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EVALUATION OF ELASTIC PROPERTIES OF MATERIALS (AN EXPERIMENTAL METHOD)

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Abstract: Modelling the behaviour of materials under different dynamic loadings requires specific mechanical characteristics and the work's aim is to determine the elastic moduli of materials. A simple versatile test rig was designed for determining the Young modulus and the shear modulus of dental materials, using probes of relatively reduced dimensions, as the economic criterion is important when referring to dental materials. Tests were performed on beams of rectangular cross-section made of dental materials as well as on metallic materials.

1. INTRODUCTION

Dental materials used for restorative dentistry must fulfil the specific requirements of biocompatibility, mechanical properties and aesthetics. Polymers, ceramics and composites are now frequently used in restorative dentistry as they perform denture teeth, veneers and filling materials. The tribological characteristics of these non-metallic materials used in biomedical applications are of primary importance and they strongly depend on the mechanical characteristics. High mechanical strength, ductility and good fatigue resistance are required and characteristics such as low coefficient of friction and resistance to wear damage are also needed. A severe restriction of dental non-metallic materials is the wear resistance which is inadequate in posterior tooth fillings, where important mastication stresses occur. Abrasion test (rubbing against abrasive paper), pin-on-disc test (sliding bodies) and scratch (penetration) tests are the most common *in vitro* tests performed for characterization of wear resistance, [1]. Radial fretting was proposed as a new method for wear characterization of ceramics. Modelling the behaviour of materials under different dynamic loadings requires specific mechanical characteristics and the first stage in our work considered the determination of the elastic moduli of materials.

2. EXPERIMENTAL TEST RIG

A simple and versatile test rig was designed for determining the Young modulus, *E* and the shear modulus, *G,* of materials, using probes of relatively reduced dimensions, as the economic criterion is important when referring to dental materials. Tests were performed on beams of rectangular cross-section made of dental materials as well as on metallic materials and the above mentioned elastic characteristics were determined.

The principle of a cantilever beam bended by a force *F* acting on the tip of the free end, [2], is presented in Figure 1 and the experimental set-up is shown in Figure 2. The cross-section of the beam is rectangular, and the axial moment of inertia of the cross-section is:

$$I_z = \frac{ba^3}{12}. \quad (1)$$

The force is given by the weight of the discs of mass *m* attached on the vertical rod:

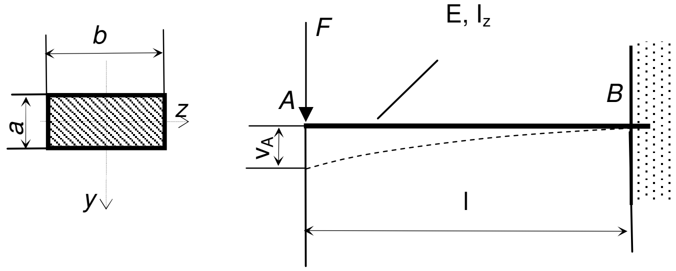
$$F = m \cdot g. \quad (2)$$

The elastic deflection of the section A, where the force is acting, is:

$$v = \frac{F \cdot l^3}{3 \cdot EI_z} \cdot \tag{3}$$

Therefore, the experimental Young modulus E_{exp} can be computed after measuring the applied force *F*, the distance *l* and reading the displacement *c* of the tip of the dial gage type sensor:

$$E_{exp} = \frac{F \cdot l^3}{3 \cdot I_z \cdot c} . \tag{4}$$



$l = 0.060\,m$

$a = 0.002\,m$

$b = 0.008\,m$

Figure 1. Sc*heme of cantilever beam under bending*

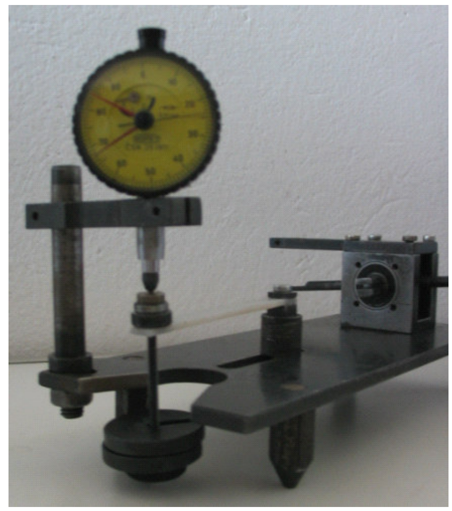
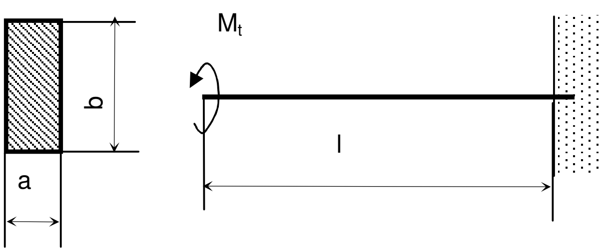


Figure 2. *Experimental device for bending*

The principle of a cantilever under torsion, [2], is shown in Figure 3. When to the cantilever beam a lever of length *R* is attached, a torque *FR can* be applied, as the experimental set-up presents in Figure 4.



$l = 0.060\,m$

$a = 0.002\,m$

$b = 0.008\,m$

$R = 0{,}05\,m,\ \beta = 0{,}281$

Figure 3. *Principle of a cantilever beam in torsion*

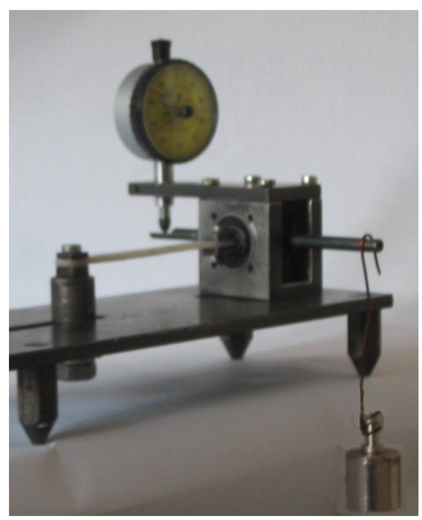


Figure 4. *Experimental set-up for torsion*

The angle of twist of the free section is then measured. Theoretically, the rotation angle of the beam in torsion is:

$$\theta = \frac{M_t l}{GI_t};\quad \theta = \frac{c}{R}, \tag{5}$$

where R is the length of the lever where the force *F* is applied by hanging a mass m and c is the length of the arch described by the lever. The torque is:

$$M_t = R \cdot F = R \cdot m \cdot g, \tag{6}$$

where *g* is the gravity. Due to small deformation domain, one can approximate the linear displacement of the tip of the transducer, with the length of the arch, *c* of angle θ, [4]; therefore, by measuring *c*, the shear modulus *G* of the beam can be computed with the relation:

$$G = \frac{m \cdot g \cdot R \cdot l}{\frac{c}{R} I_t}, \tag{7}$$

where I_t is the torsional constant for the rectangular cross-section:

$$I_t = \beta \cdot b \cdot a^3, \quad a < b. \tag{8}$$

The coefficient β can be found in literature, in tables, depending on the ratio b/a or can be found numerically, as it is shown next.

3. SHEAR STRESSES AND CALCULUS OF COEFFICIENT β FOR A RECTANGULAR CROSS SECTION OF A BEAM UNDER TORSION

Using the Prandtl function,[3], the coefficient can be numerically found,:

$$\beta = \frac{1}{3}\left[1 - \frac{192}{\pi^5} \cdot \frac{a}{b} \cdot \sum_{k=1}^{\infty} \frac{1}{k^5} \tanh\left(\frac{k\pi b}{2a}\right) \cdot \frac{1+(-1)^{k+1}}{2}\right]; \text{ for } b/a = 4; \tag{9}$$

$$\alpha = (2k+1)\frac{\pi}{2a}; \ \beta = 0{,}281.$$

It results:

$$I_t = 1{,}789 \cdot 10^{-11} m^4. \tag{10}$$

The shear stresses on the rectangular section can be computed with the relations:

$$\tau_{23}(x_1, x_2) = 4 \cdot \frac{M_t \cdot I_t}{a} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(\alpha_k)^2} \cdot \left(1 - \frac{\cosh(\alpha_k \cdot x_2)}{\cosh(\alpha_k \cdot b)}\right) \cdot \sin(\alpha_k \cdot x_1); \tag{11}$$

$$\tau_{31}(x_1, x_2) = -4 \cdot \frac{M_t \cdot I_t}{a} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(\alpha_k)^2} \cdot \frac{\sinh(\alpha_k \cdot x_2)}{\cosh(\alpha_k \cdot b)} \cdot \cos(\alpha_k \cdot x_1); \tag{12}$$

and for a particular chosen value of torque, stresses were plotted in Figures 5 and 6.

The longitudinal displacement can be computed from relation:

$$u_3(x_1, x_2) = \frac{M_t}{G \cdot I_t} \cdot \phi(x_1, x_2); \tag{13}$$

$$\phi(x_1, x_2) = x_1 \cdot x_2 - \frac{4}{a} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{(\alpha_k)^3} \cdot \frac{\sinh(\alpha_k \cdot x_2)}{\cosh(\alpha_k \cdot b)} \cdot \sin(\alpha_k \cdot x_1)$$

and is represented in Figure 7.

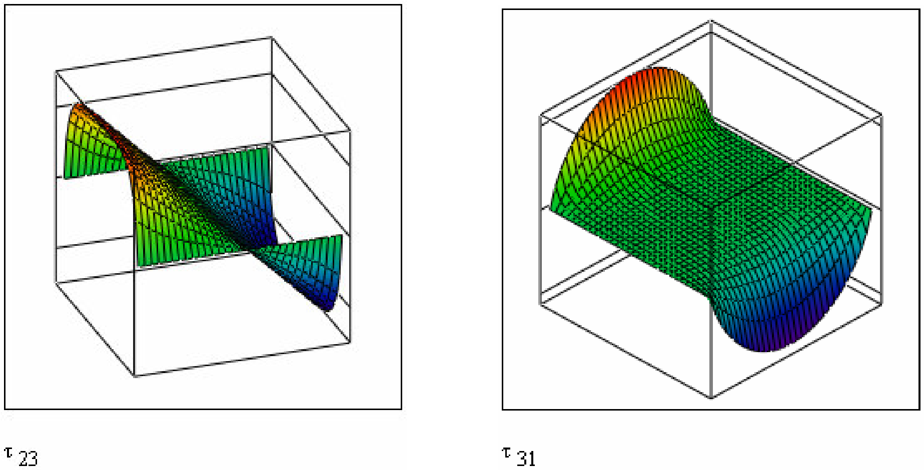


Figure 5. Torsion shear stresses on rectangular cross section

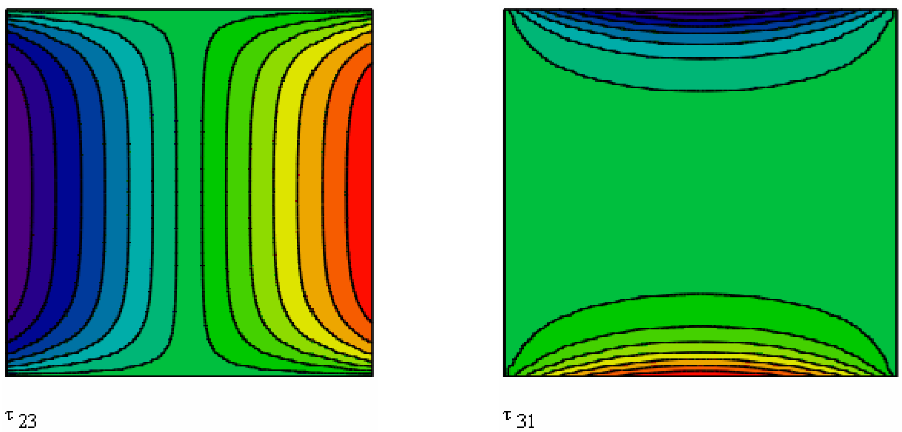


Figure 6. Torsion shear stresses on rectangular cross section: level curves

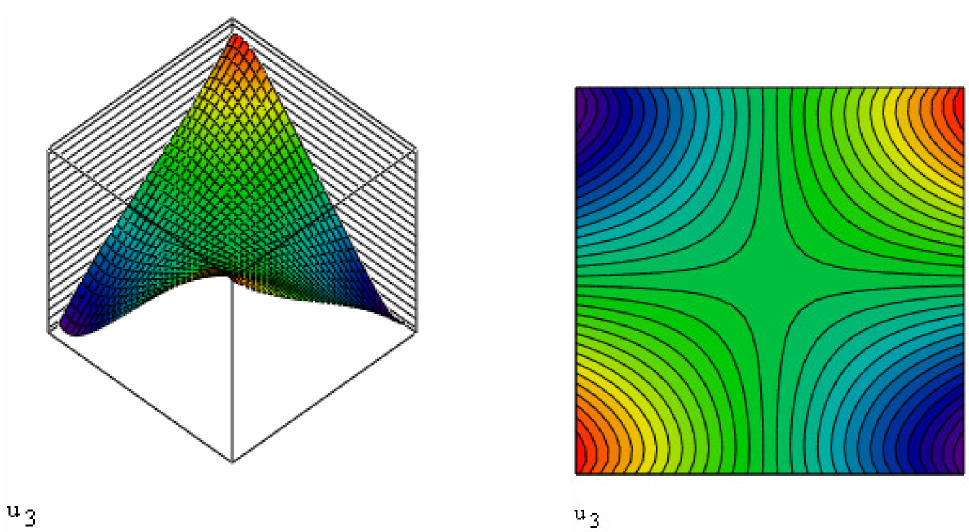


Figure 7. Longitudinal displacements for rectangular cross-section of a beam under torsion

4. EXPERIMENTAL RESULTS

Tests were carried out in a first phase on steel and aluminium beams in order to validate the method. For a steel beam, the experimental results are presented in Figure 8. The linearity of the graphs must be emphasised.

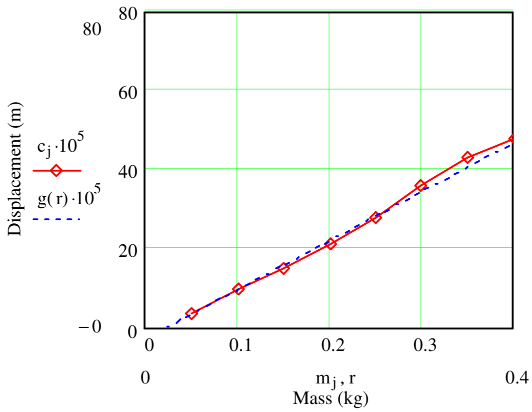


Figure 8. Displacements versus loading mass in torsion tests

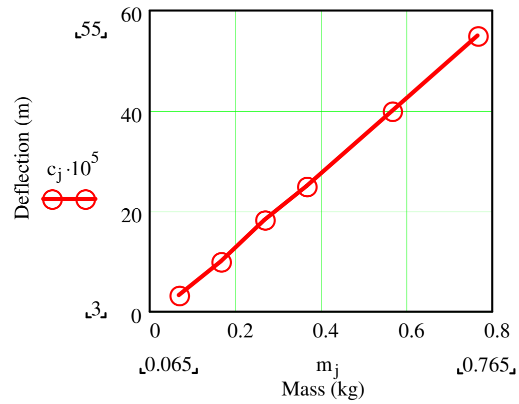


Figure 9. Displacements versus loading mass in bending tests

From relations (4) and (7), with the experimental data, it results the mean values $G_{em} = 7.91 \cdot 10^{10}$ Pa, and $E_{em} = 2.03 \cdot 10^{11}$ Pa. From theory of elasticity, we apply the relationship between E, G and ν, [2]:

$$G = \frac{E}{2 \cdot (1 + \nu)} \tag{14}$$

and, for steel, the Poisson coefficient results:

$$\nu_{exp} = \frac{E_{em}}{2 \cdot G_{em}} - 1, \quad \nu_{exp} = 0{,}286 . \tag{15}$$

The elastic characteristic obtained are considered in good agreement with the known steel characteristics, $E = 2{,}1 \cdot 10^{11}$ Pa, $G = 8{,}077 \cdot 10^{10}$ Pa and $\nu = 0{,}3$.

Similar tests made for an aluminium beam, having the geometric parameters $a = 2 \cdot 10^{-3}$ m; $b = 9{,}6 \cdot 10^{-3}$ m; $l = 0{,}053$ m; $\beta = 0{,}29$; $\alpha = 0{,}282$ and $I_t = 2{,}227 \cdot 10^{-11}$ m^4. Figure 10 shows the displacement *c* versus loading mass *m* and the linear experimental dependence is found, as expected from relation (7).

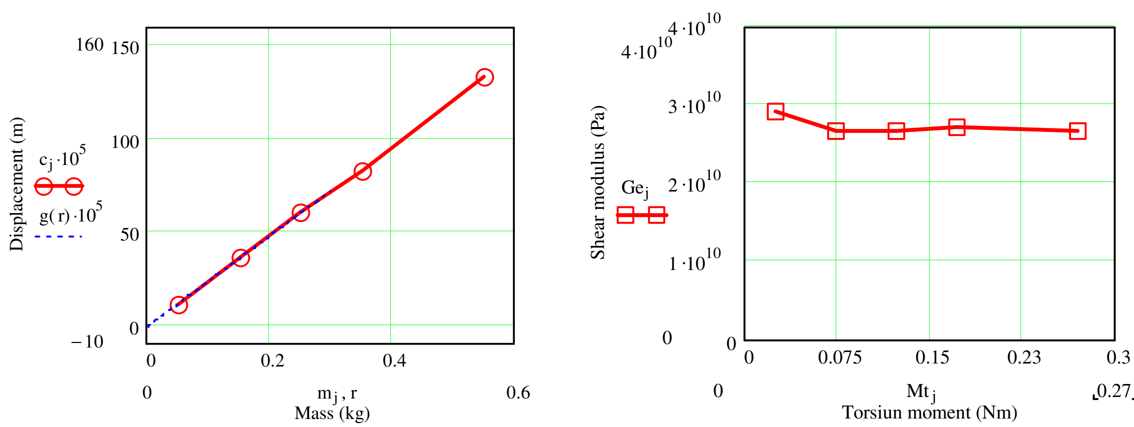


Figure 10. Results of torsion test for aluminium

The medium experimental shear modulus of aluminium was found $G_{em} = 2{,}778 \cdot 10^{10}$ Pa and it is comparable with $G_{Al} = 2.7 \cdot 10^{10}$ Pa, from literature.

The results obtained for steel and aluminium were considered to validate the proposed experimental method for determining the elastic constants of materials. Therefore, on a second stage, test on dental material were carried out.

Experimental results were obtained for a beam made of Royal dent, having the geometry described by the parameters: $a = 2 \cdot 10^{-3}$ m; $b = 8 \cdot 10^{-3}$ m; $l = 0{,}018\text{m}$; $R = 0{,}05\text{m}$; $\beta = 0{,}281$ and $I_t = 1{,}798 \cdot 10^{-11}$ m4. From torsion experiments, were obtained the results presented in Table 2 and Figure 11, where linear relation between load and deformation is observed. The mean experimental shear modulus is $G_{em} = 8{,}966 \cdot 10^8$ N/m^2.

Results obtained from bending tests are presented in Table 3 and Figure 12, for $l = 0{,}025$ m. The mean experimental value of the Young modulus is $E_{em} = 2{,}579 \cdot 10^9$ N/m^2. The Poisson coefficient, $\nu = 0{,}43$, resulted from relation (15) using the experimental values of Young modulus and shear modulus.

Table 2. Experimental results for shear modulus

c [m]	m [kg]	Mt_j [Mn]	G_{ej} [Pa]
$58 \cdot 10^{-5}$	$50 \cdot 10^{-3}$	0.025	$8.392 \cdot 10^8$
$109 \cdot 10^{-5}$	$100 \cdot 10^{-3}$	0.049	$8.931 \cdot 10^8$
$125 \cdot 10^{-5}$	$120 \cdot 10^{-3}$	0.059	$9.346 \cdot 10^8$
$159 \cdot 10^{-5}$	$150 \cdot 10^{-3}$	0.074	$9.184 \cdot 10^8$

Table 3. Experimental results for Young modulus

m [kg]	$F = m \cdot g$ [N]	c [m]	$E_{exp\,j}$ [GPa]
0,065	0,638	$20 \cdot 10^{-5}$	3,114
0,085	0,834	$28 \cdot 10^{-5}$	2,908
0,115	1,128	$45 \cdot 10^{-5}$	2,448
0,165	1,619	$70 \cdot 10^{-5}$	2,258
0,215	2,109	$95 \cdot 10^{-5}$	2,168

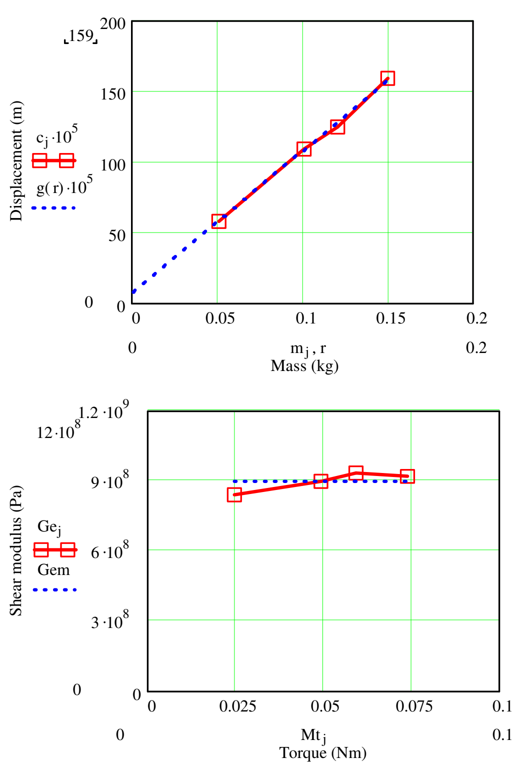


Figure 11. Experimental results from torsion: a)displacement versus loading mass; b)shear modulus versus torque

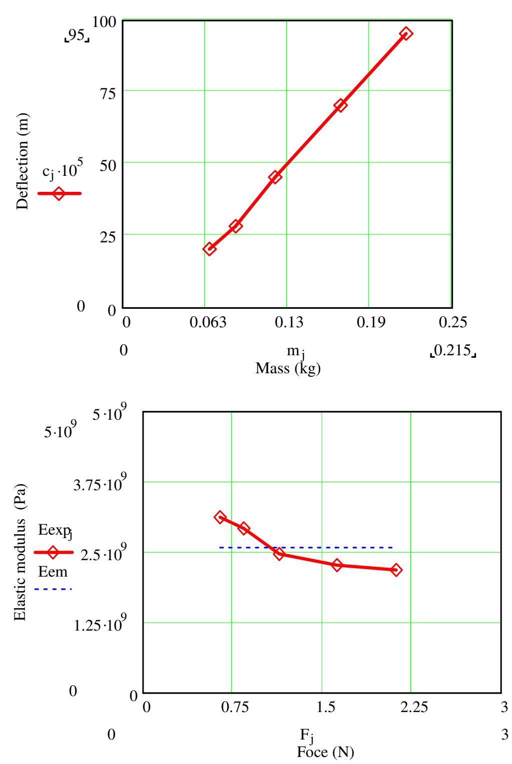


Figure 12. Experimental results from bending: a)displacement versus loading mass; b) Elastic (Young) modulus versus force

5. CONCLUSIONS

There is a deep necessity of dental material mechanical characteristics knowledge, as a large variety of such materials is used.

Standard tests like uniaxial tension, compressive test, flexural test, require samples of relatively large dimensions. For preliminary characterisation of materials in laboratory experiments, a more economic test rigs for small samples, is useful.

A test rig was designed and accomplished, having two versatile devices, for determining shear modulus from torsion tests and Young modulus from bending tests of a rectangular beam. The Poisson coefficient can be computed using the relation between elastic constants from theory of elasticity. In order to validate the method, samples of steel and aluminium were first tested and the results were in good agreement with the data from literature.

Rectangular cross-section samples were easier to be made from dental material. The exact coefficients for calculus of shear stress and torsion constant of cross section were numerically calculated and the shear stresses on section were represented.

The elastic moduli of a dental material were determined using the presented method. These elastic constants are required in order to model the mastication stresses, generated mostly by compressive forces. Further work concerns spherical probes of the same dental material investigated under conditions of Herztian contact experiments.

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