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MOMENT AMPLIFICATION FACTOR IN DESIGN OF SHOCK-ABSORBER ROD

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Abstract: *The bending of the shock-absorber rod calls for a second order analysis as it is a rather slender bar subjected to a considerable axial compressive load. The second order deflection analysis of the according mechanical model produces the theoretical moment amplification factor. The expression for the design moment amplification factor expressed by the theoretical buckling load is established. The diagrams of the buckling load are plotted.*

1. INTRODUCTION

The shock-absorber rod is subjected to both thrust and bending in a normal function of the shock-absorber. The bending stresses are usually determined through the first-order bending moment. The bending simultaneous with the compression calls for a second order deflection analysis as the rod can be considered a slender bar. The considerable axial force acts through the lateral deflection and thus produces additional bending moment which cannot be ignored. The presented paper deals with the determination of this additional moment.

It is customary to refer to the bending moment induced by the transverse load and this axial load effect as secondary bending moment and to refer to the bending moment caused by the primary bending effect as primary bending moment. The ratio of the secondary bending moment to the primary bending moment defines the moment amplification factor in a larger sense. The moment amplification factor in a limited sense is the ratio of the maximum secondary moment to the maximum primary moment. The design moment amplification factor is an approximate expression for the theoretical moment amplification factor. This paper determines the design moment amplification factor for the shock-absorber rod through the analysis of the obtained theoretical moment amplification factor. Thus the paper offers a simple formula, accompanied by an according diagram, which provides a more accurate determining of the shock-absorber rod diameter.

2.SECOND ANALYSIS

The second order analysis of the shock-absorber rod's deflection is carried out on a mechanical model shown in Fig.1a. The rod is approximated by a beam-column with fixed left end subjected to axial force *F* and transverse force F_1 acting at the free right end. The cylinder cover with the seal makes an elastically support for the rod. This support is approximated by a linear rotational spring of stiffness c_1 and a linear translational spring of stiffness c_2. The deflected beam-column is shown in Fig.1b. The elastically support is replaced by reactions $c_1\theta$ and $c_2\delta$, where θ is the slope and δ the deflection displacement at $x = a$. *The* primary bending moment diagram and the transverse force diagram are shown in Fig. 1c and Fig. 1d.

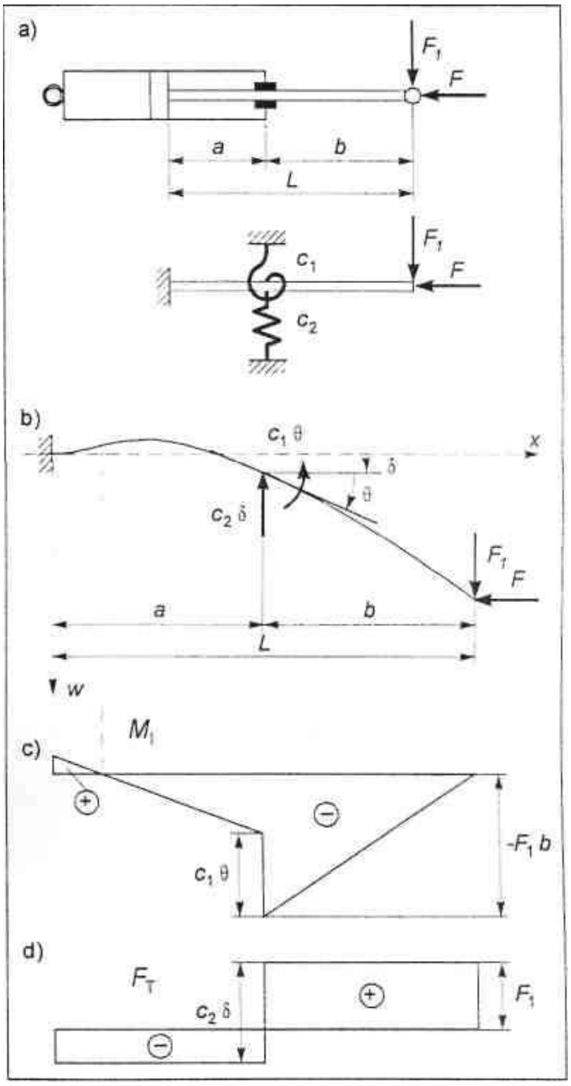

Fig.1. Mechanical model of the shock-absorber rod

Following the conditions of equilibrium of a deflected infinitesimal segment one obtains the differential equation of equilibrium of beam-columns. If a lateral distributed load is equal to zero and for small deflection, the basic differential equation of a beam-column relating the lateral deflection *w,* the axial thrust *F* and the transverse force F_T is

$$EIw''' + Fw' = -F_T \qquad (1)$$

where *EI* is the flexural rigidity of the beam-column, and ( )' = *d/dx* .The general solution to this equation is

$$w = A\cos kx + B\sin kx + C + f(x) \qquad (2)$$

where *A, B* and *C* are constants, *f(x)* is the particular solution of the differential equation and

$$k = \sqrt{\frac{F}{EI}} \qquad (3)$$

Equation (3) for the beam-column shown in Fig. 1 b takes the form

$$w_1 = C_1 \cos kx + C_2 \sin kx + C_3 - \frac{F_1}{F}x + \frac{c_2\delta}{F}x \qquad (4)$$

for 0 ≤ x ≤ *a* , and

$$w_2 = C_4 \cos kx + C_5 \sin kx + C_6 - \frac{F_1}{F}x \qquad (5)$$

for $a \le x \le L$.

The constants C_1 to C_6 are determined from the boundary conditions

$$w_1(0) = 0, \qquad w'_1(0) = 0$$
$$w_1(a) = w_2(a), \qquad w'_1(a) = w'_2(a) \qquad (6)$$

and

$$M(L) = 0, [M(a)]_L = [M(a)]_R + c_1\theta$$

which can be expressed as

$$w''_2(L) = 0, \quad -EIw''_2(a) + c_1 w'_2(a) \qquad (7)$$

The maximum secondary bending moment is (Fig. 1b)

$$M_{II\,max} = -F_1 b - F[w_2(L) - \delta] \qquad (8)$$

and the maximum primary bending moment is (Fig. 1c)

$$M_{I\,max} = -F_1 b \qquad (9)$$

Respecting δ = $w_1(a)$, one finally obtains the theoretical moment amplification factor

$$\propto_T = \frac{M_{II\,max}}{M_{I\,max}} = \frac{\operatorname{tg} u}{u}\left(1 + \frac{v\ \operatorname{tg} u + v^2 K_2 s_2}{s_1 + K_1 - v\ \operatorname{tg} u}\right) \qquad (10)$$

where

$$v = ka, \quad u = k(L-a) = kb \qquad (11)$$
$$K_1 = \frac{c_1 a}{EI}, \qquad K_2 = \frac{EI}{c_2 a^3} \qquad (12)$$
$$s_1 = \frac{v(\sin v - v\cos v + K_2 v^3 \cos v)}{2 - 2\cos v - v\sin v + K_2 v^3 \sin v} \qquad (13)$$
$$s_2 = \frac{v^2(1-\cos v)}{2 - 2\cos v - v\sin v + K_2 v^3 \sin v} \qquad (14)$$

3. DESIGN MOMENT AMPLIFICATION FACTOR

The design moment amplification factor approximates the theoretical moment amplification factor in the form

$$\propto_D = \frac{1 \pm d\frac{F}{F_c}}{1 - \frac{F}{F_c}} \qquad (16)$$

where - 0.2 ≤ d ≤ + 0.2 is the correction factor and F_c is the buckling load of the column (Petersen, 1982).

The buckling load can be determined by using the equations (4) and (5) if one inserts F_1 = 0, and by using the same boundary conditions (6) and (7). The condition of a nontrivial solution for the constants C_1 to C_6 produces a transcendental equation

$$\frac{a}{b} u \operatorname{tg} u - s_1 - K_1 = 0 \qquad (17)$$

From the smallest root u_0 of the equation (15), F_c can be determined by

$$F_c = \frac{EI}{L^2}\left(\frac{L}{b}\right)^2 u_0^2 \qquad (18)$$

The theoretical moment amplification factor has been determined for various K_1 and K_2, and for the ratios *alb* corresponding 0.1 ≤ *b/L* ≤ 0.9. The range of values of K_1 and K_2 has been determined through the analysis of F_T diagram, M_1 diagram and the first-order deflection of the beam-column. The determined values of the theoretical moment amplification factor have been approximated by the design amplification factor, defined by (15), for various values of *d.*

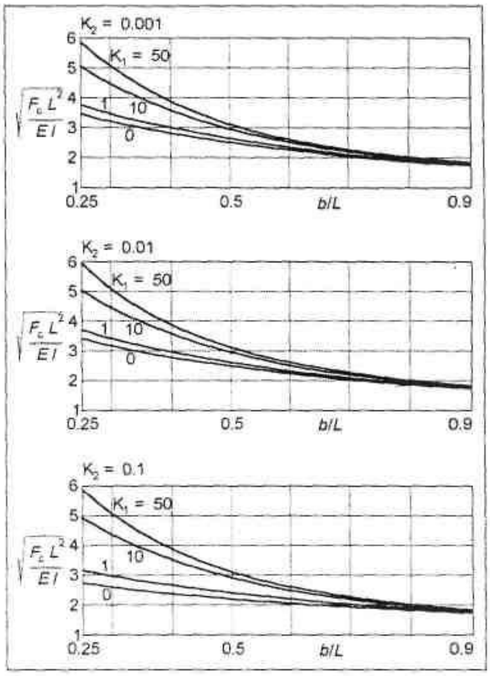

Fig. 2. The buckling load F_c as a function of b/L.

4. CONCLUSION

In the above-mentioned analysis it turned out that for F/F_C < 0.5 the theoretical moment amplification factor can be satisfactorily approximated by a design moment amplification factor

$$\alpha_D = \frac{1}{1-\frac{F}{F_C}} \qquad (19)$$

For 0.25 ≤ *b/L* ≤ 0.90 the expression (18) gives the moment amplification factor which varies from the theoretical moment amplification factor within the range from -10% (for a small rigidity of the cylinder cover: K_1 = 0, K_2 = 0.1) to +10% (for a considerable rigidity of the cylinder cover: for K_1 = 50, K_2 = 0.001) and for 0.5 ≤ *b/L* ≤ 0.9 the according variation is within the range from -2% to +10%.

5. REFERENCES

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