x h_y+q(k+1)h_y=0 \\ z=\frac{k}{2}h_z \end{cases} \qquad (3.1)$$

In order to write the equation of the conic surface, which has the directory curve a parabola, we need to determine curve. It is situated on the plane $y=(k+1)h_y$, having the vertex in M and containing the points C and G. We denote this parabola by ?, searching its equation as:

$$?:\ \begin{cases} z=a\left[(x+z\ )^2+V\right] \\ y=(k+1)h_y \end{cases} \qquad (3.2)$$

Since the vertex of this parabola is M, one gets:

$$z = -jh_x + \frac{h_x}{2},$$

$$V = \frac{k}{2} h_z.$$

Therefore, formula (3.2) becomes:

$$?: \begin{cases} z = f(x) = a\left[\left(x - jh_x + \frac{h_x}{2}\right)^2 + \frac{k}{2} h_z\right] \\ y = (k+1)h_y \end{cases} \tag{3.3}$$

The equation (3.3) should represent the required parabola.

Since C belongs to this parabola, its coordinates must verify the equation (3.3), meaning that:

$$\frac{k+1}{2} h_z = a\left[\left(q - \frac{h_x}{2}\right)^2 + \frac{k}{2} h_z\right].$$

It follows that

$$a = \frac{2(k+1)h_z}{(2q - h_x)^2 + 2kh_z}.$$

So, the equation of the parabola (3.3) that contains the points C and G is:

$$?: \begin{cases} z = \dfrac{2(k+1)h_z}{(2q - h_x)^2 + 2kh_z}\left[\left(x - jh_x + \frac{h_x}{2}\right)^2 + \frac{k}{2} h_z\right] \\ y = (k+1)h_y \end{cases} \tag{3.4}$$

Property 3.1. *The conic surface having the vertex A and parabola ? as directory curve is represented by the equation*

$$\frac{h_y\left(z - \frac{k}{2}h_z\right)}{y - kh_y} - \frac{\frac{k+1}{2}h_z}{\left(q - \frac{h_x}{2}\right)^2 + \frac{k}{2}h_z}\left(\frac{x - jh_x + \frac{h_x}{2} + q}{y - kh_y}h_y - q\right)^2 =$$

$$= \frac{\frac{k}{2}h_z^2 - \frac{k}{2}\left(q - \frac{h_x}{2}\right)^2 h_z}{\left(q - \frac{h_x}{2}\right)^2 + \frac{k}{2}h_z}.$$

Proof. The conic surface has the vertex in $A\left(j\cdot h_x - \frac{h_x}{2} - q, k\cdot h_{y,}\ \frac{k}{2}\cdot h_z \right)$. This point can be represented as the intersection of three planes parallel to the coordinate planes, as it follows:

$$\begin{cases} x = jh_x - \dfrac{h_x}{2} - q \\ y = kh_y \\ z = \dfrac{k}{2}h_z \end{cases} \quad <=> \quad \begin{cases} x - jh_x + \dfrac{h_x}{2} + q = 0 \\ y - kh_y = 0 \\ z - \dfrac{k}{2}h_z = 0 \end{cases}.$$

The equations of the generators of the cone are:

$$\begin{cases} x - jh_x + \dfrac{h_x}{2} + q = w\left(z - \dfrac{k}{2}h_z \right) \\ y - kh_y = t\left(z - \dfrac{k}{2}h_z \right) \end{cases}, \qquad ?,t\ \hat{I}\ \boldsymbol{R}.$$

The generators are supported by the parabola (3.4), which means that:

$$\begin{cases} x - jh_x + \dfrac{h_x}{2} + q = w\left(z - \dfrac{k}{2}h_z \right) & (3.5) \\ y - kh_y = t\left(z - \dfrac{k}{2}h_z \right) & (3.6) \\ z = \dfrac{\dfrac{(k+1)}{2}h_z}{\left(q - \dfrac{h_x}{2} \right)^2 + \dfrac{k}{2}h_z}\left[\left(x - jh_x + \dfrac{h_x}{2} \right)^2 + \dfrac{k}{2}h_z \right] & (3.7) \\ y = (k+1)h_y & (3.8) \end{cases}$$

Replacing (3.8) in (3.7), one gets:

$$z = \frac{h_y}{t} + \frac{k}{2}h_z. \qquad (3.9)$$

Replacing (3.9) in (3.6), it results:

$$x = jh_x - \frac{h_x}{2} - q + \frac{w}{t}h_y. \qquad (3.10)$$

Now, replacing (3.9) and (3.10) in (3.8), one obtains the compatibility condition for the system of equation, which expresses the property that the generator of the cone is supported by the parabola directory:

$$\frac{h_y}{t}+\frac{k}{2}h_z=\frac{\frac{(k+1)}{2}h_z}{\left(q-\frac{h_x}{2}\right)^2+\frac{k}{2}h_z}\left[\left(\frac{w}{t}h_y-q\right)^2+\frac{k}{2}h_z\right] \qquad (3.11)$$

The low of the movement of the generator is:

$$\frac{h_y}{t}-\frac{\frac{k+1}{2}h_z}{\left(q-\frac{h_x}{2}\right)^2+\frac{k}{2}h_z}\left(\frac{w}{t}h_y-q\right)^2=\frac{\frac{k}{2}h_z^2-\frac{k}{2}\left(q-\frac{h_x}{2}\right)^2h_z}{\left(q-\frac{h_x}{2}\right)^2+\frac{k}{2}h_z}\ . \qquad (3.12)$$

From (3.6) it results:

$$w=\frac{x-jh_x+\frac{h_x}{2}+q}{z-\frac{k}{2}h_z}\ . \qquad (3.13)$$

From (3.7), one gets:

$$t=\frac{y-kh_y}{z-\frac{k}{2}h_z}\ . \qquad (3.14)$$

Finally, the equation of the conic surface having the vertex in A and the directory curve the parabola ? , results by replacing (3.13) and (3.14) in (3.12):

$$\frac{h_y\left(z-\frac{k}{2}h_z\right)}{y-kh_y}-\frac{\frac{k+1}{2}h_z}{\left(q-\frac{h_x}{2}\right)^2+\frac{k}{2}h_z}\left(\frac{x-jh_x+\frac{h_x}{2}+q}{y-kh_y}h_y-q\right)^2=$$

$$=\frac{\frac{k}{2}h_z^2-\frac{k}{2}\left(q-\frac{h_x}{2}\right)^2h_z}{\left(q-\frac{h_x}{2}\right)^2+\frac{k}{2}h_z}. \qquad (3.15)$$

The property is completely proved.

The analytical description from this paper is useful to study the metric properties of the structures of expanded metal and to approach problems of shape optimisation. Also, this description of the grid allows us to perform detailed studies of its physical properties.

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