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ON THE SUN-EARTH ANGLES USED IN THE SOLAR TRACKERS' DESIGN. PART 1: MODELLING

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Abstract: This part of the paper presents the conclusions derived from the comparative analysis of some representative correlations that are proposed in literature for the modelling of the Sun-Earth angles. By means of these conclusions, it is established an improved modelling variant, whose correlations will be used in the solar trackers' design; on the basis of this variant, some simulations (at the most important latitudes) and their results' interpretation are made in the second part of the paper.

1. INTRODUCTION

The efficiency of a solar collector (of thermal or PV panel type) can be meaningfully increased if the collector is tracked in accordance with the sun so that the incidence angle (angle between the sun ray and the perpendicular line on the collector's plane) becomes null or very small. The achievement of this requirement supposes a modelling of the Sun-Earth angles that have to be accurate, relatively simple, without calculating difficulties and easy interpretable. In the technical literature there are many modelling variants of the Sun-Earth angles which use different reference lines and different correlations of the Sun-Earth angles [1, 2, 3].

Further on, the main conclusions derived from the comparative analysis of some representative modelling variants, that are proposed in literature [1, 2, 3] for the modelling of the Sun-Earth angles, are succinctly presented. On the basis of these conclusions, we established an improved modelling variant, whose correlations will be used in the design of the solar collectors' trackers.

2. COMPARATIVE ANALYSIS

More models for the graphical and analytical description of the Sun-Earth angles are proposed in literature. Among them, the following three are taken into consideration further: the *Goswami, Kreith and Kreider's* model [1], the *Messenger and Ventre's* model [2] and the *Stine and Harrigan's* model [3].

From the comparative analysis of these models, the following useful conclusions, on which new approaches can be developed, came out (see Figures 1, 2, 3):

a) All these models use the same definition and the same signs for the *declination* angle (noted δ_s in [1] and δ in [2, 3], see Figure 2), but describe this angle by means of different formulae that are, nevertheless, equivalent (where n and respective N are the day number during a year with January 1 being n = 1 respective N = 1) [1, 2, 3]:

$$\delta_s = 23.45^\circ \sin\frac{360^\circ(284+n)}{365};\ \delta = 23.45^\circ \sin\frac{360^\circ(n-80)}{365}; \qquad (1);\ (1')$$

$$\sin\delta = 0{,}39795\cos[0.98563(N-173)]; \qquad (1'')$$

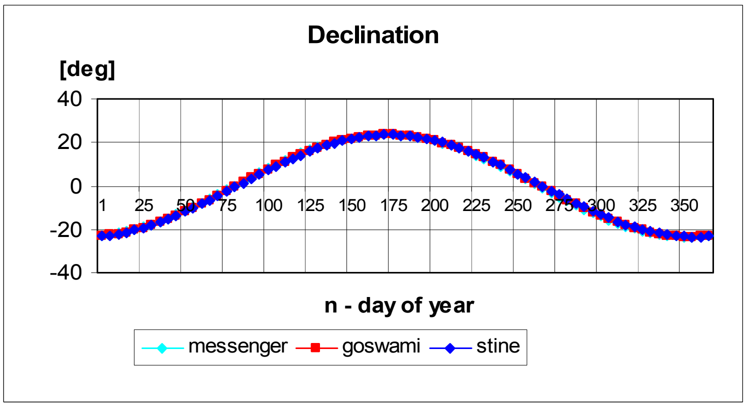


Fig. 1. *The variation of the declination (δ) depending on the day number of the year (n), calculated with the formulas of the three autors.*

b) For the *hour angle* (noted h_s in [1] and ω in [2, 3], see Figure 2), the considered models use the same definition but use different signs; after [1] and [3] this angle is negative in the morning and positive in the afternoon while [2] uses opposite signs [1, 2, 3]:

$$h_s = 15^\circ (t_s - 12);\ \omega = \frac{12 - T}{24} 360^\circ;\ \omega = 15^\circ (t_s - 12); \qquad (2);\ (2');\ (2'')$$

where t_s and respectively T are the solar time in hours.

For the *hour angle* according to the *sunrise* and respectively *sunset*, [1] and [2] give the following correlations:

$$h_{sr}\ or\ h_{ss} = \pm \cos^{-1}(-\tan L \tan \delta_s);\ \omega_s = \cos^{-1} \cdot (-\tan \varphi \tan \delta); \qquad (3);\ (3')$$

where L and respective φ represent the *latitude*.

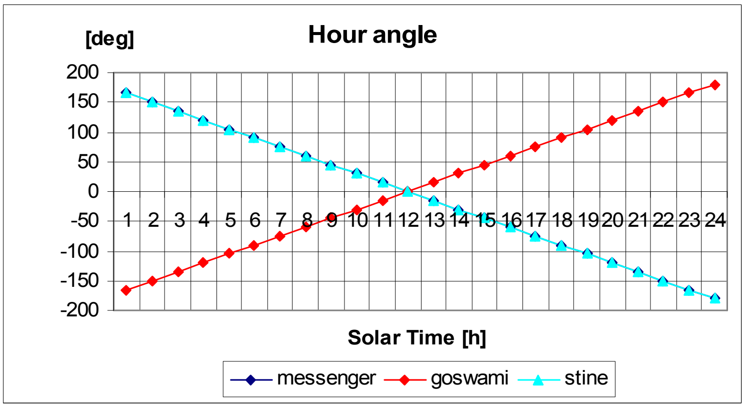


Fig. 2. *The variation of the hour angle (ω) depending on the solar time, calculated with the formulae of the three autors.*

c) All the considered models use the same definition and the same formula for the *altitude* angle (noted α everywhere, see Fig. 2) which is always positive [1, 2, 3]:

$$\sin\alpha = \sin\delta_s \sin L + \cos L \cos\delta_s \cos h_s;\ \sin\alpha = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos\omega; \qquad (4);\ (4')$$

$$\alpha = \sin^{-1}(\sin\delta \sin\phi + \cos\delta \cos\omega \cos\phi); \qquad (4'')$$

where L, φ and respective Φ represent the *latitude*.

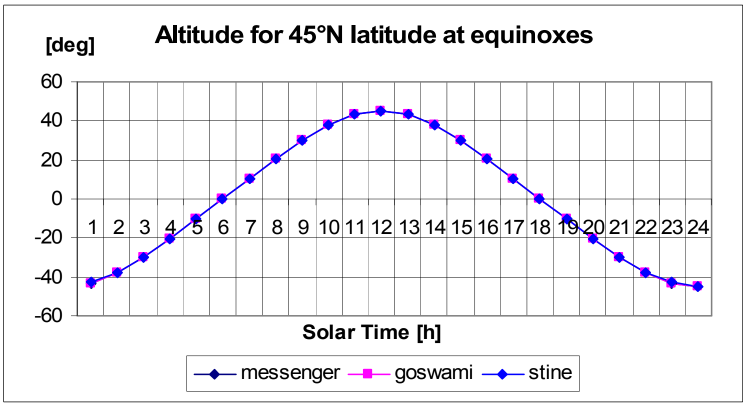


Fig. 3. *The variation of the altitude (α) depending on the solar time and season, calculated with the formulas of the three autors.*

d) For the *azimuth angle* (noted α_S in [1], ψ in [2] and A in [3], see Fig. 4), the considered models use different measures, different signs and different formulae [1, 2, 3]:

$$\sin\alpha_s = \frac{\cos\delta_s \sinh_s}{\cos\alpha};\ \cos\psi = \frac{\sin\alpha\sin\varphi - \sin\delta}{\cos\alpha\cos\varphi};\ A = \sin^{-1}\left[\frac{-\cos\delta\sin\omega}{\sin\alpha}\right]; \qquad (5);\ (5');\ (5'')$$

These formulae, which are theoretically equivalent, raise usually great calculation difficulties.

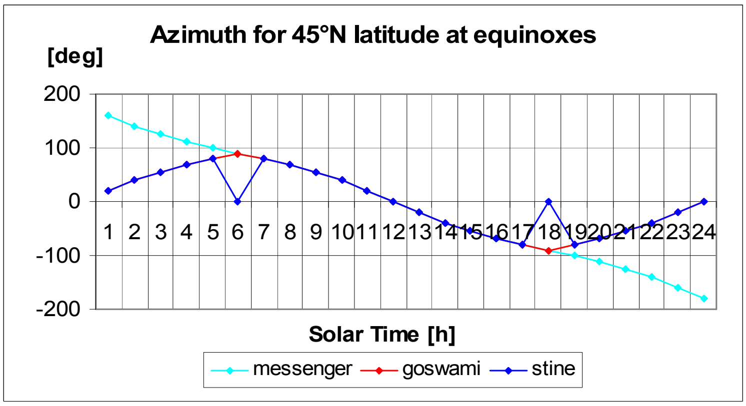


Fig. 4. *The variation of the azimuth (ψ) depending on the solar time and season, calculated with the formulae of the three autors.*

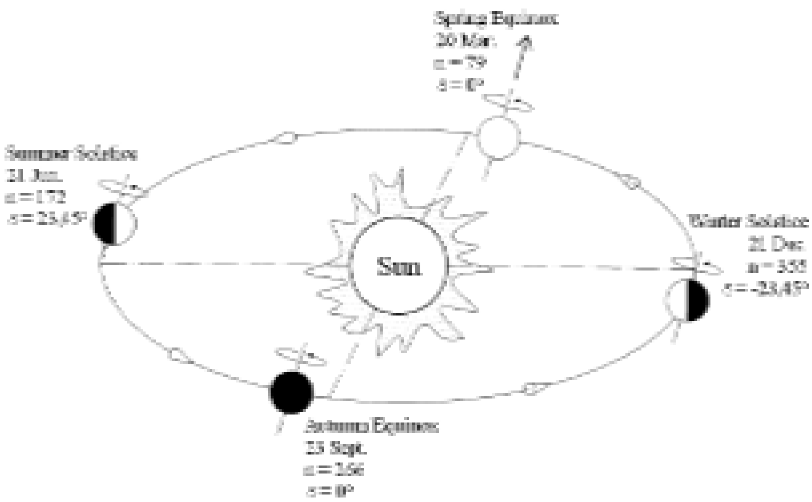
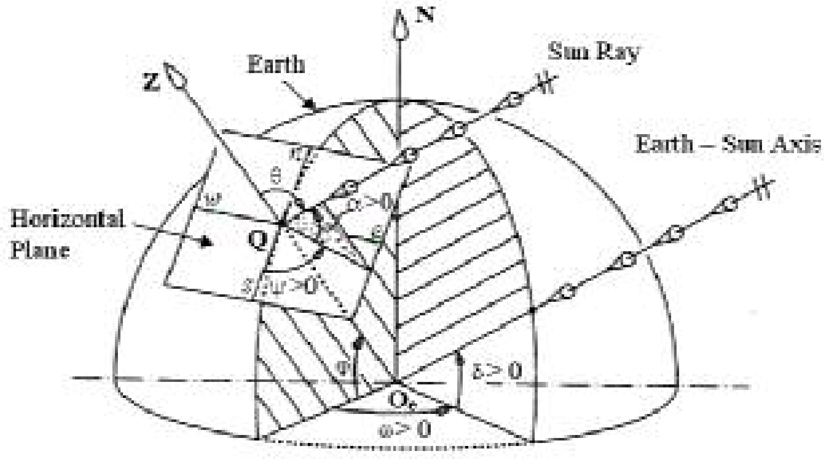
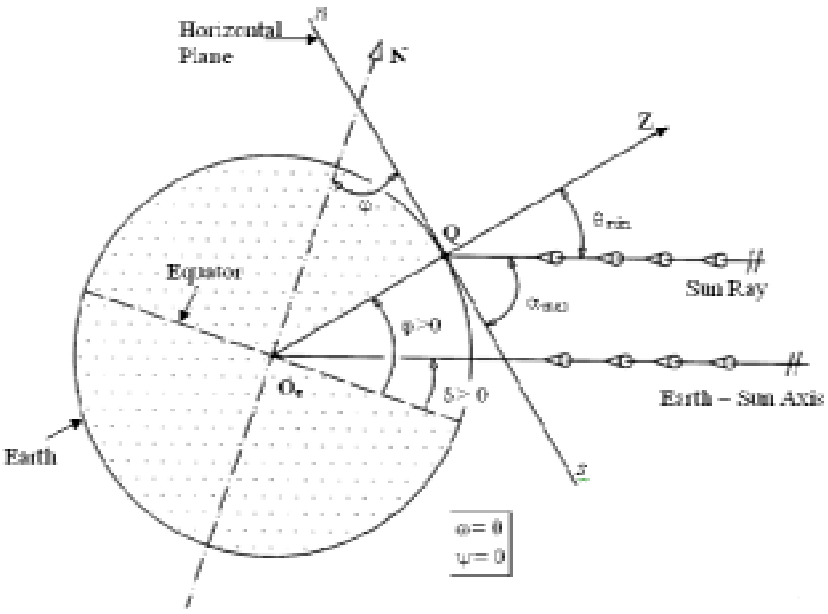


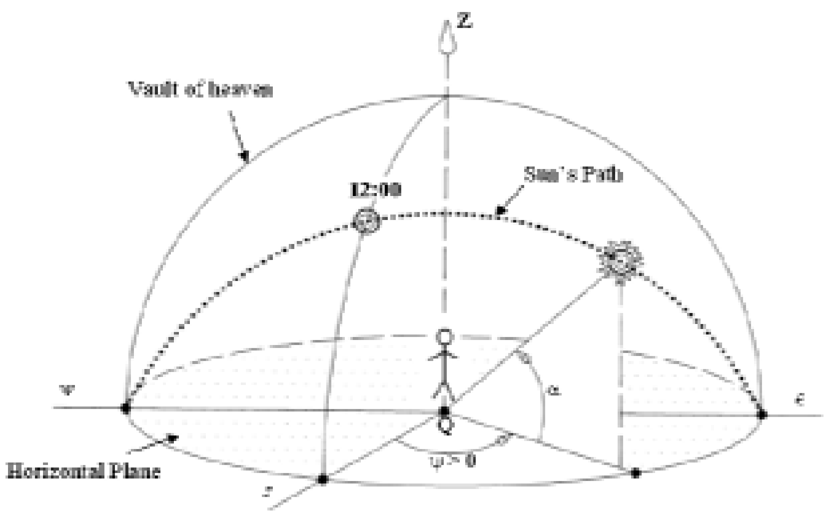
Fig. 5. *Representative relative positions Sun – Earth*



a)

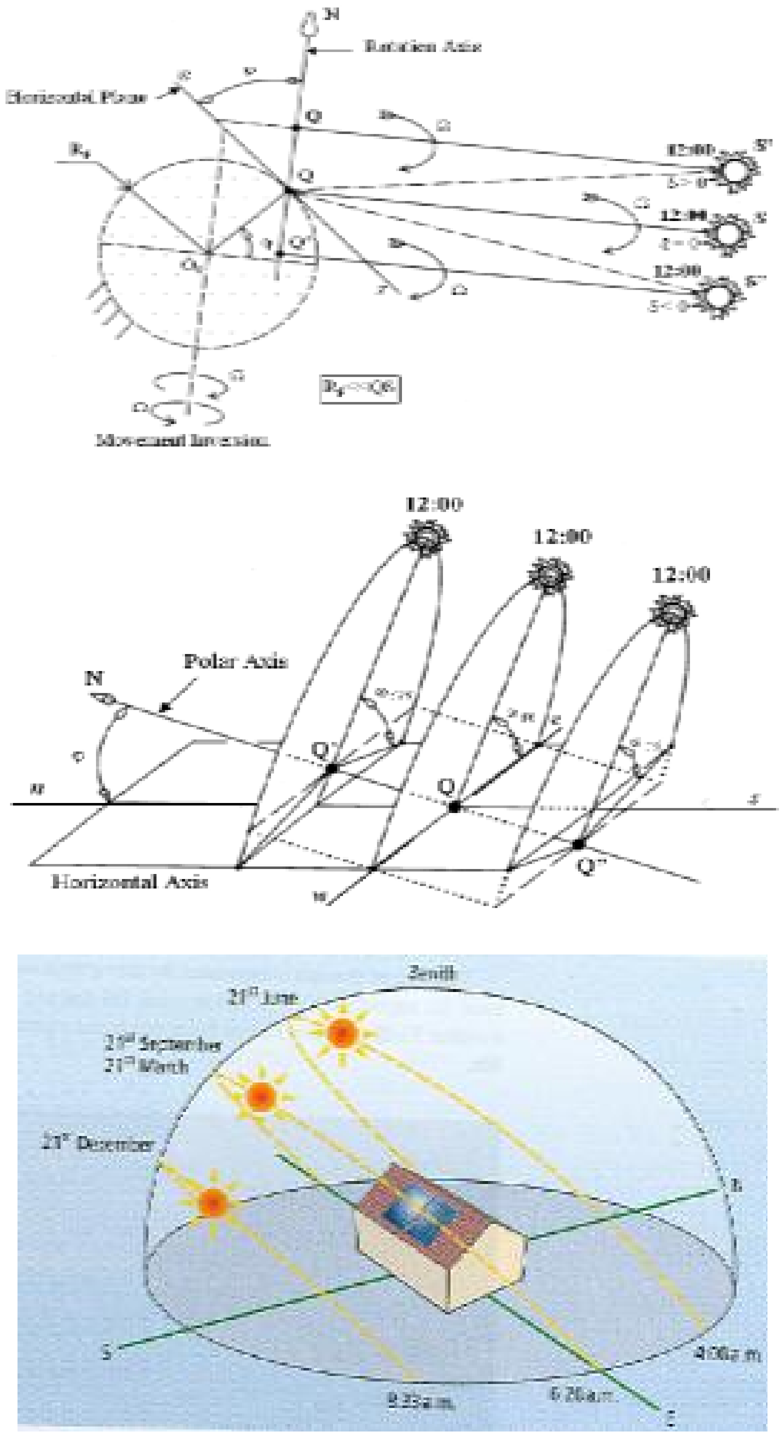


b)



c)

Fig. 6. *Earth – Sun angles: a) and b) in the equatorial system; a) and c) in the local system: Ψ, α, θ.*



a)

b)

c)

Fig. 7. *The variations of the sunrise and sunset hour angles depending on the seasons: a) movement inversion of the Sun – Earth system; b) variations of the sunrise and sunset hour angles in the winter solstice, equinox and summer solstice; c) a virtual picture after [4] of the scheme b).*

3. CONCLUSIONS

The following conclusions can be formulated based on the previous analysis:

a) The design of the solar collectors' trackers demands a modelling of the Sun-Earth angles that must be accurate, relatively simple, without calculation difficulties and easily interpretable. In the literature there are many models that describe Sun-Earth angles, but the existing models accomplish only partially meet these requirements.

b) In order to create an intuitive and unitary image on the Sun-Earth angles' measures and signs, Fig. 5, 6 and 7 propose a simplified and synthetic variant for the graphical Sun-Earth angles' description.

c) Further, in accordance with Figure 5, 6 and 7, there are recommended (for the design of the solar collectors' trackers) the following correlations, easily interpretable and *without calculation difficulties*:

$$\text{Declination: } \delta = 23.45^{o} \sin\frac{360^{o}\left(n-80\right)}{365}; \tag{6}$$

$$\text{Hour Angle: } \omega = 15^{o}\left(12-T\right); \tag{7}$$

$$\text{Sunrise, Sunset Hour Angle: } \omega_{sr,ss} = \pm\cos^{-1}\left(-\tan\varphi\tan\delta\right); \tag{8}$$

$$\text{Altitude: } \alpha = \sin^{-1}(\sin\delta\sin\varphi + \cos\delta\cos\varphi\cos\omega); \tag{9}$$

$$\text{Azimuth: } \psi = (\operatorname{sgn}\omega)\cos^{-1}\frac{\sin\alpha\sin\varphi - \sin\delta}{\cos\alpha\cos\varphi}; \tag{10}$$

where the index *sr* means *sunrise* (with the positive sign) and the index *ss* means *sunset* (with the negative sign).

By means of the previous correlations and of the Excel software, some representative simulations (at the most important latitudes from the northern hemisphere) and their results' interpretation are presented in the second part of the paper.

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