ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

STATIC TRANSMISSION ERROR

Sorin PATER, Ion BRATU, Tudor MITRAN, Nicolae POLOJIȚEF
University of Oradea, spater@uoradea.ro

Keywords: diagnostics, vibration, frequency

Abstract: The static transmission error describes the composite effect of *any* deviation of the gear teeth from perfectly formed involutes surfaces. The static transmission error is widely accepted as the principle source of vibration in gearboxes.

TRANSMISSION ERROR

Thus far, the discussion has been an idealization that has assumed that the gears are uniformly spaced, perfectly formed and completely rigid. Actual gears deviate slightly from perfect involutes surfaces and elastically deform under load. These tiny surface deviations result in an unsteady force component of the torque. The unsteady forces are transmitted as vibration from the gear, through the shaft and bearings to the gearbox casing where it is measured. It is this unavoidable vibration that we exploit to noninvasively investigate the operating condition of gearboxes while in operation.

The static transmission error describes the composite effect of any deviation of the gear teeth from perfectly formed involutes surfaces. The static transmission error is widely accepted as the principle source of vibration in gearboxes.

Mathematically, this unsteady forcing function due to the pinion is best described by a complex Fourier series with a fundamental frequency equal to the pinion rotational rate, f_r .

$$s(t)=\sum_{n=0}^{\infty} c_n e^{jn(2\pi f_r)t} \qquad (1)$$

The Fourier transform of the complex Fourier series is a one-sided pure line spectrum at multiples of the gear rotation rate.

$$S(f)=\sum_{n=0}^{\infty} c_n \delta(f-nf_r) \qquad (2)$$

We can now develop a mathematical expression for the composite vibration due to both the pinion and the gear. This is accomplished by summing two infinite complex Fourier series. Therefore, if the pinion has N teeth and the gear has M teeth, then equation 3 describes the composite vibration due to both gears.

$$s(t)=\sum_{n=0}^{\infty} c_n e^{jn(2\pi f_r)t} + \sum_{m=0}^{\infty} c_m e^{jm\left(2\pi f_r\left(\frac{N}{M}\right)t\right.} \qquad (3)$$

In equation 3, f_r is the pinion rotational frequency and $\left(\frac{N}{M}\right)f_r$ is the gear rotational frequency. The Fourier transform of the composite vibration is also a one-sided pure line spectrum.

$$S(f)=\sum_{n=0}^{\infty} c_n \delta(f-nf_r)+\sum_{m=0}^{\infty} c_m \delta\left(f-mf_r\left(\frac{N}{M}\right)\right) \qquad (4)$$

It should be clear that the two summations share a common set of frequencies. In the second summation, whenever m equals any integer multiple of M, the summations share the same frequency component. Therefore, equation 4 can be further decomposed as equation 5.

$$S(f) = \sum_{l=0}^{\infty} c_l \delta(f - lNf_r) + \sum_{n=0, n \neq lN}^{\infty} c_n \delta(f - nf_r) + \sum_{m=0, m \neq lM}^{\infty} c_m \delta\left(f - mf_r\left(\frac{N}{M}\right)\right) \quad (5)$$

If we then transform back to the time domain, we arrive at the following equation.

$$s(t) = \sum_{l=0}^{\infty} c_l e^{j(2\pi n (N f_r) t)} + \sum_{n=0, n \neq lN}^{\infty} c_n e^{j(2\pi n f_r t + \alpha_n)} + \sum_{m=0, m \neq lM}^{\infty} c_m e^{j(2\pi m f_r t + \beta_m)} \quad (6)$$

The first summation in equation 6 is composed of vibration components from both the pinion and the gear. The first summation's fundamental frequency (N f_r) is the gear mesh frequency. We have just shown that the components of vibration due to the gear mesh frequency and its harmonics are due to both the pinion and the gear. This is a fact that is neglected in the literature on gear diagnostics. In the next section, we will find that the summation over "in equation 6 is called the harmonic error signal. The second summation in equation 6 is then due solely to the pinion and the third summation is due solely to the gear. Also in the next section, we will find these two summations are called the residual error signals for the pinion and the gear. Thus, in the frequency domain, it should be possible to separate the vibration produced by different gears. Dr. Mark showed the component of the static transmission error that occurs at multiples of the gear meshing frequency is caused by *elastic tooth* deformations and the *mean* deviations of the tooth faces from perfect involutes surfaces. The remaining components of the static transmission error that occur at multiples of the gear rotational frequency are caused by the dynamic components of the tooth face deviations. Thus, we have a concrete, physical justification for using the static transmission error for gear diagnostics. The dynamic component of the static transmission error is a physical measure of any gear tooth surface deviation. This includes, but is not limited to, heavily worn teeth, missing teeth, and cracked or chipped teeth. In the next section, we will show how the residual error signal is defined as the dynamic component of the static transmission error.

GEAR MOTION ERROR

Bruce, B., [4] describe the gear motion error as the real part of the static transmission error. The gear motion error is a real signal, described by an infinite cosine series with fundamental period *f r* . The Fourier transform is then a two-sided pure line spectrum. Whereas the static transmission error was developed for predicting the amount of vibration produced by meshing gears, the gear motion error was developed for gearbox diagnostics. It should be noted that since we are dealing with real valued signals, the static transmission error and the gear motion error contain the same information and simply differ by a factor of 2. We will use the term gea*r* motion error and its interpretation throughout the remainder of this thesis. The decomposition of the composite gear motion error has three components -- the harmonic error component $s_{eh}(t)$, the residual error component due to the pinion $s_{er,p}(t)$, and the residual error component due to the gear $s_{er,g}(t)$, .

$$s(t) = s_{ch}(t) + s_{er,p}(t) + s_{er,g}(t) \quad (7)$$

Equation 7 is expressed graphically in figure 1. Notice how important good spectral resolution is when separating the vibration produced by the pinion and the gear.

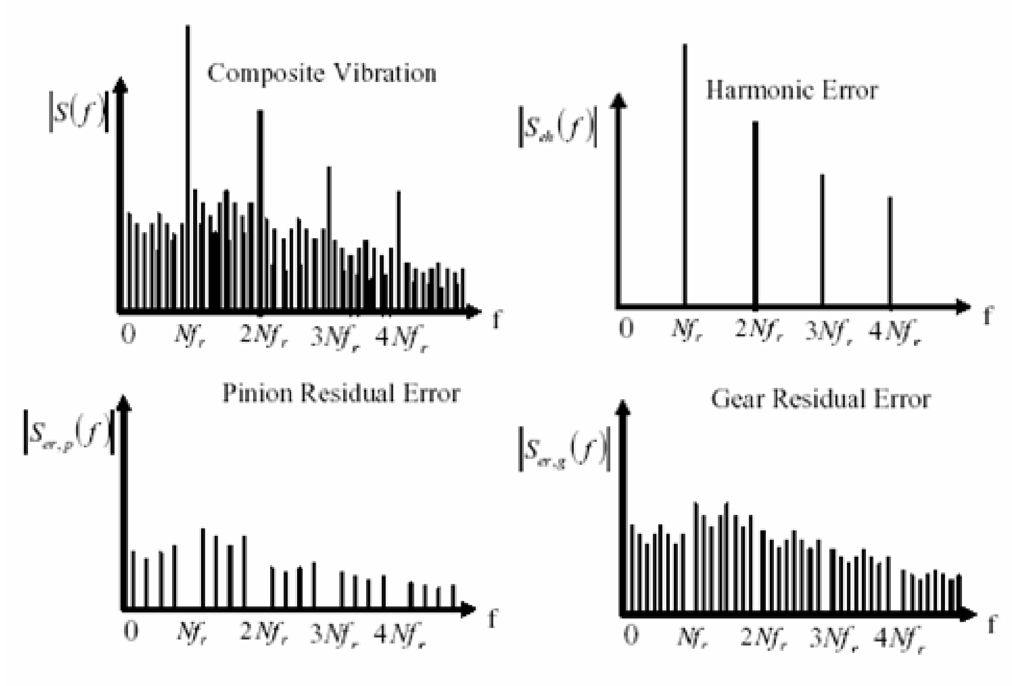

Figure 1-Graphical Representation of Equation 7 in the Frequency Domain

The situation is made increasingly difficult when monitoring multiple gear sets. It should be apparent that several frequency components in the spectrum will be sufficiently close together so that it may not be practical to resolve them with reasonably sized FFTs. In this case, it is possible for information from another gear to leak into a different gear's residual error signal. There is no easy way of dealing with this situation when performing time domain averaging. This is part of the motivation behind developing the Comblet basis function.

COMB FILTERS

A comb filter is the most natural way to extract fault information from a composite vibration signal. Why not create a wavelet that acts like a comb filter Then we can do away with the adaptive sampling needed to compute the motion error signals. This is accomplished by giving the comb filter "teeth" a small but finite pass band. Thus, even though we do not hit the shaft rate frequency bins exactly, we are still able to extract fault information. This approach also allows us to incorporate shaft speed measurement uncertainty into the filter design. This small pass band can also capture variability in the shaft speed. Additionally, a modular wavelet basis will also allow us the flexibility to neglect "ghost" frequency components that mask early fault detection in multiple gear systems. This is to say, there will be frequency bins that are contaminated by information from other vibration sources, which we can selectively neglect in our modular wavelet design. To my knowledge, the Comblet transform is the only signal processing technique for machinery diagnostics that addresses this issue.

An ideal comb filter is an unit amplitude periodically spaced impulse train in the frequency domain. A finite impulse response (FIR) comb filter is designed to increase the signal to noise ratio for periodic signals in broadband noise. The z transform of a FIR comb filter with D unity gain peaks at $\omega_k = \frac{2k\pi}{D}$ and D zeros at $\omega_k = \frac{(2k+1)\pi}{D}$

$$H_{comb}(z) = b\left(\frac{1+z^{-D}}{1-az^{-D}}\right) \tag{8}$$

where the parameters 'a' and 'b' are determined by the half power bandwidth, Δω, from the following design equation

$$\beta = \tan\left(\frac{D\Delta\varpi}{4}\right) \tag{9}$$

where

$$a = \frac{1-\beta}{1+\beta} \text{ and } b = \frac{\beta}{1+\beta}$$

REFERENCE

[1] Andrews, S.A., **Noise and Vibrations of Engines and. Transmissions**, MIMechE. University of Waster Australia, Conference Publications, 1979, p (47-57).
[2] Boyes, J.D. **Analysis Technique for Gearbox-Diagnosis Using the High Resolution FFT Analyses**, Bruel & Kjaer, Application Notes, nr.106, 1981.
[3] Brown, D.N. & Jorgensen, J.C., **Machine Condition Monitoring Using Vibration Analysis**, Bruel & Kjaer, Application Note, 1987.
[4] Bruce, B., **Precise control of vibratory stress relief**, Poolinght & Production, nov. 1983, p.(64-66).
[5] Dempsey, P.J., and Zakrajsek, J.J., **Minimizing Loan Effects on NA4 Gear Vibration Diagnostic Parameter**, NASA/TM-2001-210671, 2001
[6] Pater, S., Mitran, T., **Diagnosticarea sistemelor tehnice,** Universitatea din Oradea, 2006, ISBN 973-759-110-0