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RESEARCH CONCERNING THE ANALYSIS OF A ROBOT STRUCTURE USED FOR ORIENTATION TASKS

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Abstract: The paper presents a method that permits the analysis of a three degrees of freedom robot structure used for the orientation tasks achievement. The variation in time of the three generalized coordinates corresponding to the component active joints are determined using the rotation parameters method. Finally, the method is applied for the case when the corresponding rotation angle varies in time following a polynomial function.

1. THEORETICAL CONSIDERATIONS

In many industrial applications, the end-effector of a robot is guided to the object that is to be grasped and moved away. The location of the end-effector is controlled so that the gripper is moved to the object, while its orientation assumes an appropriate position for grasping.

In this paper, a method that permits the analysis of a three degrees of freedom robot structure (fig. 1) used for the orientation tasks achievement is presented. The variation in time of the three generalized coordinates: q_1, q_2 and q_3, corresponding to the active joints are determined using the rotation parameters method [4].

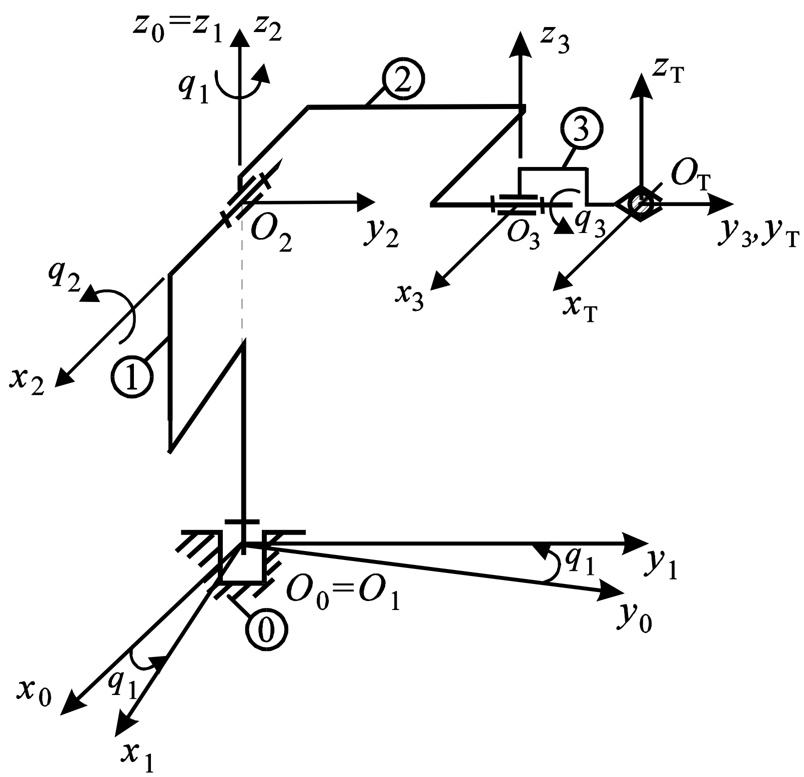

Fig. 1. Three degrees of freedom robot structure

The rotation matrix ${}^0R_T$ corresponding to the orientation of the object grasped by the robot with respect to the fixed system of coordinates $(O_0x_0y_0z_0)$ (the system of coordinates $(O_Tx_Ty_Tz_T)$ is attached to the object) can be calculated with the following relation:

$$ {}^0R_T = {}^0R_1 \cdot {}^1R_2 \cdot {}^2R_3 \cdot {}^3R_T = \begin{bmatrix} c1c3 - s1s2s3 & -s1c2 & c1s3 + s1s2c3 \\ s1c3 + c1s2s3 & c1c2 & s1s3 - c1s2c3 \\ -c2s3 & s2 & c2c3 \end{bmatrix} \quad (1) $$

where:

$$ \begin{cases} ci = \cos q_i \\ si = \sin q_i \end{cases} \quad i = \overline{1,3} \quad (2) $$

$$ {}^0R_1 = R(z,q_1) = \begin{bmatrix} c1 & -s1 & 0 \\ s1 & c1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3) $$

$$ {}^1R_2 = R(x,q_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c2 & -s2 \\ 0 & s2 & c2 \end{bmatrix} \quad (4) $$

$$ {}^2R_3 = R(y,q_3) = \begin{bmatrix} c3 & 0 & s3 \\ 0 & 1 & 0 \\ -s3 & 0 & c3 \end{bmatrix} \quad (5) $$

We consider that the system of coordinate $(O_Tx_Ty_Tz_T)$ has the same orientation as the system of coordinates $(O_3x_3y_3z_3)$ attached to the third module, so: ${}^3R_T = I_3$, where: I_3 is the unit matrix of rank three.

When the robot accomplishes an orientation task, the system of coordinates $(O_Tx_Ty_Tz_T)$ moves from an initial position, where: ${}^0R_T = {}^0R_i$, to a final position, where: ${}^0R_T = {}^0R_f$. This movement can be expressed with the rotation matrix ${}^iR_f$, where:

$$ {}^iR_f = {}^iR_0 \cdot {}^0R_f = {}^0R_i^T \cdot {}^0R_f \quad (6) $$

The change of orientation at the end-effector level can be expressed by a rotation around of an axis (Δ), with an angle θ_f. Using the rotation parameters method, the angle θ_f and the elements of the unit vector $\vec{u}$, corresponding to the axis (Δ), can be determined with the following relations [4]:

$$ \begin{cases} \theta_f = \pm \arccos \dfrac{tr\,{}^iR_f - 1}{2} \\ {}^{(i)}u^{(v)} = \dfrac{1}{2 \cdot \sin\theta_f} \cdot \left({}^iR_f - {}^fR_i \right) \end{cases} \quad (7) $$

where: $tr\,{}^iR_f$ calculates the sum of the elements from the principal diagonal of the matrix ${}^iR_f$; ${}^fR_i = {}^iR_f^T$; ${}^{(i)}u^{(v)}$ is given by:

$$ {}^{(i)}u^{(v)} = \begin{bmatrix} 0 & -{}^{(i)}u_z & {}^{(i)}u_y \\ {}^{(i)}u_z & 0 & -{}^{(i)}u_x \\ -{}^{(i)}u_y & {}^{(i)}u_x & 0 \end{bmatrix} \quad (8) $$

where: the vector ${}^{(i)}u = \left[{}^{(i)}u_x \quad {}^{(i)}u_y \quad {}^{(i)}u_z \right]^T$ contains the projections of the vector $\vec{u}$ on the axes of the system of coordinates $(O_T x_T y_T z_T)$ which is in the initial position.

When the angle of rotation θ varies from zero to θ_f, ${}^0R_T = {}^0R_i \cdot R(u,\theta)$, where the matrix $R(u,\theta)$ can be calculated with the following relation [4]:

$$R(u,\theta) = I_3 + \sin\theta \cdot {}^{(i)}u^{(v)} + (1-\cos\theta) \cdot \left[{}^{(i)}u^{(v)} \right]^2 \tag{9}$$

By taking into account the relation (1), the variation in time of the generalized coordinates: q_1, q_2 and q_3, can be determined with the following relations:

$$q_2 = (-1)^k \cdot \arcsin\left({}^0R_i \cdot R(u,\theta) \right)_{3,2} + k\pi; \quad k \in Z \tag{10}$$

$$q_1 = ATAN2\left(-\frac{\left({}^0R_i \cdot R(u,\theta) \right)_{1,2}}{\cos q_2}, \frac{\left({}^0R_i \cdot R(u,\theta) \right)_{2,2}}{\cos q_2} \right) \tag{11}$$

$$q_3 = ATAN2\left(-\frac{\left({}^0R_i \cdot R(u,\theta) \right)_{3,1}}{\cos q_2}, \frac{\left({}^0R_i \cdot R(u,\theta) \right)_{3,3}}{\cos q_2} \right) \tag{12}$$

where: ATAN2(y,x) calculates arctg(y/x) by taking into account the signs of the arguments.

2. SIMULATION RESULTS

We consider that the robot accomplishes an orientation task so that: ${}^0R_i = R(z,30°) \cdot R(x,10°)$ and ${}^0R_f = R(y,45°) \cdot R(z,20°)$. By applying the relations (7), we obtain: $\theta_f = 42.83°$; ${}^{(i)}u = \left[0.2215 \quad 0.927 \quad -0.3024 \right]^T$ (only the solution with the plus sign has been considered for the angle θ_f). The variation in time of the angle θ has the following expression: $\theta(t) = g(t) \cdot \theta_f$, where:

$$g(t) = 3 \cdot \left(\frac{t}{t_f} \right)^2 - 2 \cdot \left(\frac{t}{t_f} \right)^3 \tag{13}$$

where: $t_f = 5s$ represents the period of time during which the robot is moving.

In fig. 2÷4 a solution for the variation in time of the generalized coordinates: q_1, q_2 and q_3 is presented.

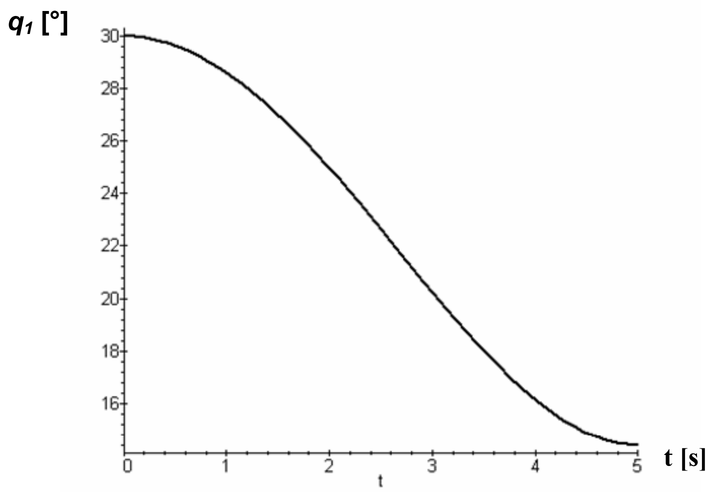

Fig. 2. The variation of q_1

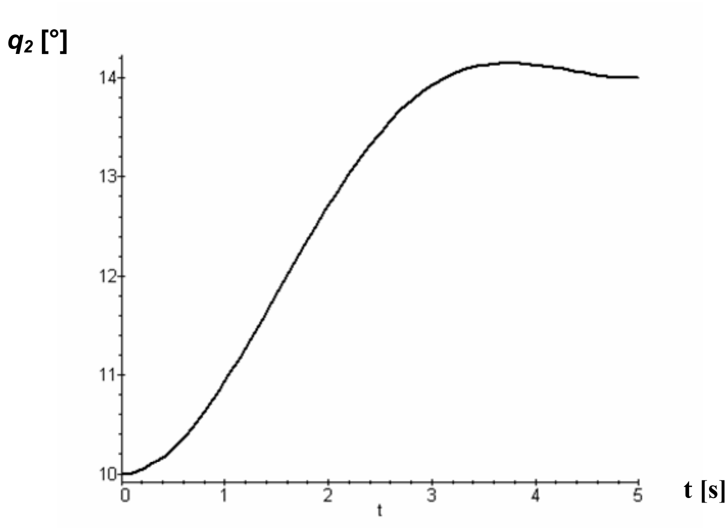

Fig. 3. The variation of q_2

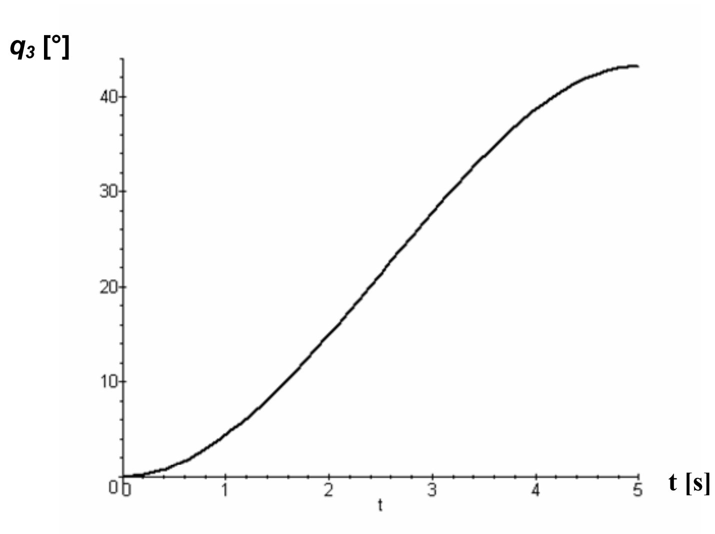

Fig. 4. The variation of q_3

3. CONCLUSIONS

The paper presents an efficient method that permits the analysis of a three degrees of freedom robots structure used for the orientation tasks achievement. The variation in time of the three generalized coordinates corresponding to the component active joints are determined using the rotation parameters method. The method can also be applied when the analyzed mechanism is a component of a more complex robot structure.

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