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NEW APPLICATIONS FOR THE PERFECT SEQUENCE TYPE FIFONACCI (SEQUENCE ϕ) IN PRODUCT DESIGN – THEORETICAL AND APPLIED CALCULUS ELEMENTS – 1

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For a harmonious result, obtained rapidly, it is recommended first to conceive a drawing and/or a 2D or even 3D model, corresponding with the artistic views of the design team. Then, starting from a modulus (or more) of the linear, angle, surface or other type of value, series of dimensions correlated with the entry dimensions, originated in (ϕ) and (1/ϕ) are obtained. The initial concept is corrected (refined) with these values.

The complete angle or segment (marked *c*) is divided in two unequal parts: a larger part, the major (marked *a*) and a smaller part, the minor (marked *b*) [RAD1981].

The condition is laid that the result of the division of the whole by the larger part be equal with the result of the division of the larger part by the smaller part $(c/a = a/b)$.

The ratio $(c/a = a/b)$ in form $a^2 = bc$ shows that the larger part (*a*) is the geometric mean of the smaller part (*b*) and the whole (*c*).

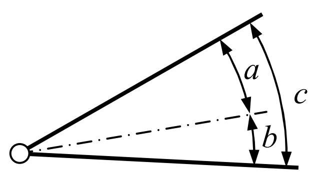
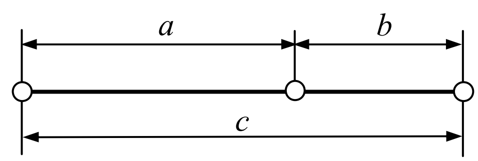
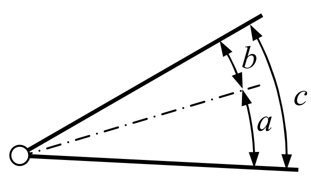
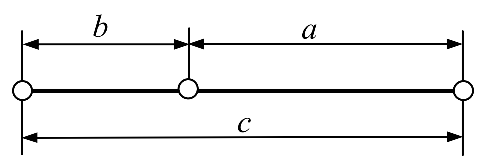
Possibility 1		
Possibility 2		

Table 1-1. Division possibilities of the angle and segment by the golden ratio ϕ

The result of the division (that is the value of the ratio) is $\phi = \frac{1+\sqrt{5}}{2}$.

In the specialty literature of different periods, the number ϕ has had several names with a pronounced poetical and mystical savour: the divine proportion = „Divina Proportione" (in Latin); the golden section = „Sectio aureea" (in Latin); the golden number = „Le nombre D'or" (in French); the golden cut „Der godene Schnitt" (in German); proportion rule = „Proportionales Gesetz" (in German); the constant proportion, the proportioned section = „Die stetige Proportin" (in German); the extreme ratio; the measure number. The name of „Sectio aureea" was first used by Leonardo DaVINCI, the name of „Divina Proportione" was used by Luca PACIOLI and the name of „Die stetige Proportion" was used by Johannes KEPLER. The symbol ϕ was proposed by . BARR and W. SCHOOLING, in the mathematical appendices of Sir Thomas COOK's work „The Curves of Life", Publishing Constable, London 1914. Mark

BARR chose the symbol ϕ for the first Greek letter of architect FIDIAS' name, who was the greatest sculptor of ancient Greece and was born in Athens around 431 BC. He used on a large scale the «divine proportion». The sculptures of Zeus from Olympia, o the great Athens (in bronze) and of the Athens inside the Parthenon are assigned to him.

I have proposed in several papers that this whole be considered as a measure of a different nature than angle or surface (even confined by a closed curve) and depending on requirements. Thus, the harmony induced by the golden section ϕ can be extended to other types of applications.

Returning to the ratio (*c/a=a/b*), it is immediately noticed that the result of the division (that is the value of the ratio) is $\phi=(1+\sqrt{5})/2$, which is the first root (positive solution) of the equation:

$$\phi^2 - \phi - 1=0 \tag{1}$$

The negative solution won't be taken in consideration because ϕ is the ratio between positive numbers. Subsequently $\phi=(1+\sqrt{5})/2=1{,}61803398874989\ldots\approx1{,}618$; the value with 3 decimals is approximated with an error of – 0,0021% versus the exact value. The numbers ϕ, ϕ^2 and $1/\phi$ have a remarkable characteristic: they have identical decimals! It has also been proven that ϕ results in an interesting limit:

$$\phi= \lim\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}\ . \tag{2}$$

The series of the FIBONACCI numbers (or the FIBONACCI sequence) is:

$$\begin{cases} a_1 = 1;\ a_2 = 1 \\ \text{with the recurrence relation } a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3 \end{cases} \tag{3}$$

That is: 1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89; 144… The main property of the recursive series is remarkable through the rapid striving of ratio between successive terms towards ϕ, respectively towards $1/\phi$. Consequently, the following limits are obtained:

$$\lim_{n\to\infty}\frac{a_n}{a_{n-1}} = \phi,\ \text{respectively}\quad \lim_{n\to\infty}\frac{a_{n-1}}{a_n} = \frac{1}{\phi} \tag{4}$$

The sequence (ϕ), with the general term (ϕ^n), $n \varepsilon N$, is defined as follows:

$$\begin{cases} a_1 = 1;\ a_2 = \phi \\ \text{with the recurrence relation } a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3 \end{cases} \tag{5}$$

That means the sequence (ϕ) is: 1; ϕ; ϕ^2; ϕ^3; ϕ^4; …; ϕ^n;…

The sequence ($1/\phi$) with the general term ($1/\phi^n$), $n\ \varepsilon\ N$, is: ($1/\phi^n$): 1; ($1/\phi$); ($1/\phi^2$);…; ($1/\phi^n$);…, with a_1 = 1; $a_2 = 1/\phi$ and the recurrent relation $a_n = - a_{n-1} + a_{n-2}$ cu n>2. At the same time, this is a series of terms in geometrical progression with the common ratio $q=1/\phi$. The geometric series corresponding to the series ($1/\phi^n$) is $\sum_{n=0}^{\infty} 1/\phi = 1+1/\phi+1/\phi^2+\ldots+1/\phi^n+\ldots$. By having the sub-unit common ratio this series is convergent. Its sum is: $S = 1/(1-1/\phi)=\phi/(\phi-1)=\phi^2$.

The (ϕ) series (as well as the ($1/\phi$) series) has the ratio of the successive terms equal with ϕ, respectively $1/\phi$, through unlike the FIBONACCI series, which reaches this ratio only with the limit. Thus from this point of view the (ϕ) series is ideal. The special property of the (ϕ) series is that it is simultaneously a recurrent additive FIBONACCI type series (but with the ratio of the consecutive terms equal ϕ) and a sequence in geometric progression (with the first term a_1=1 and the common ratio q=ϕ), fact that makes it unique [BOB2002]. For this reason the name „Ideal FIBONACCI series” is proposed for this series.

In product design obtaining harmonious shapes for the exterior of products is pursued among other matters. For this reason, in the designing stage it is recommended that the various linear and angular dimensions, surfaces and even volumes to be correlated in within the ensemble [BOC2003], with the help of the numbers obtained from the (ϕ) series, that is correlated by ($\phi^{p/q}$) with p, q Є ± 1; ± 2; ± 3; … and q ≠ 0 [POP2002].

So far, for the embossing of the exterior shapes of some mass produced products, the golden ratio ϕ, respectively curves based on ϕ, has been used in a very limited percentage. A few notable examples [ELA2001]: the Volkswagen Beetle, the Aldo ROSSI cone-shaped tea-kettle (1980-1983), the Braun coffee filter (Braun Aromaster Coffee Maker), The Braun hand blender (1987).

Several initial dimensions are proposed for the beginning (a kind of initial modules) which will then be amplified with the numbers from the (ϕ) and (1/ϕ) series. The initial dimensions and the results have of course the same measurement unit. A general dimension table is drawn up, in which the entry data are the initial modules and the exit data are the amplified dimensions. Some of these will be used minding the rest of the conditions (size, ergonomics, functional etc) at the modelling of the exterior shape of the mass produced object.

For a harmonious result, obtained rapidly, it is recommended first to conceive a drawing and/or a 2D or even 3D model, corresponding with the artistic views of the design team. Then, starting from a modulus (or more) of the linear, angle, surface or other type of value, series of dimensions correlated with the entry dimensions, originated in (ϕ) and (1/ϕ) are obtained. The initial concept is corrected (refined) with these values.

In the next example, the dimensional modules are: 64 centesimal degrees, 45 mm; 12 inch and 950 cm^2:

1 inch =	25,4 mm	[degree]	[mm]	[inch]	[cm2]
	Rate ►	**64.0**	**45.0**	**12.0**	**960.0**
Φ^7 =	29.03444	1858.2	1306.5	348.4	27873.1
Φ^6 =	17.94427	1148.4	807.5	215.3	17226.5
Φ^5 =	11.09017	709.8	499.1	133.1	10646.6
Φ^4 =	6.85410	438.7	308.4	82.2	6579.9
Φ^3 =	4.23607	271.1	190.6	50.8	4066.6
Φ^2 =	2.61803	167.6	117.8	31.4	2513.3
Φ^1 =	1.61803	103.6	72.8	19.4	1553.3
1 =	1.00000	64.0	45.0	12.0	960.0
1/Φ^1 =	0.61803	39.6	27.8	7.4	593.3
1/Φ^2 =	0.38197	24.4	17.2	4.6	366.7
1/Φ^3 =	0.23607	15.1	10.6	2.8	226.6
1/Φ^4 =	0.14590	9.3	6.6	1.8	140.1
1/Φ^5 =	0.09017	5.8	4.1	1.1	86.6
1/Φ^6 =	0.05573	3.6	2.5	0.7	53.5

$1/\Phi^7$ =	0.03444	2.2	1.5	0.4	33.1
	Result	▲	▲	▲	▲

Bibliography:

[BOB2002] BOBANCU, Ş., Şiruri cu utilizare în designul de produs corelate cu numărul de aur ϕ, obținute pe spirale logaritmice tip (ϕt), În : Bul. Simp. PRASIC 2002, Braşov, 2002.
[BOC2003] BOBANCU, Ş., CIOC, V., Inovare inginerească în design, Curs universitar, Universitatea „Transilvania” din Braşov, 274 pagini, Braşov, 2003.
[ELA2001] ELAM, K., Geometry of Design, Princenton Architectural Press, New York, 2001.
[RAD1981] RADIAN, H.R., Cartea proporțiilor. Principii şi aplicații în arhitectură şi în artele plastice, Ed. Meridiane, Bucureşti, 1981.