AN APPROXIMATE SOLUTION OF THE EQUATION OF MOTION OF A ROTARY INDUSTRIAL EQUIPMENT

Prof.dr.ing. Adrian CHIRIAC
Prof.dr.ing. Liviu BERETEU
Asist.drd.ing. Ramona NAGY
Fiz.drd. Alexandru BOLTOSI
Universitatea “Politehnica” din Timişoara

Key words: rotary industrial equipment, graphical-numerical method, recurrence formula

Abstract. In the paper, for an industrial equipment, composed by an electric motor which drives the working machine through a rigid coupling, it is elaborated a graphical-numerical method of integration of the differential equation of motion. The electric motor is of asynchronous type, having the driving torque as a function of angular velocity, and the working machine produces a resistant moment as a periodic function of position. The solution of the equation of motion is obtained under the form of a recurrence formula.

1. INTRODUCTION

It is considered the most prevalent rotary industrial equipment, whose dynamic model has the structure in figure 1, where:
- I – driving asynchronous electric motor;
- II – rigid coupling;
- III – working machine.

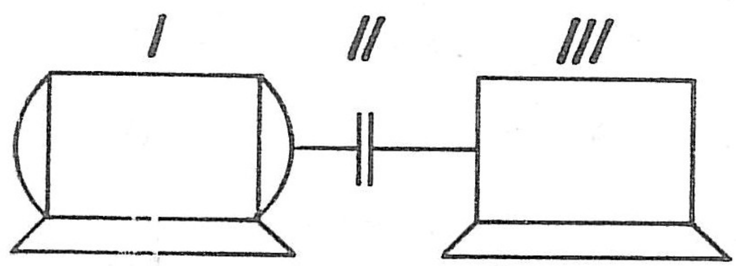

Fig. 1. Dynamic model of rotary industrial equipment

2. DIFFERENTIAL EQUATION OF MOTION

The differential equation of motion of the rotary industrial equipment is, [2],

$$J\frac{d\omega}{dt} = M_m - M_r, \tag{1}$$

where:
- J - moment of inertia, resulted by reducing the whole rotary industrial equipment, at the shaft of driving motor;
- ω - angular velocity;
- M_m - driving moment, produced by the electric motor;

- M_r - resistant moment generated by the working machine.

About the parameters of the industrial equipment, there are made the fallowing mentions:

- the moment of inertia J is constant;
- for the moment of the driving torque it is taking into account the static mechanical characteristic (figure 2), whose working portion, *AB,* is linearized; by moving the origin of abscissae in ω_k (ω_k - critical angular velocity), it can be written

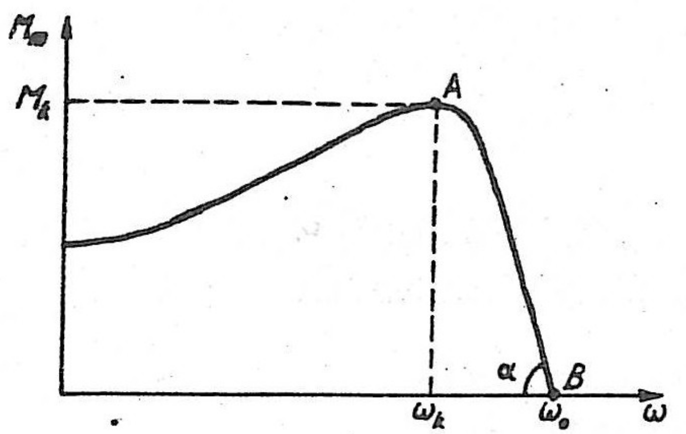

Fig. 2. Static mechanical characteristic of electric driving motor

$$M_m = M_k - \lambda\omega, \tag{2}$$

where M_k is the moment of the critical driving torque, and λ is the inclination of the static mechanical characteristic,

$$\lambda = tg\alpha = \frac{M_k}{\omega_0}, \tag{3}$$

where ω_0 is the angular velocity of synchronism of the electric motor;

- the resistant moment of the working machine is considered as a periodic function of the angle of rotation θ of the rotary industrial equipment,

$$M_r(\theta) = M_r(\theta + \Psi), \tag{4}$$

where Ψ is the angular period of the resistant moment. This situation is characteristic for a lot of equipment in different industrial branches.

Taking into account the mentions, stated above, the differential equation of motion (1) becomes

$$J\frac{d\omega}{dt} = M_m - \lambda\omega - M_r(\theta). \tag{5}$$

3. DETERMINATION OF GRAPHICAL-NUMERICAL SOLUTION

By passing from the variable *t* to variable θ, the equation (5) takes the form

$$\omega(J\frac{d\omega}{d\theta}+\lambda)=M_k-M_r. \tag{6}$$

The graphical procedure consists in the partition of the period Ψ of resistant moment in *n* equal parts, $\Delta\theta=\frac{\Psi}{n}$ (figure 3).

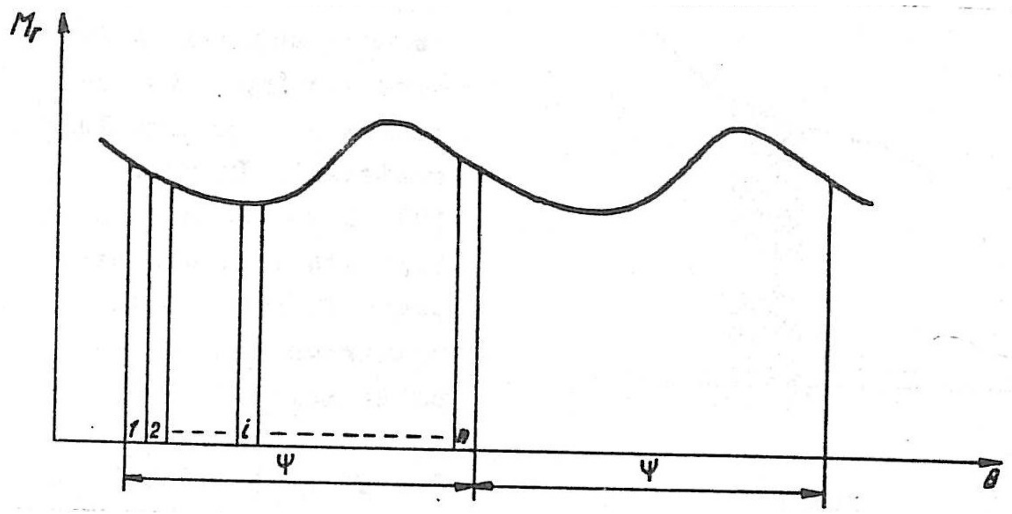

Fig. 3. Partition of characteristic of resistant moment of working machine

The number *n* of intervals must be as bigger as the resistant moment presents abrupt variations during a period and as the degree of precision is bigger. Any way, *n* must be chosen so that, on an interval $\Delta\theta$, to be possible to consider the resistant moment as constant and equal to average value on the respective interval.

For a certain interval *i*, the corresponding average value of the resistant moment is M_{ri} and the equation (6) is written

$$\omega(J\frac{d\omega}{dt}+\lambda)=M_k-M_{ri}. \tag{7}$$

By making the notations $\frac{\lambda}{J}=p,\ \frac{M_k-M_{ri}}{J}=q_i$ and separating the variables in the equation (7), this one becomes:

$$\frac{\omega}{p\omega-q_i}d\omega=-d\theta. \tag{8}$$

The numerical character of the method is given by the numerical integration of the left-hand member of the equation (8), by the method of trapezes [1]. Thus, if it is made the notation

$$f(\omega) = \frac{\omega}{p\omega - q_i},$$

then the integral of the left-hand member of the equation (8) is

$$I = \int_{\omega_i}^{\omega_{i+1}} f(\omega) d\omega, \qquad (9)$$

where ω_i and ω_{i+1} are the values of the angular velocity at the extremities of an interval *i* in which it is split the period Ψ of the resistant moment.

The integral (9) is equal to the shaded area (figure 4), which can be approximated by a trapeze, so that it can be written

$$I = f(\omega_{i+1}) + f(\omega_i)\frac{\omega_{i+1} - \omega_i}{2} = (\frac{\omega_{i+1}}{p\omega_{i+1} - q_i} + \frac{\omega_i}{p\omega_i - q_i})\frac{\omega_{i+1} - \omega_i}{2}. \qquad (10)$$

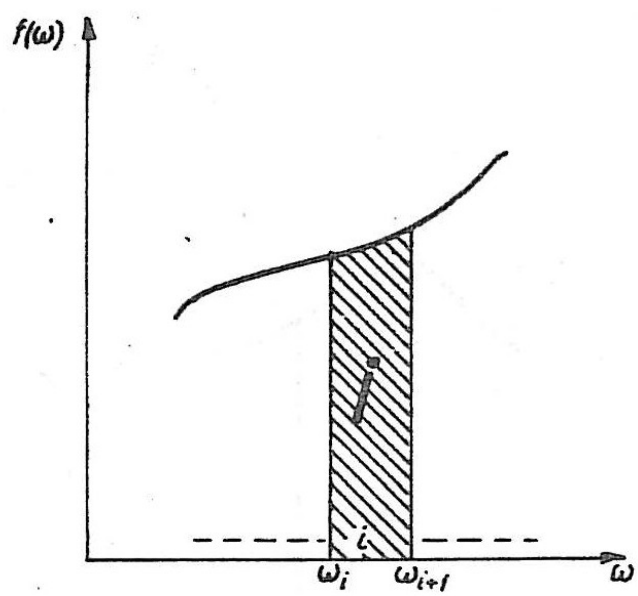

Fig. 4. Graphical aspect of method of trapezes

Taking into account of (10), the solution of the equation is

$$(\frac{\omega_{i+1}}{p\omega_{i+1} - q_i} + \frac{\omega_i}{p\omega_i - q_i})\frac{\omega_{i+1} - \omega_i}{2} = -\Delta\theta. \qquad (11)$$

4. CONCLUSIONS

From the solution (11), it is obtained the recurrence formula

$$\omega_{i+1} = \omega_i - \frac{\omega_i + p\Delta\theta + \sqrt{(\omega_i - p\Delta\theta)^2 + 2q_i\Delta\theta}}{\dfrac{2p\omega_i - q_i}{p\omega_i - q_i}}, \tag{12}$$

which gives the angular velocity ω_{i+1} at the end of an interval $\Delta\theta$, as a function of the angular velocity ω_i at the beginning of the interval *i*; evidently, ω_{i+1} constitutes, at same the time, the angular velocity at the beginning of the next interval, $i+1$.

In this way, the formula (12) can be considered as the graphical–numerical solution of the differential equation of motion (6).

REFERENCES

1. Angot, A., Complemente de matematici (traducere din limba franceză), Editura Tehnică, Bucureşti, 1966.
2. Toliyat, H., A., Analysis of Electric Machinery and Drive Systems, IEEE Transactions on Power Engineering, Vol.47, No.9, Sept. 2005, p.41-55