ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

EYE-MOVEMENT MODELLING

Daniela Mariana BARBU
Transilvania University of Brasov, Fine Mechanics and Mechatronics Department
e-mail: dbarbu@unitbv.ro

Keywords: model, eye, eye plant, eye movement, dynamics

Abstract: Modeling the eye plant in order to generate various eye movements, has been the topic among neurologists, physiologists, and engineers for a long time. The eyes rotate with three degrees of freedom: horizontal, vertical and torsional. Addressing the question of modeling the eye plant to mimic the realistic eye movements involves a three dimensional approach. A model of the neuromuscular mechanics of horizontal eye motion is developed. The model of the oculomotor system that is presented incorporates known physiological dynamics and geometry of the musculotendon complex. Muscle force development is a described by a two component version of Hill's model and consists of a passive and active contractile component. The active component includes the force-velocity and force-length characteristics of the muscle. The passive component accounts for elastic and viscous effects. Activation dynamics couple the neural controls that are appropriate for saccadic movements to the muscle mechanics.

I. INTRODUCTION

The modeling of the human ocular system and its dynamic properties have been extensively studied by neurologists, physiologists and engineers. One of the first models of eye movement was developed by Descartes in 1630, based on the principal of reciprocal innervation, a notion of paired muscular activity in which a contraction of one muscle is associated with the relaxation of the other. In 1954 Westheimer developed a linear second order approximation of eye dynamics during a saccade, in which the input to the model was assumed to be a step of muscle force. The model worked well for 12^0 saccades but not for larger such movements. In addition, the model predicted the unphysical results that the time of saccade duration would be independent of saccade magnitude and that the peak velocity would be directly proportional to saccade magnitude.

A more realistic representation of eye movement was advanced by Robinson. His linear fourth-order model could simulate saccades between 5^0 and 40^0 but the velocity profiles predicted by this model were not physically realistic. It was recognized by Westheimer and Robinson that the eye movement mechanism was inherently nonlinear issues not addressed by their work. Roughly speaking, the nonlinear features of the system can be attributed to the geometry of the system as well as the nonlinear physiological behavior of certain components that describe the extraocular muscle.

Martin and Lu developed a model of the eye system that assumed a linear model of muscle behavior but accounted for the nonlinear effects that occur when the recti muscles act in a nontangential fashion on the eyeball. They were able to construct a control law that enabled the eye to track a target through a range of both large and small displacements. The muscle model that was utilized omitted some physiological features of muscle and did not distinguish the effects of passive and active muscle behavior, a notion that will be elaborated upon later.

Another group of investigators have concentrated upon ocular models that emphasize the effects of muscular physiology upon system performance. Along these lines a sixth order nonlinear model proposed by Cook and Stark produced realistic position, velocity and acceleration profiles. This Cook-Clark-Stark model addressed the nonlinear relationship between force and velocity but ignored the force-length characteristics of muscle. This assumption was tantamount to assuming that the medial and lateral rectus muscles operate near the primary position that corresponds to looking straight ahead. Their model incorporates a force-velocity dependence into the active muscle by a velocity dependent viscosity that is experimentally determined by fitting experimental data to Hill’s equation. The model does not include any passive viscosity and moreover, the passive elasticity is lumped together with the nonmuscular suspensory passive tissue.

The model of the oculomotor system presented by Martin and Shovanec, that is presented, incorporates known physiological dynamics and geometry of the musculotendon complex. In particular the model for muscle force development is a two component version of Hill’s model and consists of a passive and active contractile component. The development allows for the inclusion of very general force-velocity and force-length characteristics in the active component. The muscle model that is utilized here includes passive elastic and viscous effects. For rapid eye movements, the passive parallel elasticity is important. In this paper, attention is focused upon saccadic eye movements which are among the fastest voluntary muscle movements the human body is capable of producing. The eye model includes activation dynamics that couple neural controls which are appropriate for saccadic movements to the muscle mechanics. It should be noted that the model which is investigated here does not account for the geometric nonlinearities. A justification for this assumption is that for saccadic movements, when motion is typically less than 30^0, the nontangential forces associated with the recti muscles do not occur.

Recent anatomical studies of extraocular muscles (EOM) demonstrate the stability of muscle paths. This is due to the fact that each rectus EOM passes through a pulley consisting of an encircling ring or sleeve of collagen. In this paper, the EOMs are modeled using the Hill type musculotendon complex and the effect of extraocular pulleys are studied. The model proposed by Martin and Schovanec in 1999, for horizontal eye movement has been used as a basic starting point. The extraocular pulleys are then introduced and analyzed mainly to study how Listing’s law is enforced and how it implements an oculomotor plant which appears commutative to the brain.

If the eye is moved from one fixation to another, in theory, there are unlimited ways to orient the axis about which the eye rotates in 3-D space. But in reality, eye is constrained in its torsional freedom. This restricts the three-dimensional space of all possible orientations to a two-dimensional subspace. Listing and Helmholtz further investigated and determined to which two-dimensional subspace the eye is restricted. Listing’s law, a specific case of more general Donders’ law, states that any physiologic eye orientation can be reached from a particular eye position known as the primary position, by rotation around a single axis, and that all such possible axes lie in a single plane known as Listing’s plane. Unless the trajectory follows a radial line passing through the primary position, the rotation axis used to move the eye from one position to another, obeying Listing’s law, tilts out of Listing’s plane. Experiments done on normal human subjects and rhesus monkeys confirm this notion, i.e., if a trajectory is orthogonal to the radial line, the ocular rotation axis tilts out of Listing’s plane by exactly half the angle of the eye’s eccentricity for saccadic and smooth-pursuit eye movements. This is known as the “half-angle rule”. Similar geometrical fact is observed for the vestibulo-ocular reflex (VOR). However the tilt angle here is only a quarter of the eye’s eccentricity, hence “quarter-angle rule”.

When pulleys were not known, Listing’s law was presumed to be enforced by a neural circuitry issuing complex commands to the extraocular muscles (EOMs). But experiments have failed to identify such a neural substrate for Listing’s law. It is also clear that the torsional component is generated somewhere downstream from the superior colliculus since the later encodes saccades as two dimensional (horizontal and vertical) rate of change of eye orientation. However, during VOR and sleep the Listing's law is violated implying that there is some kind of a neural basis.

The muscle path stability due to pulleys introduces a new mechanical basis on enforcing Listing's law. Figure 1 shows the arrangement of horizontal rectus EOMs. The rotational axis is always perpendicular to the plane containing the lines connecting pulleys with the scleral insertion. Therefore the rotational axis for straight ahead gaze in *A*, is vertical, i.e. perpendicular to the horizontal plane P_H. In *B*, the fixation is at a horizontally centered target at an elevation ϕ. The tilt of the rotational axis becomes ϕ_2 and is perpendicular to the plane containing the lines connecting pulleys with the scleral insertion and the center to the scleral insertion. This *half-angle plane* is shown as P_{ϕ_2}. During VOR, the pulleys have to be displaced in such a way that the *quarterangle rule* is satisfied. This implies the existence of a neural basis that causes the pulleys to shift posteriorly during VOR. One explanation to this phenomena is that there are separate motor neuron pools, or there is a way of adjusting the synaptic input weights in the same neuron pool, causing pulleys to move further posteriorly during VOR [1].

2. MODEL OF THE EYE

The model used here was first proposed by Martin and Schovanec [1] for horizontal saccadic eye movements. In the original model the geometrical implications due to pulleys were not considered. In this study, it is attempted to modify the model so that the resulting ocular plant would follow Listing's eye positions. The Hill-type model [3] used for the musculotendon complex, has been shown to incorporate enough complexity while remaining computationally practical.

The eye is represented as a solid sphere with moment of inertia J_G. This sphere is rotating about a fixed point due to the moments of the six extraocular muscles attached to it. But the motion of the sphere is constrained by Listing's half angle rule (Figure 1) and the muscles satisfying the isovolumic requirement [3].

The recent notion of enforcing the Listing's law due to pulley motion, can also be explained using this model. The constant volume requirement defines the pennation angle α Listing's half-angle rule requires the rotational axis to be tilted backward by an angle of $\phi/2$ as shown in Figure 1. The moment vector $\hat{m}$, which is along the rotational axis will be perpendicular to a plane given by $P_{\phi/2}$. Thus the radial distance vector from the center of the eye globe to the scleral insertion and the vector which represents the tendon force, lie on the plane $P_{\phi/2}$, i.e.,

$$\left(\vec{F} \times \vec{r}\right) \times \hat{m} = 0 \tag{1}$$

where $\vec{r}$ is the radial vector from the center of the eye to the scleral insertion. This uniquely determines the lateral and medial rectus pulley locations. The kinematics of the superior and inferior rectus muscles (which are mainly responsible for vertical eye movement) follows a similar analysis.

The dynamics the eye, which is represented as a sphere rotating about a fixed point, are described by Euler's equations,

$$\begin{aligned} M_x &= J_G \dot{\omega}_x \\ M_y &= J_G \dot{\omega}_y \\ M_z &= J_G \dot{\omega}_z \end{aligned} \tag{2}$$

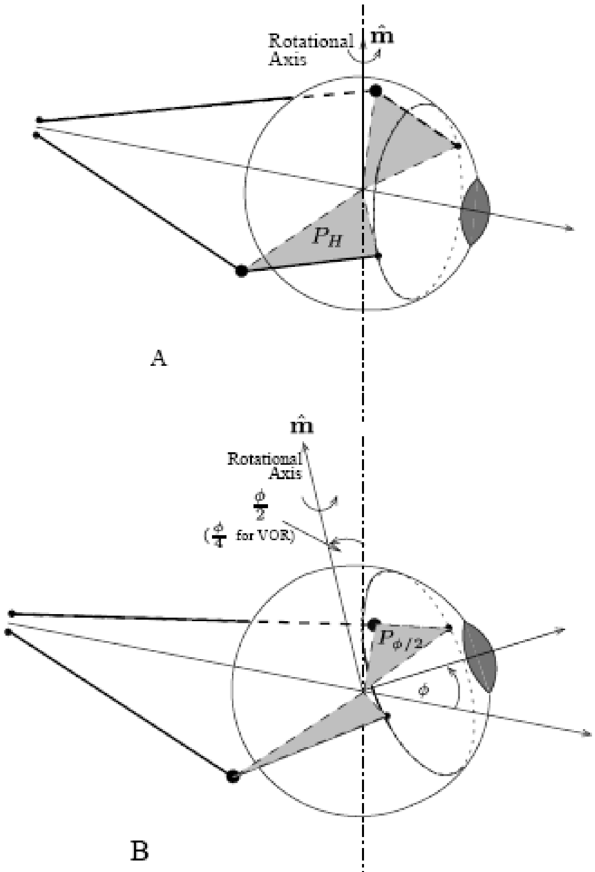

Fig. 1: Effect of pulleys on enforcing the Listing's law.
A: The rotational axis is vertical. B: When the eye is in a secondary position of elevation of an angle ϕ (EOMs - thick lines and pulleys - dark dots).

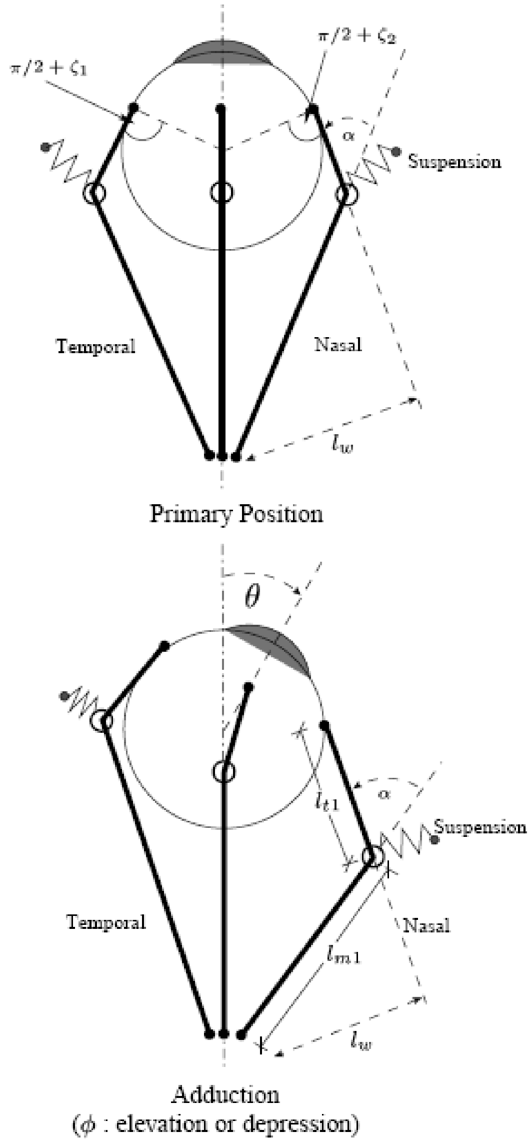

Fig. 2. Superior view of the eye showing the shifts in horizontal rectus pulley posi-tion required to satisfy half-angle rule in tertiary positions of adducted elevation and depression. Pulleys are shown as rings.

This can be written in terms of the six moments generated by each muscle and a passive moment produced by orbital tissues, as,

$$\begin{bmatrix}\dot{\omega}_x\\ \dot{\omega}_y\\ \dot{\omega}_z\end{bmatrix}=\frac{1}{J_G}\left(\sum_{i=1}^{6}M_i+M_p\right)=\frac{1}{J_G}\left(\sum_{i=1}^{6}\vec{F}_{ti}\times\vec{r}_i+M_p\right) \quad (3)$$

In order to obtain the model in rotational velocities $\dot{\theta}$, $\dot{\phi}$ and ψ, the notion of Eulerian angles can be used. Thus taking $\vec{\omega}=\left(\omega_x\omega_y\omega_z\right)^T=[\theta\phi\psi]^T$,

$$\vec{\omega}=\underbrace{\begin{pmatrix}0 & \cos\theta & \cos\phi\sin\theta\\ 0 & \sin\theta & \cos\theta\phi os\theta\\ 1 & 0 & \sin\phi\end{pmatrix}}_{M}\dot{\overline{\theta}} \quad (4)$$

i.e., $\vec{\omega} = M\dot{\bar{\theta}}$; $\dot{\vec{\omega}} = \dot{M}\dot{\bar{\theta}} + M\ddot{\bar{\theta}}$. Let $\bar{\theta}_1 = \bar{\theta}$; $\bar{\theta}_2 = \dot{\bar{\theta}}$

Then,

$$\dot{\bar{\theta}}_1 = \bar{\theta}_2$$

$$\dot{\bar{\theta}}_2 = \ddot{\bar{\theta}} = M^{-1}\left[\dot{\omega} - \dot{M}\dot{\bar{\theta}}\right] = M^{-1}\left[\frac{1}{J_G}\left(\sum_{i=1}^{6}\vec{F}_{ti}\times\vec{r}_i + M_p\right) - \dot{M}\bar{\theta}_2\right] \quad (5)$$

The torsional component ψ can be eliminated using

$$\psi = \cos^{-1}\left(\frac{\sin\theta\sin\phi}{1+\cos\theta\cos\phi}\right) \quad (6)$$

which is due to Listing's law.

In order to satisfy the Listing's law, the rotational axis shown in Figure 1 should remain perpendicular to the plane containing the lines connecting pulleys with the scleral insertion and the center to the scleral insertion. For this to happen, the pulleys move in such a way that the muscle maintains a constant volume by keeping l_w (Fig. 2) constant throughout [3].

3. CONCLUSION

In order to satisfy the Listing's law, the rotational axis shown in Figure 1 should remain perpendicular to the plane containing the lines connecting pulleys with the scleral insertion and the center to the scleral insertion. For this to happen, the pulleys move in such a way that the muscle maintains a constant volume by keeping lw constant throughout [3].The purpose of the Listing's law is however still not very clear. One most suggesting idea is that it reduces the computational or physical work of some system [1].

REFERENCES

[1] Martin, C., Schovanec, L., (1999), The control and mechanics of human movement systems, Progress in Systems and Control Theory, nr. 25/1999;
[2] Tomassone, R., (1991), Biometrie: modelisation de phenomenes biologiques, Paris;
[3] Valenta, J., (1993), Biomechanics, Ed. Elsevier, Amsterdam.

This paper was financed thru CNCSIS grant no. AT133/ theme 1/ 2006