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DYNAMIC MODELLING OF N-CARDAN TRANSMISSIONS WITH SHAFTS IN SPATIAL CONFIGURATION.
Part II. THE ALGORITHM OF DYNAMIC MODELLING

Codruța JALIU, Radu SĂULESCU, Livia HUIDAN
Transilvania University of Bv, cjaliu@unitbv.ro

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Abstract: A general algorithm for the dynamic modelling of Cardan transmissions is presented in this paper. This algorithm is based on the assembling of analytical models (of modular type), previously established, and it is exemplified on a representative 2-Cardan transmission, with values from companies' catalogues. The modelling is made in the premise that the intermediary Cardan shafts are elastic; it is also analyzed the case in which the intermediary shaft is rigid.

1. INTRODUCTION

A library of *modular correlations* regarding the forces from the *typical modules* of Cardan transmissions was elaborated in the first part of this paper; the establishment of an algorithm for n-Cardan transmissions and the simplification of the modelling of forces and moments become possible by assembling these correlations.

A general algorithm for the establishment of forces and moments from the classical 2-Cardan transmissions, statically determined it is further presented in the premise that the intermediary shaft is elastic, by assembling the kinematical correlations with the modular ones. The technical data that are used in modelling this transmission are taken from companies' catalogues [3]. Firstly, for a distinct highlighting of the elastic intermediary shaft influence, the inertial effects are neglected. The proposed generalized algorithm is applied on the example of the 2-Cardan transmission from Fig. 1.

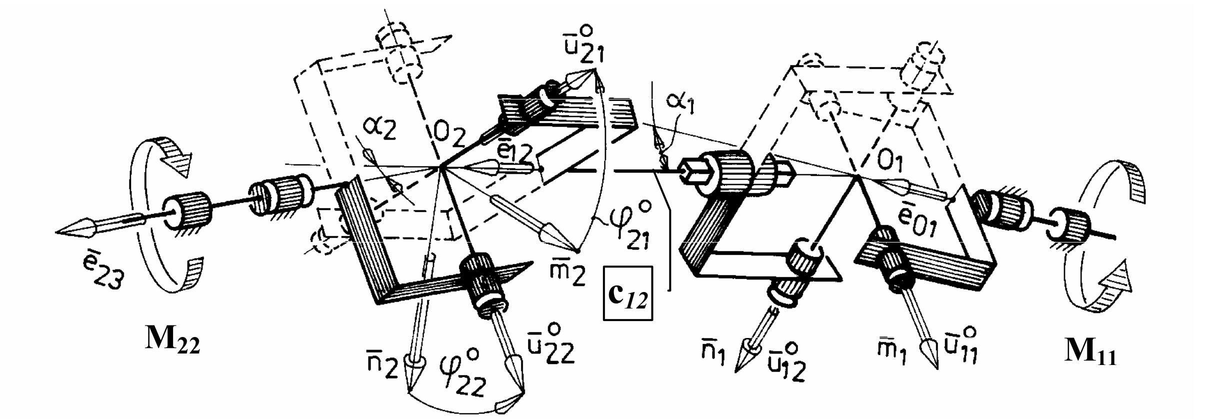

Fig. 1. *Variant of 2-Cardan statically determined transmission, with an elastic intermediary shaft*

2. NOTATIONS AND PREMISES

The notations and premises used in the previous paper are further valid. The following two premises are added: The intermediary shaft is considered elastic and the inertia moment is neglected. The elasticity of the intermediary shafts is described through the rigidity coefficient:

$c_{ij} = T_{j1}/\xi_{ij}$ [Nm/rad],

in which T_{j1} represents the moment of rotation for the $j1$ fork, and ξ_{ij} designates the angular deformation, meaning the angular difference between the rotation angles of the ij intermediary shaft forks:

$$\xi_{ij} = \varphi_{i2} - \varphi_{j1} \quad (1)$$

For the intermediary shafts made of steel tube, the rigidity coefficient and the inertia moment have a well-known expression [7]:

$c = \dfrac{\pi \cdot G \cdot \left(D^4 - d^4\right)}{32 \cdot l}$ [Nm/rad], in which:

• l_{ij} [mm] represents the length of the intermediary Cardan shaft,
• G = 81000 [N/mm^2] represents the module of transversal elasticity for steel,
• D, d [mm] represents the external diameter and internal diameter, respectively, of the intermediary Cardan shaft.

3. ADAPTATION OF KINEMATICAL CORRELATIONS TO THE PREMISE OF ELASTIC SHAFTS

Unlike the kinematical modelling of the n-Cardan transmissions with rigid elements, in the modelling of the transmissions with elastic intermediary shafts, the equalities $\varphi_{i2} = \varphi_{j1}$ are transformed in equalities of $\varphi_{j1} = \varphi_{i2} - \xi_{ij}$ type; the other correlations remain unchanged. For exemplification, the kinematical correlations necessary in modelling the 3-Cardan transmissions with elastic intermediary shafts are further presented:

$$\varphi_{11}^0 = 0, \ \varphi_{i2}^0 = \operatorname{arctg}\left(\frac{\operatorname{tg}\varphi_{i1}^0}{\cos\alpha_i}\right), \ \varphi_{j1}^0 = \beta_{ij} + \varphi_{i2}^0 - \nu_{ij} - 90°; \quad (2)$$

$$\varphi_{i2} = \operatorname{arctg}\left(\frac{\operatorname{tg}\left(\varphi_{i1} + \varphi_{i1}^0\right)}{\cos\alpha_i}\right) - \varphi_{i2}^0, \ \varphi_{j1} = \varphi_{i2} - \xi_{ij}; \quad (3)$$

$$\operatorname{tg}\theta_{i1} = \operatorname{tg}\alpha_i \cdot \sin\left(\varphi_{i1} + \varphi_{i1}^0\right), \ \operatorname{tg}\theta_{i2} = \frac{\sin\alpha_{i1} \cdot \cos\left(\varphi_{i1} + \varphi_{i1}^0\right)}{\sqrt{1 - \sin^2\alpha_i \cdot \cos^2\left(\varphi_{i1} + \varphi_{i1}^0\right)}}; \quad (4)$$

Taking into account the fact that the changes of the moments of rotation for an intermediary shaft is usually reduced in relation to its medium value, the simplification $\xi \approx$ constant can be admitted; therefore, the angular speeds and accelerations can be deduced with the following relations:

$$\omega_{i2} = \omega_{i1} \cdot \frac{A_i}{1 + \left(A_i^2 - 1\right) \cdot \sin^2\left(\varphi_{i1} + \varphi_{i1}^0\right)}, \quad A_i = \frac{1}{\cos\alpha_i}, \ \omega_{j1} \approx \omega_{i2}; \quad (5)$$

$$\varepsilon_{i2} = \frac{A_i \cdot \varepsilon_{i1} + \omega_{i1} \cdot \omega_{i2} \cdot \left(1 - A_i^2\right) \cdot \sin 2\left(\varphi_{i1} + \varphi_{i1}^0\right)}{1 + \left(A_i^2 - 1\right) \cdot \sin^2\left(\varphi_{i1} + \varphi_{i1}^0\right)}, \ \varepsilon_{j1} \approx \varepsilon_{i2}. \quad (6)$$

4. ADAPTATION OF THE GENERAL ALGORITHM TO THE PREMISE OF ELASTIC INTERMEDIARY SHAFTS

On the base of the *modular correlations* established in the first part, the following general algorithm for the modelling of the forces and moments from the statically determined 2-Cardan transmissions is further proposed (Fig. 1):
There are known: the kinematical scheme of the n-Cardan transmission and its *state quantities*:

φ_{11}, ω_{11}, ε_{11} (*independent motion*), $\varphi_{11}^0 = 0$; α_1, α_2; ν_{12}; β_{12}; l_{12}; J_{12}; r; a_{11}, b_{11}, J_1;…; a_{ij}, b_{ij}, J_n; $F_{\mu ij}$; M_{n2} (*independent moment*).

• The quantities φ_{i1}^0, φ_{i2}^0; φ_{i1}, φ_{i2}; θ_{i1}, θ_{i2}, ω_{ij} and ε_{ij} for the component couplings are established [8].
• Starting from the output shaft *n2* and based on rel. (1 part - I), the moments of rotation ($T_{i1,2}$), bending moments ($B_{i1,2}$) and resultant moments ($K_{i1,2}$) that load each Cardan shaft fork are successively established in terms of the resistant moment M_{n2}.
The *resultant* bending moment of an intermediary shaft *ij* is obtained through a vectorial summing of the bending moments from the *ij* shaft forks: $\overline{B}_{i2}$ and $\overline{B}_{j1}$ (see Fig. 5 part I).
• The axial ($A_{i1,2}$) and radial ($H_{i1,2}$ şi $R_{i1,2}$) forces of each shaft's forks are established, establishing, at the beginning, the axial and radial forces for the forks *i*2 and *j*1 of the unsupported telescopic shaft *„ij“,* on the base of rel. (5, part I);
• The reactions from the extremal shafts' bearings (input and output) are calculated with rel. (9, part I).
On the base of the forces and moments from the Cardan forks (*T*, *B*, *K*; *A*, *R*, *H*) and based on rel. (3 and 4, part I), the forces that are specific to the contact zones between the forks and the crosses are established.

In the premises of elastic intermediary shafts, the relations in which interferes the influence of rigidity c are modified. Thus, the moment of rotation on a Cardan shaft, considered elastic, becomes:

$$T_{j1} = \xi_{ij} \cdot c_{ij} \tag{7}$$

For exemplification, the algorithm for 2-Cardan transmissions like those from Fig. 1, in a modular variant, is presented synthetically in Fig. 2, as a flow chart.

In the case in which the intermediary shaft is considered rigid, its elasticity ξ_{12} becomes null, the presented algorithm being applicable in this kind of situations, as well.

Calculus programs can be elaborated on the base of the proposed algorithm, using different commercial software. This kind of program, elaborated in Excel, was run for a representative set of numerical data for the considered 2-Cardan transmission; this data are written above the Figures. As an example of numerical simulation, diagrams for the bending and rotation moments from this transmission are further presented (see Fig 1 and 2).

5. NUMERICAL SIMULATIONS

After running the program which was elaborated on the base of the previous algorithm and the Excel software, a set of representative diagrams was obtained, part of which being illustrated in Fig. 3 and 4. These diagrams illustrate the variations of the angular differences (Fig. 3, a and 4, a) and rotation moments (T_{ij}) (Fig. 3,a$_1$, a$_2$ and 4,a$_1$, a$_2$) for the analyzed example, both in dynamic conditions (considering the inertia moments of the Cardan shafts) and in static conditions (neglecting the inertial effects). The considered state parameters are written above each Figure.

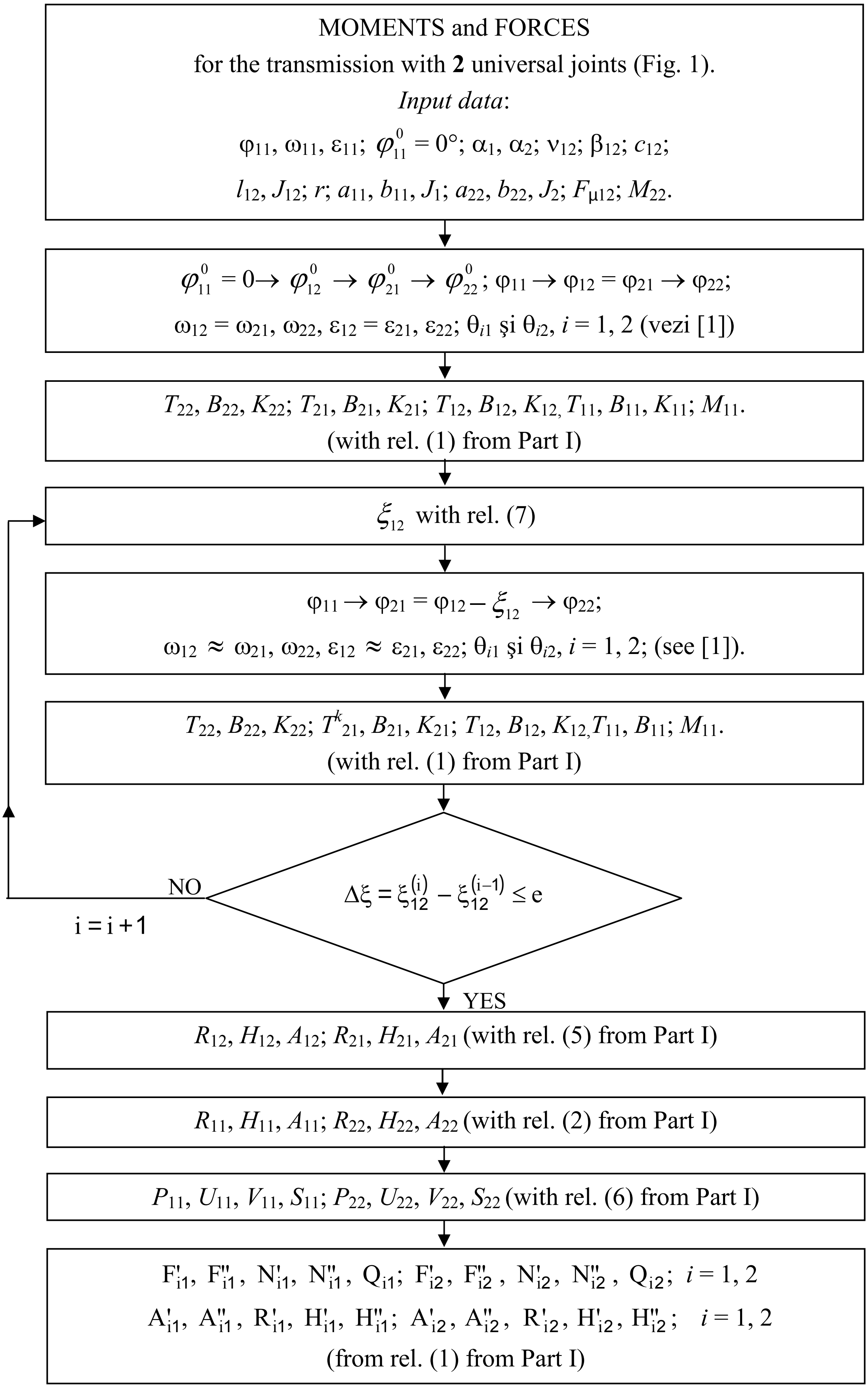

Fig. 2. *The flow chart of the algorithm for the modular modelling of a 2-Cardan transmission, in the premise of elastic intermediary shaft*

$$\alpha_1 = \alpha_2 = 35^o, \varphi_{11}^0 = 0^o, \nu_{12} = 0^o, \beta_{12} = 180^o,$$
$$\omega_{11} = 1[rad/s], \varepsilon_{11} = 0[rad/s^2], M_2 = 150[Nm]$$

$$c_{12} = \infty[Nm/rad], J_{12} = 0[kgm^2], \quad c_{12} = \infty[Nm/rad], J_{12} = 3{,}41 \cdot 10^{-4}[kgm^2]$$

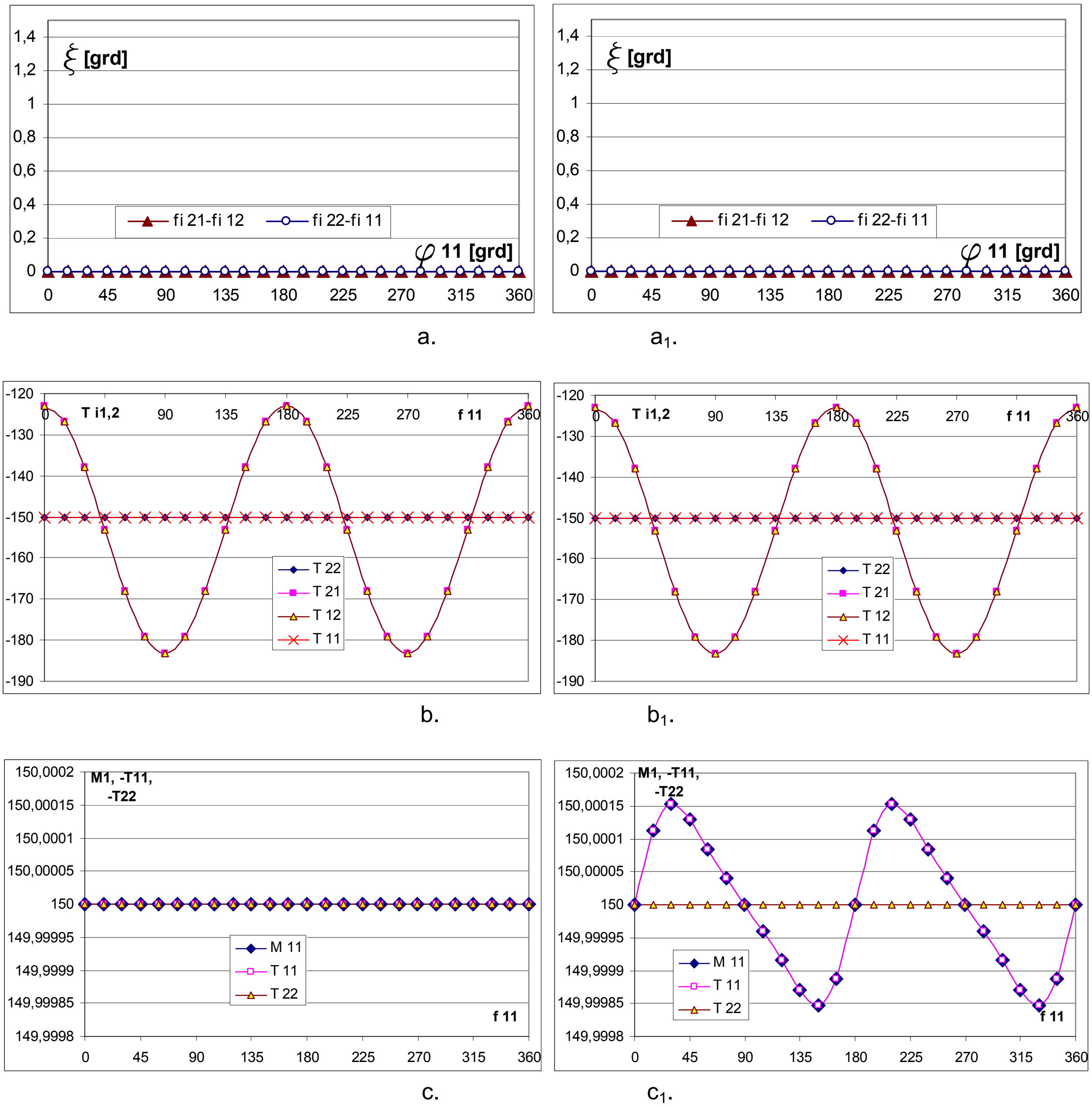

Fig. 3. *The variations of the angular differences between the intermediary shaft's forks (ξ_{12}) and the rotation moments (T_{ij}), in the premise of considering a rigid intermediary shaft: without inertial effects (a ...c) and with inertial effects (a_1...c_1).*

$$\alpha_1 = \alpha_2 = 35^o, \varphi_{11}^0 = 0^o, \nu_{12} = 0^o, \beta_{12} = 180^o,$$
$$\omega_{11} = 1[rad/s], \varepsilon_{11} = 0[rad/s^2],\ M_2 = 150[Nm]$$

$$c = 9526{,}896[Nm/rad], J_{12} = 0[kgm^2],\ c = 9526{,}896[Nm/rad], J_{12} \neq 0[kgm^2]$$

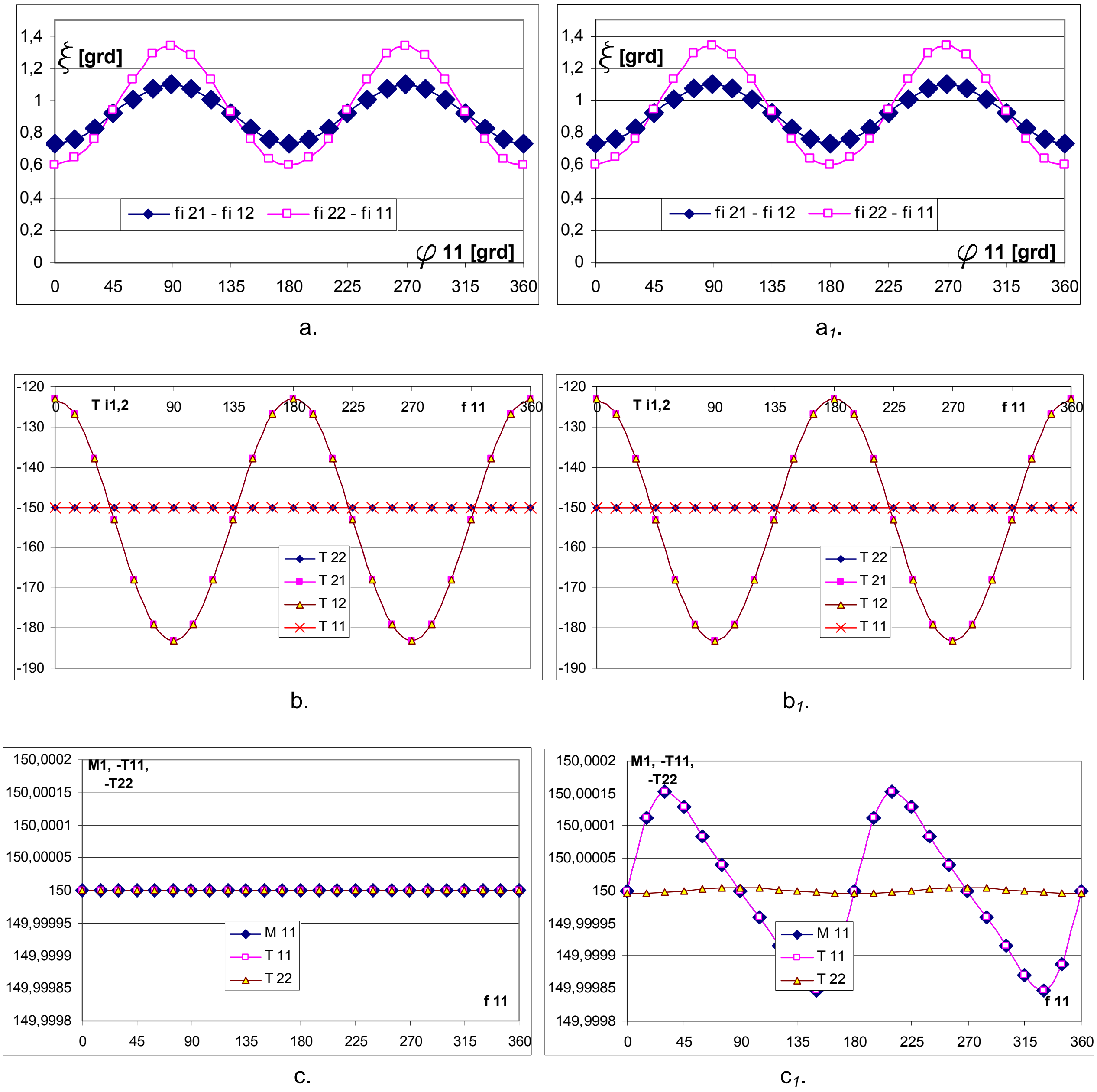

Fig. 4. *The variations of the angular differences between the intermediary shaft's forks (ξ_{12}) and the rotation moments (T_{ij}), in the premise of considering an elastic intermediary shaft: without inertial effects (a ...c) and with inertial effects (a_1...c_1).*

From these numerical simulations result, on one side, the variations of the angular differences between the intermediary Cardan shaft's forks and of the rotation moments that load the elements of the 2-Cardan transmission in terms of the rotation angle of the input shaft and, on the other side, the influence of the Cardan shafts' inertia on the rotation moments, by nullifying the inertia moments J_1, J_{12} si J_2 .

The comparative analysis of the diagrams from Fig. 3 and 4 highlights the fact that the influence of the Cardan shafts' inertia, in normal conditions of use is practically negligible. Thus, in static conditions (Fig. 3,c and 4,c), the input rotation moment (T_{11}) coincides, in module, with the output rotation moment (M_{22}=150 Nm); in dynamic conditions (Fig. 3,c_1 and 4,c_1), the moment T_{11} has small sinusoidal oscillations that are practically negligible.

6. CONCLUSIONS

In the conditions of the considered numerical example, it can be observed that the elasticity of the intermediary shaft influences mainly the transmission's kinematics and has a reduced influence on the rotation moments (see Fig. 4,a, a_1 and c_1). Therefore, the main conclusions that result from the numerical simulations systematized in Fig. 3 and Fig. 4, can be synthetically formulated as follows:

- The analyzed transmission is homokinetic due to the fact that the input and output rotation moments coincide in module, in the conditions of neglecting friction and inertial effects (T_{22}=T_{11} – see Fig. 3,c and 4,c); in the mean time, the amplitude of the rotation moments of the intermediary shafts' forks (T_{12}, T_{21}) describes the dynamic load of the same shaft.
- While the extremal Cardan shafts are loaded with constant or approximately constant rotation moments, the intermediary shaft is loaded with an oscillating rotation moment, of cosinusoidal type, whose amplitude is of approx. 13 % from the medium value.
- In usual applications, the axial inertia moments of the Cardan shafts have insignificant influence on the transmission forces and moments (see Fig. 3,c_1 and 4,c_1).
- According to Fig. 4,a, and a_1 the variations of the angular differences that describe the rotation deformations of the intermediary shaft ($\varphi_{21} - \varphi_{12}$), are reduced, in relation to their medium variations; thus, it is justified the simplifying premise ξ_{ij} =constant (particular case when the intermediary shaft is considered rigid).
- The influences of the intermediary shafts' elasticity are highlighted in Fig. 4,c_1, cumulated with their inertial effects.
- The dynamics of the n-Cardan transmissions can be approached similarly.

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