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A MATHEMATICAL FORMING OPERATION ON THE IMPACTS'INFLUENCE IN VIBRATING MILLS' WORKING THAT ARE USED IN CONSTRUCTION MATERIALS INDUSTRY

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ABSTRACT

During a vibrating mill's working , between milling bodies, grist and the mill's walls some mutual impacts and frictions are appearing. These phenomena's influence on the milling process efficiency is focused in this paper.

1. INTRODUCTION

During a vibrating mill's working, the mill's body vibrations are transmitted to the mill's filling consisting of balls or short rods that, beyond the vibrating movements, will also performe rotations around its proper axis. So, once introduced into the vibrating mill, the material that is going to be milled, is submitted to some recurred impacts and to some frictions.

There are constructive alternatives of vibrating mills – for exemple those that have an inner pipe – whose milling bodies' movement and milling bodies-grist's impact force are reduced; on the other hand, the weight of impact interaction between balls, interior pipe, grist and the mill's walls is significantly increasing.

2. MOVEMENT'S DIFFERENTIAL EQUATIONS DETERMINATION

In the eccentric vibrating mill's case, a dynamic model is proposed in figure 1, which shows a rigid body excited by a rotary perturbing force, linear elastically suspended, having the damping proportional to speed values, and whose movement can be described using three coordinates (*x*, *y* and *z*).

Considering the impact interaction between milling bodies (balls or cylindrical rods) and grist (in some constructive alternatives there is an inner pipe, too) very important, the specialists are estimating that the perturbing force has two components, one determined by the rotary perturbing force and other one due to the mentioned impact.

This shocks' existence is reflected also in the movement differential equation so that, in its right member, there is a term that takes into account this phenomenon:

So:

$$m\ddot{y} + c_v\dot{y} + k_v y = m_e r\omega^2 \sin\omega t + f_s(t) \qquad (1)$$

in which:

m is the mass being in the vibrating process;

c_v - the rubber buffers' damping coefficient in vertical direction;

k_v - elastic elements' stiffness coefficient in vertical direction;

m_e - the eccentric's mass;

ω -the perturbing force's angular frequency;

r -the unbalance distance (the eccentricity);

$f_s(t)$ -the function that's characterizing the shock's influence.

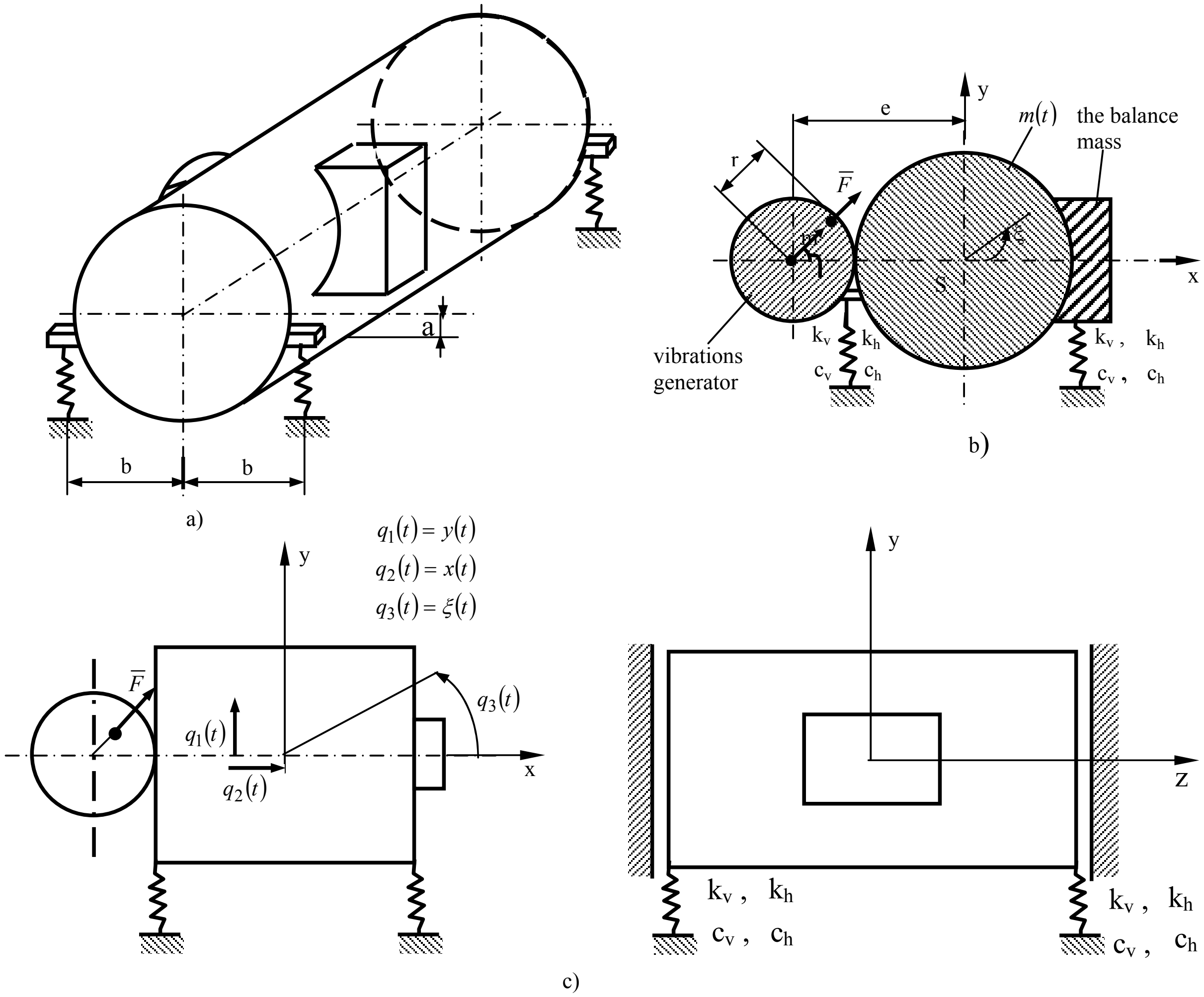

Fig. 1 The dynamic model proposed for a vibrating mill having the vibrations generator laterally mounted

Because the vibrations are uncoupled in vertical and horizontal directions, and the vibrating process' analytical study is analogous, we'll analyse only the movement's differential equation in vertical direction, described by equation 1.

3. THE $f_s(t)$ TERM'S ASSESSEMENT

The analysed speciality literature confirms the fact that the eccentric mass m_e and the vibrating mill are oscillating almost in phase opposition reverse. The vibrations' measurements recordings revealed that the relative movement between the milling bodies and the milling pipe takes place also in phase opposition reverse relating to vibrating mill's deflection, y.

For these reasons we can infer that $f_s(t)$ has the same functional form and the same frequency as the perturbing force, thus it could be estimated so:

$$f_S(t) = F_S \cdot \sin\omega \quad t \tag{2}$$

Therefore, the effect of f_s is overlapping on the inertial excitation „strenghtening" the mill's vibration.

In accordance with this assessment function's structure, the differential equation (1) acquires the following simplified form:

$$m\ddot{y} + c_v\dot{y} + k_v y = \left(m_e r\omega^2 + F_s\right)\sin\omega t \tag{3}$$

The differential equation (3) is resolved and this form solution is obtained:

$$y(t) = \hat{y}\cdot\sin[\omega t - \varphi_D(\eta)] \tag{4}$$

in which the vibration's measured amplitude is:

$$\hat{y} = \hat{y}_0 + \hat{y}_s \tag{5}$$

where

$$\hat{y}_0 = \frac{m_e r}{m}V(\eta) \tag{6}$$

is the vibration's theoretical amplitude

and y_s is the vibration's additional amplitude caused by shocks:

$$y_s = \frac{F_s}{m\omega^2}A(\eta) \tag{7}$$

The folowing notations appear in relations (4), (6) and (7):

$$V(\eta) = \frac{\eta^2}{\sqrt{\left(1-\eta^2\right)^2 + 4D^2\eta^2}} \text{ - the increment function} \tag{8}$$

$$\varphi_D(\eta) = arctg\frac{2D\eta}{1-\eta^2} \text{ – the phase difference between the lack of balanced} \tag{9}$$

excitation, $m_e \cdot r \cdot \omega^2 \cdot \sin\omega t$, and the mill's vibration, *y(t)*, in which:

$$\eta = \frac{\omega}{\omega_y} \text{ - the synchronizing relationship between the perturbing frequency } \omega \tag{10}$$

and the vibrating mill's self angular frequency in vertical direction;

$$\omega_y = \sqrt{\frac{k_v}{m}} \text{ - the vibrating mill's self angular frequency in vertical direction} \tag{11}$$

$$D = \frac{c_v}{2\cdot m\cdot\omega_y} \text{ - the damping factor} \tag{12}$$

Taking into account the relations (6) and (7), the equation (5) can be solved also depending on F_s, the percution's amplitude, so that we'll obtain:

$$F_s = \left[\frac{m\cdot y}{A(\eta)} - m_e\cdot r\right]\omega^2 \tag{13}$$

In accordance with relation (13), the percution's amplitude can be determined depending on mill's vibration amplitude $\hat{y}$.

4. THE RESEARCHES' EXPERIMENTAL RESULTS

Between 1998 – 2000 the authors were observing a vibrating mill with cylindric rods type HUMBOLDT-PALLA U35 found at SC DUCTIL SA Buzău, used to obtain very fine powders wanted in tip electrodes' fabrication technology.

Using $\varphi = 20\,mm$ cylindric rods, for *1000 rot/min* normal speed (without inner pipe), some values for F_s were obtained depending on mill's filling degree and on the characteristic's value of the dynamic unbalance system $m_e r$ (the static moment).

These values allowed the plotting of graph showed in figure 2.

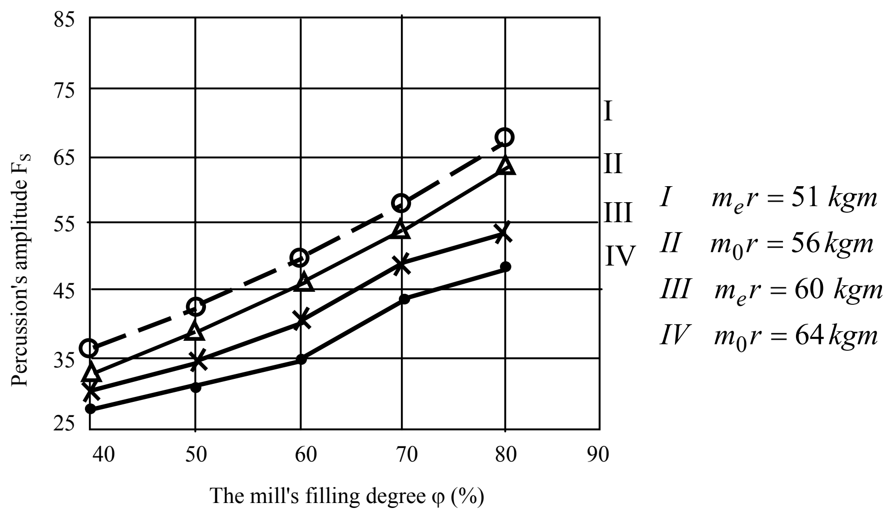

Fig.2. The F_s graph

5. THE MATHEMATICAL ASSESSMENT OF F_s TERM

Reporting the term F_s to the perturbing force's amplitude $m_e r\omega^2$, the dimensionless parameter S_u can be defined so:

$$S_u = \frac{F_s}{m_e r\omega^2} \tag{14}$$

Thus, the differential equation (3) will become:

$$m\ddot{y} + c_v\dot{y} + k_v y = m_e r\omega^2(1 + S_u)\sin\omega t \tag{15}$$

So, the dimensionless variable S_u specifies how many percents are to be added to the perturbing force's amplitude $m_e r\omega^2$ in case of taking also into account the existing shocks' influence.

In order to describe the vibrating mill's dynamic behaviour characterized by (15) we have to know the S_u function's variation according to the mill's filling degree φ and to the perturbing force's amplitude. Thus, a dimensionless quantity u is introduced:

$$u = \frac{m_e r \omega^2}{\left(m_e r \omega^2\right)_I} \tag{16}$$

In the definition above, the denominator refers to the perturbing force amplitude's smallest value.

The problem consists in representing the function $S_u = S_u(u, \varphi)$.

Due to the experiments done, some values for S_u were obtained depending on the mill's filling degree φ and on the perturbing force's amplitude $m_e r \omega^2$ corresponding to the normal speed $n = 1000 rot/\min$ (fig.2). This figure focuses that, according to $\varphi \in$[40%, 80%], the function $S_u = S_u(u, \varphi)$, can be represented like a straight line, the variation being almost balanced.

Therefore, aiming the assessment of F_S term, we can consider the dimensionless variable S_u having the following form:

$$S_u(u, \varphi) = a(u)(\varphi - 40) + b(u) \tag{17}$$

In the expression above, $a(u)$ represents the $f_1(u)$ function corresponding to the angular coefficient of each straight line apart, and $b(u)$ represents the $f_2(u)$ function corresponding to the ordinate at the fixed point for each straight line apart (φ is the mill's filling degree in percents).

The $a(u)$ and $b(u)$ coefficients will be determined so that the root mean square sum from the points to the straight line represents a minimum value.

Table 1 presents the coefficients of the $S_u(u, \varphi)$ function, using cylindric rods without inner pipe (running conditions: $n = 1000 rot/\min$, $\phi = 20 mm$).

Table 1

The eccentric's mass position	I	II	III	IV
$m_e r \omega^2$ (kN)	558,71	613,48	657,306	701,127
u	1,00	1,09	1,176	1,254
a	1,07	1,166	1,258	1,341
b	61,74	67,279	72,584	77,376

Each of $a(u)$ and $b(u)$ coefficients is represented as a fuction of the dimensionless quantity u , so that:

$$a(u) = p_3 u^3 + p_2 u^2 + p_1 u + p_0$$
$$b(u) = p_3 u^3 + p_2 u^2 + p_1 u + p_0 \tag{18}$$

In table 2 the coefficients p_i (i=1,2,3) are presented, both for $a(u)$ and for $b(u)$, being determined using the same approximate method mentioned above.

Table 2

	p_3	p_3	p_3	p_3
a	-	-0,803	2,687	-0,85
b	201,11	-842,04	2107,61	-503,75

So, in case cylindric rods are used :

$$S_u(u,\varphi)=\left(-0{,}8u^2+2{,}7u-0{,}85\right)(\varphi-40)+\left(201{,}1u^3-842u^2+2108u-504\right) \quad (19)$$

In accordance with figure 2, since the maximum value of $S(u)$ function appears for the mill's filling degree around $\varphi=80\%$, the $S_{u,\max}(u)$ is obtained considering $\varphi=80\%$ in the (19)-th equation.

7. CONCLUSIONS

In any type of vibrating mill working we can't neglect the effects produced by shocks appearing between the mill's chamber and the filling material (grist and milling bodies). The shocks focused by this paper are directly influencing the analysed vibrating mill's dynamic behaviour.

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