# CALCULUS METHODOLOGY OF ROTARY TABLES INCLUDED IN POSITIONING AND FEEDING MACHINE TOOLS KINEMATICS CHAINS

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**Abstract:** To realize feeding and positioning in circular motion, many types of machine tools use rotary tables in the structure of boring and milling machine and metal cutting center. Applications are referring at positioning or circular feed kinematics chains. Main stages of calculus for rotary tables that have coupling with face teeth are presented. On the basis of specific data are determined power of the drive motor, design and verification of teeth coupling, hydraulic motor and verification of DC motor included. The methodology offers to mechanical engineers an algorithm to define and dimensioning kinematics chain with rotary table.

### **1. INTRODUCTION**

The rotary tables ( $M_r$ ) are used frequently in the structures of boring and drilling machine, milling machine and metal cutting center. These are realized in different constructive variants with different drive possibilities. Applications are referring at positioning or circular feed kinematics chains. The circular motion of the mobile elements, which support workpiece, is accomplished on horizontal or vertical plane in both senses. Ensemble of  $M_r$  is fixed on the longitudinal saddle of machine tool or it is integrated in this one [1].

From constructive point of view,  $M_r$  are composed by circular casting, carcass, drive kinematics chain (electric motor, belt transmissions, gears), blocking and unblocking mechanisms, coupling, simple joints for circular or straight motion, elements for clearance compensation, position encoder. Some of  $M_r$  have the surfaces which allow guidance and workpiece pallet fixation, when machine tool is integrated into FMS.

DC motors drive most of rotary tables used in the structure of CNC machine tool. The speed of rotary table is  $n_{Mr} = 0,003 \dots 2 \ rpm$  in circular feed motion and  $n_{Mr} \approx 4 \ rpm$  in positioning. The coupling with face teeth is used to realize rigidity and precision of positioning for the rotary table. Angular precision is  $\pm 1" \dots \pm 3"$  for special cases and  $\pm 3" \dots \pm 12"$  for high precision domain.

Blocking mechanism has mechanical action (helical or disc springs) and unblocking linear motion is realized with a hydraulic motor.

### 2. CALCULUS METHODOLOGY

The stages of calculus presented in the table 1 are considering the dimensioning and verification activities of main constructive elements of rotary tables.



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Stages of calculus	Calculus relations	Recommendations	
1	2	3	
1. Total torsion moment on the rotary table shaft <i>M<sub>R</sub></i> ,	$\begin{split} \boldsymbol{M}_{R} &= \boldsymbol{M}_{din} + \boldsymbol{M}_{fr} + \boldsymbol{M}_{st};\\ \boldsymbol{M}_{din} &= \left(\frac{\boldsymbol{m}_{Mr} \cdot \boldsymbol{R}_{Mr}^{2}}{2} + \sum_{i=1}^{n} \boldsymbol{m}_{sfi} \cdot \boldsymbol{R}_{i}^{2}\right) \boldsymbol{\varepsilon}_{a}, \end{split}$	$\mathcal{E}_a = \frac{\mathcal{O}_r}{4};$	
$M_{din}$ – dynamic moment do it by inertial forces, in <i>N</i> · <i>m</i> ; $M_{fr}$ –friction moment into bearing of rotary table, in <i>N</i> · <i>m</i> ; $M_{st}$ –static moment do it by unbalanced masses, in <i>N</i> · <i>m</i> :	where $m_{Mr}$ -mass of rotary table, in $kg$ ; $R_{Mr}$ -rotary table radius, in $m$ ; $m_{sfi}$ -workpieces masses turns by $M_{r}$ , in $kg$ ; $R_{r}$ position of the workpieces	$t_a$ $\omega_r$ –angular stable speed, in <i>rad/s</i> ; $\omega_r$ =0,51 <i>rad/s</i> ; $t_a$ –acceleration time, in <i>s</i> ; $t_a$ =0,20,4 <i>s</i> ;	
$R_1$ $M_R$ $R_n$ $S_{fn}$ $S_{fn}$ $R_{fr}$ $R_{fr}$ $M_r$ Hearth coupling	from turn axes, in <i>m</i> ; $\varepsilon_a$ –angular acceleration or deceleration (the biggest value) of rotary table, in <i>rad/s</i> <sup>2</sup> ; $M_{fr} = \mu (m_{Mr} + \sum_{i=1}^{n} m_{sfi})g \cdot R_{fr}$ , where $\mu$ -friction coefficient; g –gravity acceleration, in <i>m/s</i> <sup>2</sup> ; $R_{fr}$ –friction radius, in <i>m</i> ;	$\mu$ =0,0080,01 for friction bearing case; $\mu$ =0,10,2 for slide friction case;	
$R_{st}$	$M_{st} = m_{sf} \cdot g \cdot R_{st}$ , where $m_{sf}$ -workpiece mass in unbalanced position, in $kg$ ; $R_{st}$ -length between workpiece	<i>M<sub>st</sub></i> is considered only for rotary table with horizontal axis;	
	and foldry axis, in <i>m</i> ,		
2. Power of the motor drive $P_a$ , in $kW$ ; $F_y$ $F_x$ $M_r$ $M_r$ $M_r$ $M_E$ $M_E$	$P_{a} = \frac{M_{R} \cdot n_{Mr}}{9550 \cdot \eta},$ where $n_{Mr}$ -stable speed of the rotary table, in <i>rot/min</i> ; $n_{Mr} = 30\omega_{r} / \pi$ ; $\eta$ -output of the kinematics chain for rotary table turning;	$\eta$ =0,780,92 (product of the outputs of the kinematics chain mechanisms).	

Table 1 Calculus methodology

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1	2	3
· ·	- Tangential force calculus:	
<b>3. Hearth coupling calculus</b> -Outside diameter has	$F_t = M_{cp} / R_m$ , where $F_t$ is tangential force according $M_{cp}$ moment, in $N$ ; $M_{cp}$ –total moment of cutting process, in $N \cdot m$ ; $R_m$ –average radius of the coupling, in $m$ ; $f_t = F_t / z$ ,	Diameter series: 20, 25, 28, 32, 36, 40, 45, 48, 50, 56, 63, 71;
the maximal value normalized up to outside diameter of the rotary table;	where <i>f<sub>t</sub></i> is tangential force transmitted by one tooth, in <i>N</i> ; <i>z</i> –teeth number of the coupling; - <i>Initial force calculus</i> :	<i>z</i> =24240;
-Verifying the Hearth coupling:	$F_a = (1,52) F_t \frac{\sin 2\alpha}{2},$ where <i>E</i> is initial axial force in <i>N</i> :	
-Torsion stress calculus; -Initial force	$\alpha$ -inclination angle of teeth, in <i>deg</i> ; $f_a = F_a / z$ , where $f_a$ is axial force transmitted by one tooth, in <i>N</i> ;	
-Verification of screws coupling fixing; -Verification of	-Verification of screws coupling fixing: $\tau_{fs} = F_t / (n \cdot A_1);$ $\tau_{fs} \le \tau_a;$ where $\tau_{fs}$ is screw cutting stress,	<i>n</i> = 818;
contact pressure on the teeth; -Verification of the base tooth bending.	in $N/mm^2$ ; <i>n</i> –number of screws; $A_1$ –single screw surface section, in $mm^2$ ; $\tau_a$ -accepted stress, in $N/mm^2$ ;	τ <sub>a</sub> =4060 <i>N/mm</i> ²;
	-Verification of contact pressure on the	
	<i>teeth</i> : $p = \frac{\frac{F_a}{2 \cdot \tan \alpha} + \frac{M_{cp}}{2 \cdot R_m}}{2(R - r) \cdot 0,68 h_t \cdot c_p} \le p_a,$ where <i>p</i> -contact pressure, in <i>N/mm</i> <sup>2</sup> ; <i>R</i> , <i>r</i> -outside, inside radius of teeth, in <i>mm</i> ; <i>h</i> <sub>t</sub> -high of tooth, in <i>mm</i> ; <i>c</i> <sub>p</sub> -real contact spot, in %; <i>p</i> <sub>a</sub> -accepted pressure, in <i>N/mm</i> <sup>2</sup> :	c <sub>p</sub> =4060%; p <sub>a</sub> =80120 N/mm <sup>2</sup> ;
Reference teeth angle ( $\alpha$ ):	-Verification of the base tooth bending: $\sigma_{i} = \frac{6 \cdot f_{i} \cdot h_{i} \cdot z^{2}}{\pi \cdot (R+r) \cdot (R-r)^{2}} \leq \sigma_{ai};$ where $\sigma_{i}$ is real bending stress, in <i>N/mm</i> <sup>2</sup> ; $\sigma_{ai}$ –accepted bending stress, in <i>N/mm</i> <sup>2</sup> ;	$\sigma_{ai}$ = 4050 <i>N/mm</i> <sup>2</sup> for carbon steel; $\sigma_{ai}$ =90140 <i>N/mm</i> <sup>2</sup> for alloy steel case;

## Table 1 (continuation)

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Table 1 (continuation)

1	2	3		
	-Total assurance axial force ( $F_{ts}$ ); $F_{ts} = F_a \cdot c_a$ , where $c_a$ –safety coefficient;	<i>c<sub>a</sub></i> =1,52;		
<b>4. Fixing system calculus</b> with STAS 8216-80 considering.	-Disc springs are fixed initial at the deformation $f_1 = (0,30,5) \cdot h$ , where <i>h</i> is maximal deformation of the single spring; -Load on single disc spring, that corresponding of the deformation $f_1$ is:			
	$F_1 = \frac{4E}{1-\mu^2} \cdot \frac{s^4}{\alpha D_e^2} \cdot \frac{f_1}{s} \cdot \left[ \left( \frac{h}{s} - \frac{3f_1}{2s} \right) + 1 \right],$	$E=2,1.10^{\circ} N/mm^{2};$ $\mu =0,3;$ $\frac{4E}{23.10^{\circ}}=9.23.10^{\circ}$		
Disc springs; Main dimensions:	springs; nensions: where <i>E</i> is Young modulus, in <i>N/mm</i> <sup>2</sup> ; $\mu$ – Poisson coefficient; <i>s</i> –thickness of disc spring, in <i>mm</i> ;			
$b_e$	$\alpha = \frac{\left[ (\delta - 1) / \delta \right]^2}{\pi \left( \frac{\delta + 1}{\delta - 1} - \frac{2}{\ln \delta} \right)} \text{ where } \delta = \frac{D_e}{D_i};$			
Charge diagram:	$D_e$ –outside diameter of spring, in <i>mm</i> ; $D_i$ –inside diameter of spring, in <i>mm</i> ; -Number of springs in group is according to relation $n_a ≥ F_{ts} / F_1$ , where $n_a$ is rounded as first integer value; -Number of spring groups to be able that assure linear displacement of hydraulic motor and coupling opening: $i = (h_t + h_s) / (f - f_1)$ ,	<i>h<sub>s</sub></i> =13 <i>mm</i> ; <i>f</i> is according with STAS 8216-80		
	$I = (h_t + h_s) / (t - t_1),$ where $h_t$ is high of the teeth Hearth coupling, in $mm$ ; $h_s$ –security high for coupling opening, in $mm$ ; $f$ –displacement of spring that corresponding at 0,75· $h$ compression, in $mm$ ; -Free length of spring groups: $L_0 = i [l_0 + (n_a - 1)s],$ where $l_0$ is free length for single spring, in mm; -Length of the spring groups when the springs are compressed to 0,75· $h$ is: $L = L_0 - i \cdot f$ ; -Length of the spring groups after hydraulic motor assembly: $L_2 = L_0 - i \cdot f_1;$		<i>i</i> is superior rounded as integer value;	

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Table 1 (continuation)

1	1 2		
5. Hydraulic motor drive calculus	-Force for Hearth coupling opening ( $F_H$ ), in $N$ : $F_H \ge (m_{Mr} + m_{sf})g + F_{fr} + n_a \cdot F$ , where $F_{fr}$ is friction force into hydraulic motor, in $N$ ; $F$ –standardized force when the spring is compressed to 0,75· $h$ ;		
Main dimensions:	-Diameter of the hydraulic cylinder ( <i>D</i> ), in <i>mm</i> : $D = \sqrt{\frac{4F_H}{p_H \cdot \pi}}, \text{ where } p_H \text{ is}$ recommended work pressure for auxiliary chain drive, in <i>N/mm</i> <sup>2</sup> .	р <sub>н</sub> =25 N/mm <sup>2</sup> ;	
d D Using example:	-Thickness of hydraulic cylinder ( $\delta_H$ ), in <i>mm</i> : $\delta_H \ge \frac{p_H \cdot D}{2 \cdot \sigma_{at}}$ , where $\sigma_{at}$ is admitted stress for traction solicitation, in <i>N/mm</i> <sup>2</sup> ;	σ <sub>at</sub> =110130 <i>N/mm</i> ²;	
$Z_2$ Mr $S_T$ $Z_3$ MHL $p_{H}$ $Z_1$ $M_E$	-Necessary flow for hydraulic pump ( $Q_p$ ), in <i>l/min</i> : $Q_p = (1,11,15) \cdot 10^{-3} \cdot v_H \cdot \frac{\pi D^2}{4}$ , where $v_H$ is speed value for hydraulic motor drive, in <i>m/min</i> ;	v <sub>H</sub> =24 m/min;	
	-Diameter of pipe ( $d_c$ ), in <i>mm</i> : $d_c = \sqrt{4 \cdot 10^2 Q_p / (v_o \cdot \pi)}$ , where $v_o$ is speed of the oil into pipes system; -Tank volume, $V=4Q_p$ ; -Power of the electric motor for	v <sub>o</sub> =120240 m/min;	
	pump drive ( $P_{ME}$ ), in <i>kW</i> : $P_{ME} = \frac{1}{60} \cdot p_H \cdot Q_p$ .		

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2 3 1 -Kinematics values verification:  $n_{\min}^{ME} \leq n_{Mr} / i_R \leq n_{\max}^{ME}$  , where  $n_{\min}^{ME}$  and  $n_{\max}^{ME}$  are minimal 6. Verification of electric motor for kinematics chain speed of the motor drive and maximal drive speed respectively:  $i_R$  –kinematics transfer ratio for all mechanisms between electric motor and rotary table; Kinematics chain example: -Nominal torque verification  $(M_n^{ME})$ , in  $N \cdot m$ :  $J_{Sf}$ Ш  $M_n^{ME} \ge k_s \cdot (2-\eta) \cdot M_R \cdot i_R;$  $J_{p}$  $k_{\rm s} = 1, 1...1, 2;$ where  $k_s$  is the safety coefficient; -Starting torgue verification  $J_{z3}$  $(M_p^{ME})$ , in  $N \cdot m$ ):  $M_{p}^{ME} = \left[J_{I} + J_{m} + (J^{II} + J^{III} \cdot i_{2}^{2}) i_{1}^{2}\right] \cdot \frac{\pi n_{ME}}{30 \cdot t_{a}^{ME}}$ For an cylindrical MCC body:  $J_{i}$ where  $n_{ME}$  and  $t_a^{ME}$  are speed and J =*m⋅*R<sup>2</sup>/ 2, where acceleration time characteristics of  $i_1 = k / z_{rm}$ , where k is m is the mass and R electric motor; outside radius of number of threadlike teeth  $J^{\prime\prime}$  -inertial moment of axis II; cylinder. for worm gear; *z<sub>rm</sub>* –number of tooth for worm wheel;  $J^{II} = J_{z2} + J_{rm} + J_{II};$  $M_{p}^{ME}$  must to be  $i_2 = z_2 / z_3$ , where  $z_2$  and  $z_3$  $J_{z2}$  -inertial moment of  $z_2$  pinion,  $J_{rm}$  – are numbers of teeth for inertial moment of worm wheel,  $J_{ll}$  lowest than folder pinion and gear in spur inertial moment of shaft II; value of the motor gears.  $J^{III}$  is inertial moment of axis III; drive.  $J^{III} = J_{z3} + J_p + J_{Sf};$  $J_{z3}$  –inertial moment of  $z_3$  wheel,  $J_p$  – inertial moment of rotary table part,  $J_{Sf}$ -inertial moment of workpiece considering axis III that reference.

### Table 1 (continuation)

### 3. CALCULUS EXAMPLE

Horizontally rotary table with following characteristics:  $\omega_r = 1.5 \text{ rad/s}; t_a = 0,25 \text{ s}; R_{Mr} = 0.6 \text{ m}; m_{Mr} = 250 \text{ kg}; m_{Sf} = 50 \text{ kg}; R_i = 0.5 \text{ m};$  **Total torsion moment** on the rotary table shaft ( $M_R$ );  $M_{din} = 431.2 \text{ N·m}; \mu = 0,01; R_{fr} = 100 \text{ mm}; g = 9,81 \text{ m/s}^2; M_{fr} = 2,9 \text{ N·m}; M_R = 434.1 \text{ N·m}.$ **Power of the motor drive** ( $P_a$ );  $\eta = 0,8; n_{Mr} = 30 \omega_r / \pi = 14,3 \text{ rot/min}; P_a = 0,81 \text{ kW};$ 

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*Hearth coupling data*; *z*=144; *h*<sub>t</sub>=5,16 *mm*;  $\alpha$ =30°; *R*=280 *mm*; *r*=225 *mm*; *M*<sub>cp</sub>=900 *N*·*m*; -Tangential force calculus:  $R_m$ =(*R*+*r*)/2=252.5 *mm*; *F*<sub>t</sub>=3564 *N*; *f*<sub>t</sub>=24,7 *N*;

-Initial force calculus:  $F_a$ =2700 N; n=10 screws M10;  $f_a$ =18,7 N;

-Verification of screws coupling fixing:  $\tau_{fs}$ =3,4 *N/mm*<sup>2</sup> <40 *N/mm*<sup>2</sup>;

-Verification of contact pressure on the teeth:  $c_p=40\%$ ; p=14,3 N/mm<sup>2</sup> <80 N/mm<sup>2</sup>;

-Verification of the base tooth bending:  $\sigma_i$ =3,2 *N/mm*<sup>2</sup> <40 *N/mm*<sup>2</sup>;

*Fixing system calculus*; disc springs B80 with following characteristics:  $D_e$ =80 mm;  $D_i$ =41mm; s=3,0 mm; h=2,3 mm;  $l_0$ =5,3 mm; F=10700 N;

-Total assurance axial force  $F_{ts}$ = 4725 *N*; safety coefficient,  $c_a$ =1,75; initial deformation,  $f_1$  = 0,3·*h* =0,69 *mm*;

-Load on single disc spring,  $F_1$ =2430 N; number of springs in group,  $n_a$ =2; number of groups *i*=7; Lengths of spring groups:  $L_0$ =6 mm; L=46 mm;  $L_2$ =53 mm;

Hydraulic motor drive calculus;  $F_{fr}$ =20 N;  $p_H$ =2,0 N/mm<sup>2</sup>;  $\sigma_{at}$ =110 N/mm<sup>2</sup>;

-Force for Hearth coupling opening,  $F_H$ =24500 N;

-Diameter of the hydraulic cylinder *D*=120 *mm*; thickness of hydraulic cylinder  $\delta_H$ =3 *mm* (≥1,4 *mm*); diameter of pipe  $d_c$ =6 *mm*;  $v_H$ =3 *mm/min*.

-Necessary flow for hydraulic pump,  $Q_p \ge 25,9 \ l/min$ ; power of the electric motor for pump drive  $P_{ME}$ =0,86 kW;

**DC motor drive** of rotary table:  $i_R = i_1 \cdot i_2 = 1/30 \cdot 1/10 = 1/300$ ;  $M_n^{ME} \ge 2 N \cdot m$ ;  $M_n^{ME} \ge 4 N \cdot m$ .



For the above case studied, was defined a mathematical function for the parameter  $M_R$ in the case of using  $m_{Mr}=250 \ kg, \ R_i=R_{Mr}$ 0.1;  $\varepsilon_a = 7.5 \ m/s^2; \mu =$  $0.01; R_{fr} = 0.1 m.$ Using the software Mathcad Professional, in the figure 1 is presented the influence of the radius of rotary table (x scale) on the variation of the total torsion moment  $M_R$ .(function f(x)).

Fig.1 Total torsion moment

In the table 2 are presented main characteristics for an example of DC motors to drive the rotary table.

Rate speed	Rated output <i>P<sub>N</sub></i>	Rated Torque <i>M</i> <sub>N</sub>	Maximum speed	Order No.	Rated current I <sub>N</sub>	Efficiency	Armature resistance at 120°C
1170 rpm	1,09 kW	8,0 N·m	1400 rpm	1G#5 102-0 ED	5,4 A	63%	10.7 Ω

Table 2. Characteristics of the Siemens DC motors 1G5

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## 4. CAD OF ROTARY TABLE

On the basis of relations exposed in the table 1, were conceived software calculus programs to design the main elements founded in the structure of rotary table. Were considered a DC motor drive, the coupling, the disc springs and hydraulic motor.

The ensemble drawing of rotary table is presented in the figure 2 and it was obtained using AutoCAD release 14.



Fig. 2 Rotary table

## **5. CONCLUSIONS**

The methodology presented offers to mechanical designers a unitary algorithm for defines positioning kinematics chain that is using rotary table. At the end of calculus example were resulted consistent results, according with those from rotary table, which equips a real machine tool. In order to obtaining final dimensions for ensemble is necessary to take into account-standardized values. These are chosen using constructive criterion or technological and fitting criteria.

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