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ON THE DYNAMICS OF AN INDUSTRIAL EQUIPMENT WITH SPEED VARIATOR

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Abstract. In the paper it is studied the dynamics of an industrial equipment, composed by an electric motor which drives the working machine through a speed variator. For a linear law of control of the transmission ratio and for a linear variation of the resistant moment, it is determined the law of motion of the driving shaft of the speed variator. It is also established the analogy between the behavior of the speed variator within the framework of the industrial equipment and a system of automatic control.

1. INTRODUCTION

It is considered an industrial equipment, whose dynamic model has the structure in figure 1, where:

- I – driving electric motor;
- II – speed variator;
- III – working machine.

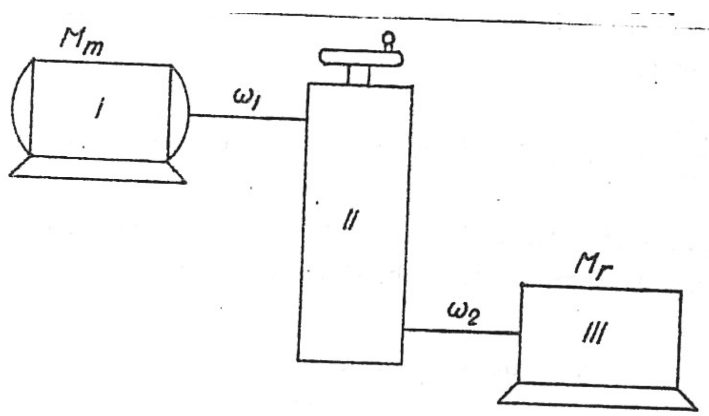

Fig. 1. Dynamic model of industrial equipment

2. DIFFERENTIAL EQUATION OF MOTION

The differential equation of motion of the industrial equipment is, [2],

$$(J_1 i^2 + J_2)\frac{d^2\theta}{dt^2} + J_1 i \frac{di}{dt}\frac{d\theta}{dt} = M_m i - M_r, \tag{1}$$

where:

- J_1 - moment of inertia, resulted by reducing, at the driving shaft of speed variator, of the rotor of motor and driving element of variator;

- J_2 - moment of inertia, obtained by reducing, at the driven shaft of speed variator, of the driven part of variator and working machine;
- $i = \frac{\omega_1}{\omega_2}$ - transmission ratio of speed variator;
- ω_1, ω_2 - angular velocities of two shafts;
- θ - angle of rotation of driven shaft of speed variator;
- M_m - driving moment, produced by the electric motor;
- M_r - resistant moment of working machine.

About the parameters of industrial equipment, there are made the fallowing mentions:
- the moments of inertia J_1 and J_2 are constant;
- the transmission ratio is a linear function of time,

$$i = a + bt, \tag{2}$$

where *a* is the initial value of transmission ratio, and *b* is the inclination of characteristic, figure 2;

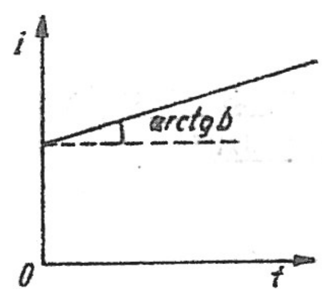

Fig. 2. Dependence on time of the transmission ratio

- the driving moment is constant;
- the resistant moment is a linear function of position,

$$M_r = p + q\theta, \tag{3}$$

where *p* and *q* are positive constants; if the resistant moment is not a linear function of position, the study can be made by linearization on portions the load diagram.

By introducing the expressions (2) and (3) in the differential equation (1), this one becomes

$$[J_1(a+bt)^2]\frac{d^2\theta}{dt^2} + J_1 b(a+bt)\frac{d\theta}{dt} + q\theta = M(a+bt) - p. \tag{4}$$

3. DETERMINATION OF LAW OF MOTION

The differential equation (4) comes to an exact analytical integration by the changing of variable [1]

$$a + bt = \frac{1}{n} shnz, \tag{5}$$

where z is the new variable, and $n = \sqrt{\frac{J_1}{J_2}}$. With the help of this changing of variable, in the equation (4), the term containing the derivative of first order disappears. By introducing, also, the notations $\frac{q}{b^2 J_2} = \Omega^2$, $\frac{M_m}{b^2 J_2} = F$, $\frac{p}{b^2 J_2} = r$, the equation (4) takes the form

$$\frac{d^2\theta}{dz^2} + \Omega^2\theta = F\frac{shnz}{n} - r. \tag{6}$$

The general solution of the equation (6) is

$$\theta = A\cos\Omega z + B\sin\Omega z + F\frac{shnz}{(\Omega^2 + n^2)n} - \frac{r}{\Omega^2}. \tag{7}$$

By returning to the variable *t,* by the substitution (5), the solution (7) becomes

$$\theta = A\cos\frac{\Omega}{n} arg\, shn(a + bt) + B\sin\frac{\Omega}{n} arg\, shn(a + bt) + F\frac{a + bt}{\Omega^2 + n^2} - \frac{r}{\Omega^2}. \tag{8}$$

The constants of integration *A* and *B* are determined from the initial conditions, $t = 0,\ \theta = \theta_0,\ \omega_2 = \omega_{20}$, where:

$$\begin{aligned} \omega_2 = \frac{d\theta}{dt} = -b\frac{\Omega}{\sqrt{1 + n^2(a + bt)^2}}\{A\sin[\frac{\Omega}{n} arg\, shn(a + bt)] - \\ - B\cos[\frac{\Omega}{n} arg\, shn(a + bt)]\} + F\frac{b}{\Omega^2 + n^2}. \end{aligned} \tag{9}$$

By putting the initial conditions, there are obtained the constants of integration:

$$\begin{aligned} A &= (\frac{r}{\Omega^2} - F\frac{a}{\Omega^2 + n^2} + \theta_0)\cos(\frac{\Omega}{n} arg\, shna) - \frac{\sqrt{1 + n^2a^2}}{\Omega}(\frac{\omega_{20}}{b} - F\frac{1}{\Omega^2 + n^2})\sin(\frac{\Omega}{n} arg\, shna), \\ B &= (\frac{r}{\Omega^2} - F\frac{a}{\Omega^2 + n^2} + \theta_0)\sin(\frac{\Omega}{n} arg\, shna) + \frac{\sqrt{1 + n^2a^2}}{\Omega}(\frac{\omega_{20}}{b} - F\frac{1}{\Omega^2 + n^2})\cos(\frac{\Omega}{n} arg\, shna). \end{aligned} \tag{10}$$

By introducing the constants of integration in the expression (8) of the angle θ and returning to the initial notations, it is obtained the law of motion of driven element of the speed variator,

$$\theta = K\,sin[\frac{1}{b}\sqrt{\frac{q}{J_1}}\,arg\,sh\sqrt{\frac{J_1}{J_2}}(a+bt)+\Psi\,] - \frac{p}{q} + (a+bt)\frac{M_m}{q+b^2 J_1}, \tag{11}$$

where

$$K = \sqrt{A^2 + B^2},\;\; tg\Psi = \frac{A}{B}.$$

The law of motion of driven element of the speed variator is represented in figure 3.

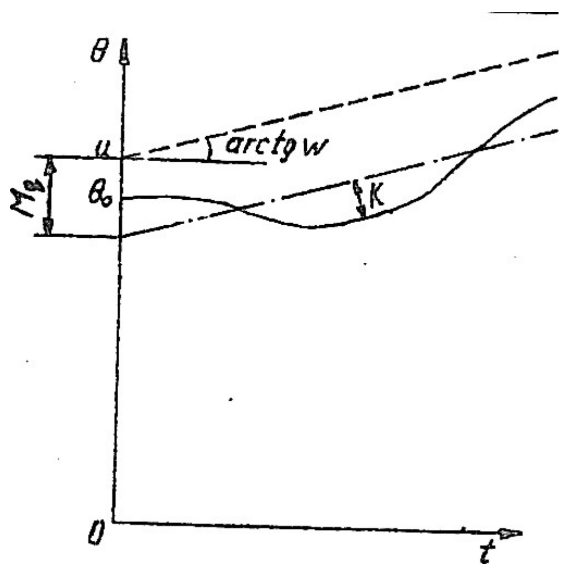

Fig. 3. Law of motion of driven element of speed variator

4. CONCLUSIONS

By comparing the laws of variation (2) and (11), and the corresponding diagrams in figures 2 and 3, it can be affirmed that, in the established conditions, i.e. when the driving moment is constant and the resistant moment is a linear function of position, within the framework of the industrial equipment, the speed variator has a behavior of a system of automatic control, [3], [4].

In truth, by using the terminology of automatics, a system of automatic control receives at the input a certain quantity, which is transformed in a certain way, characterized by the transfer function, and then, it is furnished at the output (figure4).

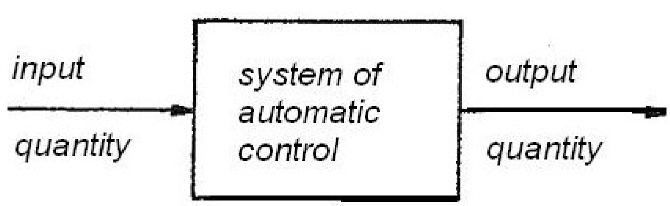

Fig. 4. Block diagram of system of automatic control

If the system of automatic control is ideal, it does not introduce any error, i.e. it transforms the input quantity in strict conformity to the transfer function:

output quantity=input quantity x transfer function. (12)

If in the right-hand member of the equation (12), other terms intervene, these ones constitute the error of the system of automatic control.

By returning to the relations (2) and (11), for the variator it can be done the diagram in figure 5, where the input quantity is the transmission ratio, and the output quantity is the angle of rotation of the driven shaft.

Fig. 5. Block diagram of speed variator

The transfer function is constituted by the constant $\frac{M_m}{q+b^2J_1}$. If in the relation (11) would be present only the last term which, as has been stated above, represents the input quantity $(a+bt)$, multiplied by the transfer function $(\frac{M_m}{q+b^2J_1})$, the variator would have the behavior of an ideal system of automatic control.

But, in addition to this term, in (11) also appears the constant term $(-\frac{p}{q})$ which, because of the fact that it is constant, is named static error and the harmonic term of amplitude *K,* which being variable, is called dynamic error of the system of automatic control.

In figure 3, with dashed line, it is represented the variation of the output quantity θ, non affected by the errors of the system of automatic control and by continuous line, the real variation. The notations that were made in figure 3 are $u = a\frac{M_m}{q+b^2J_1}$ and $w = b\frac{M_m}{q+b^2J_1}$.

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