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THE STABILITY OF THE PORTABLE MASTS

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Abstract: The masts are slim constructions, with reduced stiffness, which must support big axial loads. In this case, the phenomena of stability (buckling) become decisive for the masts sizing and they must be analysed using methods as exact as possible. In this paper, the authors present a method for obtaining the stability equation in the case of the masts having elastic links. Based on this equation, they can determine the buckling length, the critical buckling load and the buckling safety coefficient for the mast.

1. GENERAL ASPECTS

The portable masts are spatial structures made from beams, building, in general, two parts. During the transportation, the two parts are introduced one in another. During the functioning, these parts realize a structure having the form shown in figure 1, having a $\alpha = 3 \cdots 5^{o}$ inclination in comparison with the vertical axis. They are provided with four anchors: two for the superior edge of the superior part and two for the superior edge of the inferior part.

In order to accomplish a small weight structure, with a reduced material consuming, the masts are built from materials with high mechanical characteristics and reduced beams cross-sections. So, they obtain slim constructions, with reduced stiffness, which can support big axial loads. In this case, the buckling phenomena become decisive for the masts sizing and they must be analysed using exact specific methods.

In the designing process, the study of the general stability of a portable mast is made using its schematisation as a rigid fixed at the superior part beam. In the YZ plan (fig.1), this is a console type and the study of buckling is not a complicated problem.

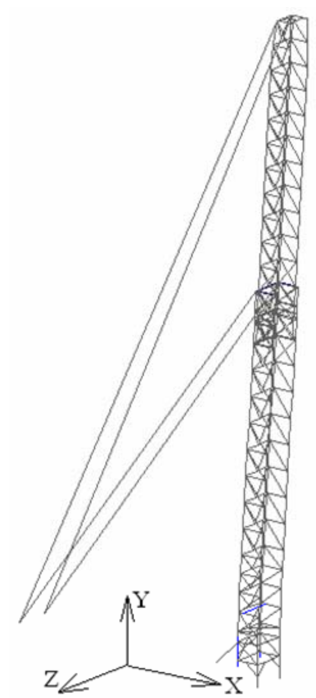

Fig.1. The calculus model for a mast

2. THE STABILITY OF THE MAST IN THE XY PLAN

In the XY plan, the mast is a rigid fixed base beam with elastic links given by anchors (fig. 2). This kind of schematisation increases the difficulty of the study. The mast stability is studied using the 2-nd order non-linear theory and in that case the equilibrium equations are written on the deformed form of the structure (fig.3).

In the following, the authors will present a general method for the stability equation determination. Using that equation, they will determine the buckling length l_{fz}, the critical load P_{cr} and the buckling safety coefficient of the mast in that plan.

From figure 3 they can observe that, on the deformed form of the mast, schematised as a beam, the anchors elongate with the following quantities:

$$\Delta l_1 = v_1 \sin\gamma_1; \quad \Delta l_2 = v_2 \sin\gamma_2 \tag{1}$$

where v_1 and v_2 are the mast displacements along the local axis y.

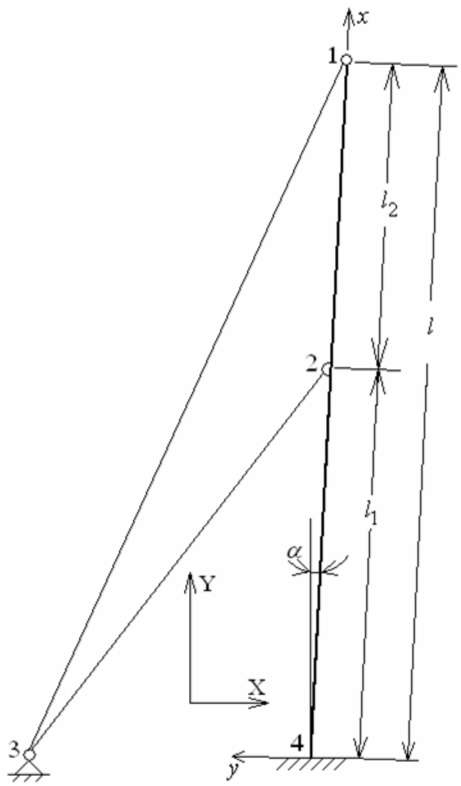

Fig.2.

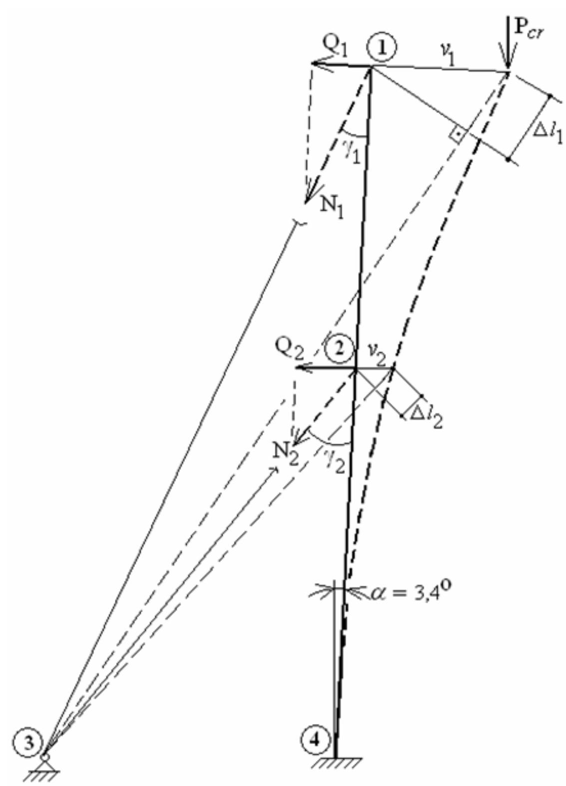

Fig.3

In accordance with the axial forces N_1 and N_2 (fig.3) developed in anchors, the elongations values are determined using the expressions from the "Strength of Materials":

$$\Delta l_1 = \frac{N_1 l_{1-4}}{EA_{1-4}}; \quad \Delta l_2 = \frac{N_2 l_{2-4}}{EA_{2-4}} \tag{2}$$

The axial forces components N_1 and N_2 (fig.3) along the perpendicular direction on the mast axis have the following values:

$$Q_1 = N_1 \sin\gamma_1; \quad Q_2 = N_2 \sin\gamma_2 \tag{3}$$

Using the mathematical expressions (1), (2) and (3), they obtain:

$$v_1 = \frac{\Delta l_1}{\sin\gamma_1} = \frac{N_1 l_{1-4}}{EA_{1-4}} \cdot \frac{1}{\sin\gamma_1} = \frac{Q_1 l_{1-4}}{EA_{1-4}} \cdot \frac{1}{\sin^2\gamma_1}$$
$$v_2 = \frac{\Delta l_2}{\sin\gamma_2} = \frac{N_2 l_{2-4}}{EA_{2-4}} \cdot \frac{1}{\sin\gamma_2} = \frac{Q_2 l_{2-4}}{EA_{2-4}} \cdot \frac{1}{\sin^2\gamma_2} \quad (4)$$

In accordance with a calculus simplification, they introduce the notations u_1 and u_2 as a ratio between the mast elasticity bending coefficients (l^3/EI) and the anchors traction elasticity coefficients (l_{1-4}/EA_{1-4} and l_{2-4}/EA_{2-4}):

$$u_1 = \frac{l^3}{EI} \cdot \frac{EA_{1-4}}{l_{1-4}} \cdot \frac{1}{\sin^2\gamma_1}$$
$$u_2 = \frac{l^3}{EI} \cdot \frac{EA_{2-4}}{l_{2-4}} \cdot \frac{1}{\sin^2\gamma_2} \quad (5),$$

where l represents the mast length and I the bending moment of inertia along the z axis.

So, the equations (4) become:

$$v_1 = \frac{Q_1}{u_1} \cdot \frac{l^3}{EI}; \quad v_2 = \frac{Q_2}{u_2} \cdot \frac{l^3}{EI} \quad (6).$$

They use the following notations:

$$k = \frac{P}{EI}; \quad \nu = kl \quad (7)$$

and they obtain $EI = \dfrac{Pl^2}{\nu^2}$.

Using these results, the equations (6) can be written as it follows:

$$v_1 = \frac{Q_1 l}{u_1} \cdot \frac{\nu^2}{P}; \quad v_2 = \frac{Q_2 l}{u_2} \cdot \frac{\nu^2}{P} \quad (8).$$

In order to obtain the stability equation, they use the expression of the linear displacement in the 2-nd order theory, particularized for the 1-4 beam, where $v_0 = 0; \varphi_0 = 0; T_0 = Q_1 + Q_2$:

$$v = -\frac{M_0}{P}(1 - \cos kx) + \frac{Q_1 + Q_2}{kP}(kx - \sin kx) - \frac{Q_2}{kP}\left[k(x - l_2) - \sin k(x - l_2)\right]_{x > l_2} \quad (9)$$

Using it, they obtain the expression of the bending moment in the 2-nd order theory, and, considering (7), this one becomes:

$$M = -EI \cdot v'' = M_0 \cos kx - \frac{Q_1 + Q_2}{k}\sin kx + \frac{Q_2}{k}\sin k(x - l_2)_{x > l_2} \quad (10)$$

In the linear displacement expression (9) the unknown $M_0, Q_1,$ and Q_2 intervene and these ones can be determined using the limit conditions:

- at $x = l$, $M = 0$ and $v = v_1$; at $x = l_2$, $v = v_2$.

From the limit condition $x = l$, $M = 0$ results:

$$M_0 = (Q_1 + Q_2)l \cdot \frac{tg\nu}{\nu} - Q_2 l \cdot \frac{\sin kl_1}{\nu \cos \nu} \qquad (11)$$

In allowance with the expressions (11) and (8), the limit conditions: for $x = l$, $v = v_1$ and for $x = l_2$, $v = v_2$ lead to the following homogeneous algebraic system in the unknown quantities Q_1 and Q_2:

$$\begin{aligned} A \cdot Q_1 + B \cdot Q_2 = 0 \\ C \cdot Q_1 + D \cdot Q_2 = 0 \end{aligned} \qquad (12)$$

in which:

$$\begin{aligned} A &= kl_2 - \sin kl_2 - (1 - \cos kl_2) tg\nu \\ B &= kl_2 - \sin kl_2 - (1 - \cos kl_2) tg\nu + \frac{\sin kl_1}{\cos \nu}(1 - \cos kl_2) - \frac{\nu^3}{u_2} \\ C &= \nu - tg\nu - \frac{\nu^3}{u_1} \\ D &= \nu - tg\nu - kl_1 + \frac{\sin kl_1}{\cos \nu} \end{aligned} \qquad (13)$$

The homogeneous algebraic system admits the solution $Q_1 = Q_2 = 0$, which does not present a buckling situation for the mast. In order to have different from zero solutions for the algebraic system, its determinant mast have the zero value:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 0 \qquad (14)$$

(14) represents the stability equation for the mast with elastic links (with anchors). The equation has an infinite number of solutions, the first one being ν_{cr}. Knowing the critical parameter, then they can determine the real buckling length of the mast using the known expression:

$$l_f = \frac{\pi}{\nu_{cr}} l \qquad (15).$$

Then, varying with the buckling domain (elastic or elastic-plastic domain), they can determine the critical buckling load.

3. THE STABILITY CALCULUS FOR A MAST

It is thought a portable mast having the form shown in figure 1 and the scheme presented in figure 2. With a view a static calculus, the mast have been divided in finite elements type “beam”. In the absence of anchors, they determined the real transversal linear displacement of the mast along the X-axis. Using its equivalence with the transversal displacement from a console edge, they made obvious the equivalent bending moment of inertia of the mast and the resulted value was $I = 6{,}789 \cdot 10^9 mm^4$. Similarly, they determined the mast equivalent area using the equality between the linear axial displacements of the real mast and those from the scheme shown in figure 3 and the result was: $A_m = 16048 mm^2$. Using these values, they can determine the inertia radius $i_m = \sqrt{\frac{I}{A_m}} = 650{,}41 mm$.

The mast beams are made from OL52 steel. For this material, the buckling coefficient value, in accordance with the buckling domain, is $\lambda_0 = 100$.

The mast has the following lengths: $l = 33856 mm$; $l_1 = 14816 mm; l_2 = 19040 mm$. The anchors have the following areas and lengths: $A_{1-4} = A_{2-4} = 1200 mm^2$ $l_{1-4} = 38564 mm; l_{2-4} = 25206 mm$. The other geometrical elements from figure 3 are: $\gamma_1 = 21{,}445^0$ and $\gamma_2 = 32{,}582^0$.

Having the previously presented elements and using the expressions (5), they determined the parameters $u_1 = 23{,}776$ and $u_2 = 78{,}915$. Also, the expressions from (13) can be given only in accordance with the ν parameter as it follows: $kl_1 = kl \cdot \frac{l_1}{l} = 0{,}737\nu$ and $kl_2 = 0{,}652\nu$.

After solving the equation (14), for which they used the MATHCAD program, the result was $\nu_{cr} = 4.0725$. With this value, using (15), they obtained the buckling length $l_f = 0.771 l$. The buckling coefficient of the mast has the value $\lambda = \frac{l_f}{i_m} = 46$ and, because it is inferior to the value $\lambda_0 = 100$, it results that the buckling happens in the elastic-plastic domain.

According to the critical coefficient $\nu_{cr} = 4.0725$, using (9) the deformed mast buckling form can be drawn. Using MathCAD, this form is shown in figure 4.

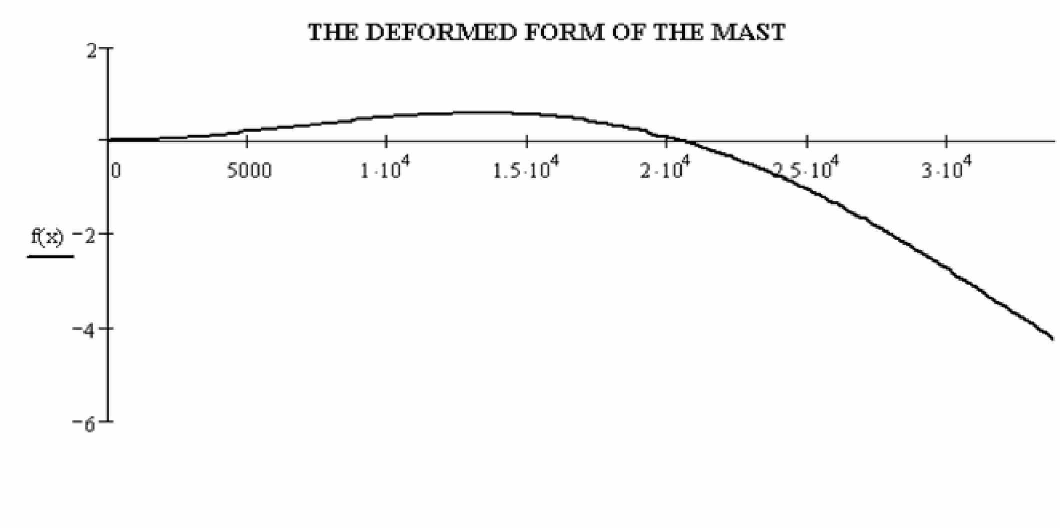

Fig. 4

For the buckling in the elastic-plastic domain, the critical buckling stress can be determined with the *Tetmajer-Iasinski* formula [1], $\sigma_{cr} = a - b \cdot \lambda$. For the mast material, $a = 577$, $b = 3{,}74$ [1] and it results $\sigma_{cr} = 404{,}96 MPa$ and then the critical buckling load

$P_{cr} = \sigma_{cr} \cdot A_m \cong 6500 kN$. Toward the maximum load of the mast (the probation load), $F_p = 2000 kN$, it results a safety coefficient $c = P_{cr} / F_p = 3{,}25$.

3. CONCLUZII

The portable masts are slim constructions, with reduced stiffness, which must support big axial loads. In this case, the phenomena of stability (buckling) become decisive for the masts sizing and they must be analysed using methods as exact as possible. This work presented a method in order to obtain the stability equation in the case of the masts having elastic links given by the resistance anchors. This method can be easily applied in projection calculus and allows the critical buckling load determination.

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