ON THE STABILITY OF THE CENTRELESS GRINDING PROCESS

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Keywords: grinding, stability, rigidity, waveform

Abstract
In this paper we shall discuss the problem of the stability for the centreless grinding. We obtain the equation of the machine tool's response and the equation, which defines the dynamic depth of cut. We also present the characteristic equation of the centreless grinding, written for the repulsive force. In the paper the reader can also find a numerical application. Finally, we present method to increase the zone of the work-piece's velocities for which the dynamic rigidity is a positive one, and also the minimal value needed for the residual flexibility.

FORMULATION OF THE PROBLEM

We shall study the dynamical behavior of a centreless grinding machine tool drawn schematically in Figure 1. We made the following notations:

– φ – the angle formed by the mobile axis Ox, which coincides with OB in the initial position, and OB, where B is the contact point between the work-piece and its support, and, O is the center of the work-piece;

– φ_1, φ_2 – the angles between OB and the lines of the centers OO_1 and respectively OO_2, with O_1 the center of the grinding wheel and O_2 the center of the driving wheel;

– ω_w, ω_1, ω_2 – the angular speeds of the work-piece, grinding wheel and driving wheel, respectively;

– θ – the angle formed by the work-piece's support with the horizontal direction;

– β, α – the angles formed by the lines OO_1 and OO_2 with the horizontal direction;

– R_1, R_2 – the radii of the grinding wheel and driving wheel, respectively.

The wave form of the work-piece $r_w(\varphi)$ can be described as a function of its value both at its previous revolution, $r_w(\varphi-2\pi)$, and of depth of cut, $t_w(\varphi)$,

$$r_w(\varphi)=r_w(\varphi-2\pi)-t_w(\varphi). \tag{1}$$

Considering now both the waveform of the work-piece at the contact point of the technological system, and the work-piece's deformation, the depth of cut reads

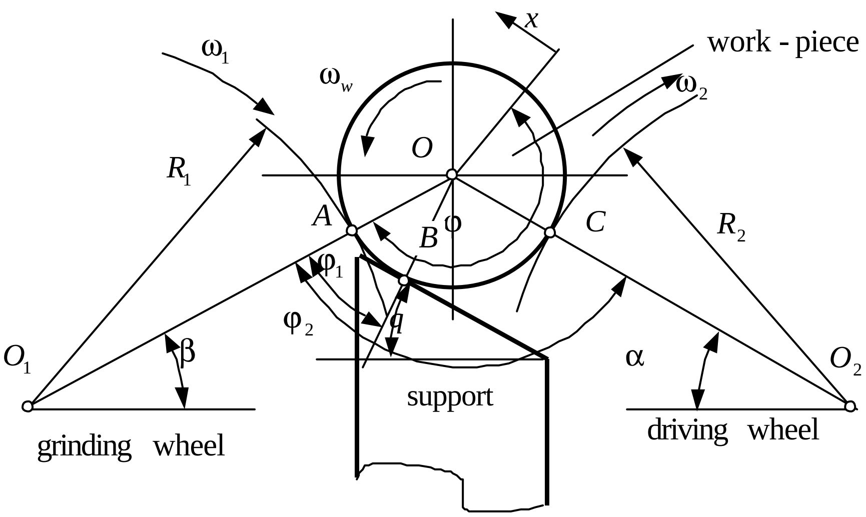

Fig. 1. The model for the centreless grinding

$$t_w(\varphi) = r_w(\varphi - 2\pi) - \varepsilon' r_w(\varphi - \varphi_1) + (1-\varepsilon) r_w(\varphi - \varphi_2) - s_i(\varphi), \tag{2}$$

where ε' and $1-\varepsilon$ represent the ratios between the depth of cut and the amplitudes of the work-piece's wave form at the points B and C respectively, and by $s_i(\varphi)$ we marked the variation of the relative distance between the work-piece and the grinding wheel, variation due to the structural flexibility of the technological system. From Figure 1, one can write the following relations:

$$\varepsilon' = \frac{\sin(\alpha+\beta)}{\cos(\theta-\alpha)} = \frac{\sin \varphi_2}{\sin(\varphi_2 - \varphi_1)}, \tag{3}$$

$$1-\varepsilon = \frac{\cos(\theta+\beta)}{\cos(\theta-\alpha)} = \frac{\sin \varphi_1}{\sin(\varphi_2 - \varphi_1)}. \tag{4}$$

MACHINE TOOL'S RESPONSE

Applying now the Laplace transformation to equations (1) and (2), we obtain:

$$R_w(s) = R_w(s) e^{-2\pi \frac{s}{\omega_w}} - T_w(s), \tag{5}$$

$$T_w(s) = R_w(s) e^{-2\pi \frac{s}{\omega_w}} - \varepsilon' R_w(s) e^{-\varphi_1 \frac{s}{\omega_w}} + (1-\varepsilon) R_w(s) e^{-\varphi_2 \frac{s}{\omega_w}} - S_i(s). \tag{6}$$

In relations (5) and (6) we marked by s the complex variable attached to Laplace transformation. Replacing now the relation (5) in relation (6), results

$$R_w(s) - \varepsilon' R_w(s) e^{-\varphi_1 \frac{s}{\omega_w}} + (1-\varepsilon) R_w(s) e^{-\varphi_2 \frac{s}{\omega_w}} - S_i(s) = 0. \tag{7}$$

The response of the grinding machine can be written

$$\{\mathbf{X}\} = [\mathbf{h}(s)]\{\mathbf{F}\}, \tag{8}$$

where $\{\mathbf{X}\}$ and $\{\mathbf{F}\}$ are the vectors displacement and force, respectively, in two orthogonal directions at the points between the work-piece and machine-tool, and $[\mathbf{h}(s)]$ is the transfer matrix given by

$$[\mathbf{h}(s)] = \sum_{r=1}^{N_m} \frac{\{\mathbf{F}_{\mathbf{r}}\}\{\mathbf{F}_{\mathbf{r}}\}^T}{m_r\left(s^2 + \omega_r^2 + 2\xi_r \omega_r s\right)}. \tag{9}$$

In relation (9), $\{\mathbf{F}_{\mathbf{r}}\}$ defines the r th mode of vibration, N_m represent the number of vibration modes taken into account, and m_r, ω_r and ξ_r marks the modal mass, pulsation and viscous damping corresponding to the r th mode of vibration, respectively. Vector $\{\mathbf{F}\}$ is given by the expression

$$\{\mathbf{F}\} = \{\mathbf{P}\} F_n , \tag{10}$$

where F_n is the normal component of the grinding force, and $\{\mathbf{P}\}$ is a vector, which links the components of the forces at other points and in other directions and the normal component of the grinding force.

THE DYNAMIC DEPTH OF CUT

The variation of the depth of cut due to the elastic deformation of the machine tool can be written as

$$S_i(s) = \{\mathbf{C}\}^T \{\mathbf{X}\} , \tag{11}$$

$\{\mathbf{C}\}^T$ being a vector of geometric relations between the displacements of the machine tool at the contact point and the variation of the depth of cut. Assuming that between the components of the cutting force and the depth of cut there exist linear relations, then the grinding normal force can be expressed as

$$F_n = K T_w(s) , \tag{12}$$

where K is the rigidity of the grinding process.

THE CHARACTERISTIC EQUATION

From relations (8), (10), (11) and (12) we find

$$S_i(s) = \{\mathbf{C}\}^T [\mathbf{h}(s)] \{\mathbf{P}\} K T_w(s) . \tag{13}$$

Replacing $[\mathbf{h}(s)]$ from relation (9) and $T_w(s)$ from relation (5) in expression (12), results the following expression

$$S_i(s) = K \sum_{r=1}^{N_m} \left[\frac{\frac{1}{m_r} \{\mathbf{C}\}^T \{\mathbf{F}_{\mathbf{r}}\} \{\mathbf{F}_{\mathbf{r}}\}^T \{\mathbf{P}\}}{s^2 + \omega_r^2 + 2\xi_r \omega_r s} R_w(s) \left(e^{-2\pi \frac{s}{\omega_w}} - 1 \right) \right] . \tag{14}$$

One obtains the characteristic equation of the centreless grinding written for the repulsive force,

$$\begin{aligned} R_w(s) = {} & \varepsilon' R_w(s) e^{-\varphi_1 \frac{s}{\omega_w}} - (1-\varepsilon) R_w(s) e^{-\varphi_2 \frac{s}{\omega_w}} + \\ & + K \sum_{r=1}^{N_m} \left[\frac{\frac{1}{m_r} \{\mathbf{C}\}^T \{\mathbf{F}_{\mathbf{r}}\} \{\mathbf{F}_{\mathbf{r}}\}^T \{\mathbf{P}\}}{s^2 + \omega_r^2 + 2\xi_r \omega_r s} R_p(s) \left(e^{-2\pi \frac{s}{\omega_w}} - 1 \right) \right] . \end{aligned} \tag{15}$$

We shall select K as a parameter and we shall look for pure imaginary solutions for equation (15),

$$s = i n \omega_w \tag{16}$$

where i is the imaginary unit, $i^2 = -1$, and n is a positive real number defining the number of waves. One obtains the relation

$$1=\varepsilon' e^{-in\varphi_1}-(1-\varepsilon)e^{-in\varphi_2}+K\sum_{r=1}^{N_m}\left[\frac{\frac{1}{m_r}\{\mathbf{C}\}^T\{\mathbf{F}_{\mathbf{r}}\}\{\mathbf{F}_{\mathbf{r}}\}^T\{\mathbf{P}\}}{-n^2\omega_p^2+\omega_r^2+2\xi_r\omega_r(in\omega_p)}\left(e^{-2\pi in}-1\right)\right]. \quad (17)$$

We shall also assume that the structure has no damping, i.e.

$$\xi_r=0 \quad (18)$$

for $r=\overline{1,N_m}$. In addition, we denote

$$\frac{1}{K_D(\omega)}=\sum_{r=1}^{N_m}\frac{\frac{1}{m_r}\{\mathbf{C}\}^T\{\mathbf{F}_{\mathbf{r}}\}\{\mathbf{F}_{\mathbf{r}}\}^T\{\mathbf{P}\}}{\omega_r^2-\omega^2}. \quad (19)$$

From relation (19) keeping into account that the parameter K take the critical value denoted K_{cr} , it follows the equation

$$1=\varepsilon' e^{-in\varphi_1}-(1-\varepsilon)e^{-in\varphi_2}+\frac{K_{cr}}{K_D(n\omega_w)}\left(e^{-2\pi in}-1\right). \quad (20)$$

Relation (20), separating the real and imaginary parts, leads us to

$$1-\varepsilon'\cos(n\varphi_1)+(1-\varepsilon)\cos(n\varphi_2)=\frac{K_{cr}}{K_D(n\omega_w)}\left[\cos(2\pi n)-1\right]; \quad (21')$$

$$\varepsilon'\sin(n\varphi_1)-(1-\varepsilon)\sin(n\varphi_2)=-\frac{K_{cr}}{K_D(n\omega_w)}\sin(2\pi n). \quad (21'')$$

In system (21) we shall now consider that n is an integer number. In this situation the two equations take the forms

$$1-\varepsilon'\cos(n\varphi_1)+(1-\varepsilon)\cos(n\varphi_2)=0;\ \varepsilon'\sin(n\varphi_1)-(1-\varepsilon)\sin(n\varphi_2)=0. \quad (22)$$

System (22) has solution only if

$$\varphi_2=\frac{\lambda_2}{\lambda_1}\varphi_1, \quad (23)$$

with λ_1, λ_2 integers. The number of waves is given by

$$n=1+\frac{2\pi\lambda_2}{\varphi_1}=1+\frac{2\pi\lambda_1}{\varphi_2}. \quad (24)$$

The equations of the system (22) are identical to those obtained for a rigid machine tool and they correspond to the so-called geometric problem. Practically, we take into account only integer numbers, which lead to a small number of waves. Therefore, the number of waves is independent upon the value of the parameter. The geometric errors of the work-piece corresponding to this number of waves can not be reduced. In such conditions, the regeneration of the form defects is possible without the aid of the dynamic cutting forces, and the process becomes independent upon both the dynamic characteristic of the machine tool and the rigidity of the cut.

APPLICATION

Determine the number of waves in the case of small values for the angle θ, and $\varphi_1=\frac{\pi}{72}[\text{rad}]$.

Solution. If angle θ has small values, then

$$\varphi_2=2\varphi_1 \quad (i)$$

and we obtain:

$$\lambda_1 = 2;\ \lambda_2 = 1. \qquad \text{(ii)}$$

It follows

$$n = 1 + \frac{2\pi\lambda_2}{\varphi_1} = 1 + \frac{2\pi \cdot 1}{\frac{2\pi}{5}} = 1 + 5 = 6. \qquad \text{(iii)}$$

DYNAMIC RIGIDITY AND RESIDUAL FLEXIBILITY

The best stability is obtained wen the dynamic rigidity of the machine tool, $K_D(n_r\omega_w)$, is positive for any frequency. For the angular speed of the work-piece for which

$$\omega_w > \frac{\omega_0}{n_r}, \qquad (25)$$

for which the rigidity is negative, the system is always unstable for roots associated to positive coefficients $C(n_r)$. The increasing of the zone where the dynamic rigidity is positive can be made in two ways:

– increasing the pulsation ω_r;

– increasing the residual flexibility of the machine-tool structure.

First way leads to an increasing of the stability for a small number of waves; the effect is not satisfactory for a greater number of waves. The second way has a satisfactory effect no matter the number of waves. We shall write the dynamic flexibility of the machine tool as

$$\frac{1}{K_D(\omega)} = \frac{1}{K_0\left(1 - \frac{\omega^2}{\omega_0^2}\right)} + \frac{1}{K_R}, \qquad (26)$$

where K_0 appears from the first mode of vibration, and K_R appear from the others mode of vibration. The conditions asked to obtain intervals of angular speeds without roots which lead to instability, impose

$$\frac{\omega_s}{n_r} \le \frac{\omega_0}{n_r - 1}, \qquad (27)$$

where ω_s is the superior limit at which the dynamic rigidity becomes negative. If we shall consider two consecutive roots, n_{r-1} and n_r, the previous inequality offers us the minimal value for the residual flexibility,

$$\frac{1}{K_R} \ge \frac{1}{K_0} \frac{1}{\left(\frac{n_r}{n_{r-1}}\right)^2 - 1}. \qquad (28)$$

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