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PARETO APPROACH IN MULTI-OBJECTIVE OPTIMAL DESIGN OF HELICAL COMPRESSION SPRINGS

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Abstract: In this paper, we propose a population-based evolutionary multi-objective optimization approach based on the concept of Pareto optimality, in order to design compression springs. The goals of the optimization were to minimize the weight of the spring and maximize, in the same time, the number of loading cycles working under an imposed safety factor for fatigue. The problem is also subjected to six geometrical, technological, and related to the strength of material constraints. The Pareto optimal set was obtained by running a new genetic algorithm inspired by Non-Dominated Sorting Genetic Algorithm II (NSGA-II) implemented in *Cambrian v.1.0* software.

1. INTRODUCTION

The optimal design of springs is not a novelty. The mono-objective problem was firstly described and solved with traditional optimization methods by Arora [1] and Belegundu [2]. Later, Coello Coello [3] using a special GA had obtained better results. Deb [5] described a multi-objective design problem, but there were used two objective functions with an obvious and simple inverse proportionality.

2. PROBLEM FORMULATION

The goal of this paper is to optimal design a helical compression spring (figure 1) made of oil tempered wire (ASTM A229 / SAE J315). In table 1 the main input design data are presented:

Table 1. Spring specification

Parameter	Designation	Value	Units
Minimum load	F_{max}	40	N
Maximum load	F_{min}	500	N
Stroke (working range)	h	50	mm
Wire steel density	ρ	$7.87 \cdot 10^{-6}$	Kg/mm^3
Young's modulus	E	$2.06 \cdot 10^5$	MPa
Rigidity modulus	G	$0.78 \cdot 10^5$	MPa
End treatment	-	squared (closed)	-
End support parameter	v	0.5	-
Coefficients related to the wire material [2]	C_1	$3.72 \cdot 10^5$	-
	C_2	$1.152 \cdot 10^5$	-
	C_3	$0.625 \cdot 10^5$	-
	A_1	– 0.19	-
	B_1	– 0.1845	-
Safety factor for fatigue	SF_f	1.1	-

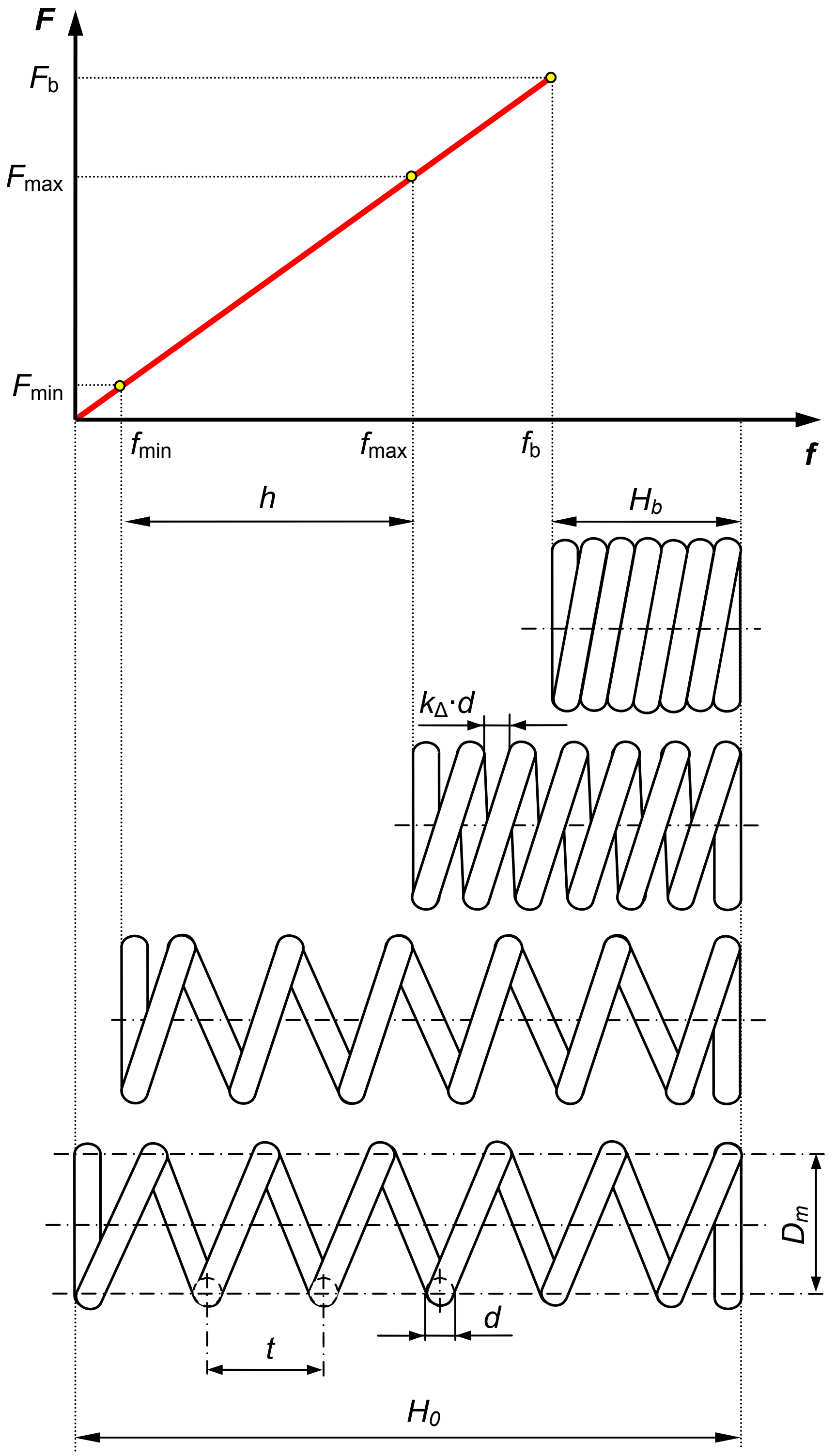

Figure 1. Load vs. spring deflection

3. OPTIMAL DESIGN OF THE COMPRESSION SPRING

Multi-objective optimization (also called multi-criteria optimization, multi-performance or vector optimization) can be defined as the problem of finding [7]: “a vector decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term *optimize* means finding such a solution which would give the values of all the objective functions acceptable to the designer."

In these terms, we want to solve multi-objective optimization problems of the form:

$$\text{Minimize } \left[f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})\right] \tag{1}$$

subject to the *m* inequality constraints:

$$g_i(\vec{x}) \leq 0,\ i = \overline{1, m} \tag{2}$$

and the *p* equality constraints:

$$h_i(\vec{x}) = 0,\ i = \overline{1, p} \tag{3}$$

where *k* is the number of objective functions $f_i : \mathbf{R}^n \rightarrow \mathbf{R}$. We call $\vec{x} = \left[x_1, x_2, \ldots, x_n\right]^T$ the vector of decision variables. We want to determine from among the set $\Im$ of all sets of numbers which satisfy (2) and (3) the particular set $\vec{x}^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$ which yields the optimum values of all the objective functions.

It is rarely the case that there is a single point that simultaneously optimizes all the objective functions. Therefore, we normally look for *trade-offs*, rather than single solutions when dealing with multi-objective optimization problems. The notion of *optimality* is therefore, different in this case. The most commonly adopted notion of optimality is called Pareto optimality. We say that a vector of decision variables $\vec{x}^* \in \Im$ is Pareto optimal if there does not exist another $\vec{x} \in \Im$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = \overline{1, k}$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one *j*.

Unfortunately, the concept of Pareto optimality almost always gives not a single solution, but rather a set of solutions called the *Pareto optimal set*. The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called *non-dominated*. The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the *Pareto front*.

In the following paragraphs we will identify and propose the variables (genes), the objective functions, and the constraints which will be aggregated in the multi-objective optimization program.

3.1. GENES (VARIABLES) AND PARAMETERS

After a closely analysis of the optimization problem we proposed 3 genes as follows:

Gene 1: *d* – wire diameter (real discrete variable): 64 values (table 2) of oil tempered wire according to ASTM A229 / SAE J315.

Gene 2: *i* – spring index (real continuous variable in the range of 4 … 16):

$$i = \frac{D_m}{d} \tag{4}$$

where:
d – wire diameter [mm];
D_m – spring mean diameter [mm];

Gene 3: k_Δ – coefficient of the distance between coils, at the maximum load (real and continuous variable in the range of 0.1 … 0.5).

$$k_\Delta = \frac{t_{F\max}}{d} \tag{5}$$

Table 2. Wire diameter (possible values of the Gene 1)

d [mm]	**d** [mm]	**d** [mm]	**d** [mm]	**d** [mm]	**d** [mm]	**d** [mm]	**d** [mm]
0.55	1.00	1.50	2.20	3.00	3.60	4.25	5.50
0.60	1.10	1.55	2.25	3.10	3.70	4.30	5.60
0.65	1.12	1.60	2.30	3.20	3.75	4.50	6.00
0.70	1.20	1.70	2.40	3.25	3.80	4.75	6.20
0.75	1.25	1.80	2.50	3.30	3.90	4.80	6.30
0.80	1.30	1.90	2.60	3.35	4.00	5.00	6.40
0.85	1.32	2.00	2.70	3.40	4.10	5.20	6.50
0.90	1.40	2.10	2.80	3.50	4.20	5.30	7.00

Gene 3: k_Δ – coefficient of the distance between coils, at the maximum load (real and continuous variable in the range of 0.1 … 0.5).

$$k_\Delta = \frac{t_{F\max}}{d} \tag{6}$$

Taking into consideration the spring specification (design input data) and the above mentioned genes, one could compute the following important parameters:

- Spring stiffness (spring rate) [N/mm]:

$$c = \frac{F_{\max} - F_{\min}}{h} \tag{7}$$

- Minimum deflection [mm]:

$$f_{\min} = \frac{F_{\min}}{c} \tag{8}$$

- Maximum deflection [mm]:

$$f_{\max} = \frac{F_{\max}}{c} \tag{9}$$

- Spring mean diameter [mm]:

$$D_m = i \cdot d \tag{10}$$

- Spring outside diameter [mm]:

$$D_e = D_m + d \tag{11}$$

- Spring inside diameter [mm]:

$$D_i = D_m - d \tag{12}$$

- Number of active coils of the spring:

$$n = \frac{G \cdot d^4}{8 \cdot c \cdot D_m^3} \tag{13}$$

- Number of end coils of the spring:

$$n_r = \begin{cases} 1.5 & \text{if} \quad n \le 7 \\ 2 & \text{otherwise} \end{cases} \tag{14}$$

- Total number of spring coils:

$$n_t = n + n_r \tag{15}$$

- Spring solid length [mm]:

$$H_b = n_t \cdot d \tag{16}$$

- Pitch of unloaded spring [mm]:

$$t = d + \frac{f_{max}}{n} + k_{\Delta} \cdot d \quad (17)$$

- Spring free length [mm]:

$$H_0 = H_b + n \cdot (t - d) \quad (18)$$

- Helix angle of the unloaded spring [rad]:

$$\alpha_0 = a\tan\left(\frac{t}{\pi \cdot D_m}\right) \quad (19)$$

- Wire length [mm]:

$$l_s = \frac{\pi \cdot D_m \cdot n_t}{\cos\alpha_0} \quad (20)$$

- Deflection at solid limit [mm]:

$$f_b = H_0 - H_b \quad (21)$$

- Load at solid limit [N]:

$$L_b = c \cdot f_b \quad (22)$$

- Slenderness ratio:

$$\lambda = \frac{H_0}{D_m} \quad (23)$$

- Coefficients of the critical slenderness ratio:

$$c_{f1} = \frac{E}{2 \cdot (E - G)} \quad (24)$$

$$c_{f2} = \frac{2\pi^2 \cdot (E - G)}{E + 2 \cdot G} \quad (25)$$

- Critical slenderness ratio:

$$\lambda_{critic} = \frac{\sqrt{c_{f2}}}{\nu} \quad (26)$$

- Theoretical buckling deflection:

$$f_f = \begin{cases} H_0 \cdot c_{f1} \cdot \left(1 - \sqrt{1 - \frac{c_{f2}}{\nu^2 \cdot \lambda^2}}\right) & \text{if} \quad \lambda > \lambda_{critic} \\ H_0 & \text{otherwise} \end{cases} \quad (27)$$

- Amplitude of the loading cycle [MPa]:

$$\tau_a = \frac{\tau_{t\_max} - \tau_{t\_min}}{2} \quad (28)$$

- Mean of the loading cycle [MPa]:

$$\tau_m = \frac{\tau_{t\_max} + \tau_{t\_min}}{2} \quad (29)$$

- Stress factor (Wahl factor):

$$k_W = \frac{4 \cdot i - 1}{4 \cdot i + 4} + \frac{0.615}{i} \quad (30)$$

- Maximum share stress [MPa]:

$$\tau_{t\_max} = \frac{8 \cdot k_W \cdot i \cdot F_{max}}{\pi \cdot d^2} \quad (31)$$

- Minimum share stress [MPa]:

$$\tau_{t\_\min} = \frac{8 \cdot k_W \cdot i \cdot F_{\min}}{\pi \cdot d^2} \tag{32}$$

- Shear ultimate strength [MPa]:

$$S_{us} = C_2 \cdot \left(\frac{d}{25.4}\right)^{A_1} \cdot \frac{4.448}{25.4^2} \tag{33}$$

- Shear yield strength [MPa]:

$$S_{ys} = C_3 \cdot \left(\frac{d}{25.4}\right)^{A_1} \cdot \frac{4.448}{25.4^2} \tag{34}$$

- Safety factor for yielding:

$$SF_y = \frac{S_{ys}}{\tau_a + \tau_m} \tag{35}$$

3.2. OJECTIVE FUNCTIONS

The objective functions chosen for this optimization model are the mass of the spring and the number of working cycles (for a certain factor of safety for fatigue, SF_f).

$$m = l_s \cdot \frac{\pi \cdot d^2}{4} \cdot \rho \quad \rightarrow \quad \min \tag{36}$$

$$N_c = \left(\frac{C_2}{C_1} \cdot \frac{\tau_a}{\dfrac{S_{us}}{SF_f} - \tau_m} \right)^{\frac{1}{B_1}} \quad \rightarrow \quad \max \tag{37}$$

3.3. CONSTRAINTS

The solutions of the optimization program have to satisfy the following constraints:

C1: The share tension has to be less than the allowed share stress, i.e. the factor of safety for yield should be greater than an imposed value.

$$SF_y \geq 1.1 \tag{38}$$

C2: The load at spring solid length must be larger than the maximum load (with at list 10%).

$$F_b \geq 1.1 \cdot F_{\max} \tag{39}$$

C3: The minimum clearance at maximum load should be at list 0.5 mm.

$$k_\Delta \cdot d \geq 0.5 \tag{40}$$

C4: The minimum clearance at maximum load should be at list 0.5 mm.

$$f_{\max} \geq f_f \tag{41}$$

C5-6: The pitch of coils should be within an imposed range.

$$\frac{D_m}{4} - 0.2 \leq t \leq \frac{2 \cdot D_m}{3} \tag{42}$$

3.4. RESULTS

The optimal Pareto set was obtained using *Cambrian v.1.0* software belonging to the Optimal Design Centre of the Technical University of Cluj-Napoca and the resulted Pareto front is plotted in figure 3. The extreme values are shown in the first part of table 3.

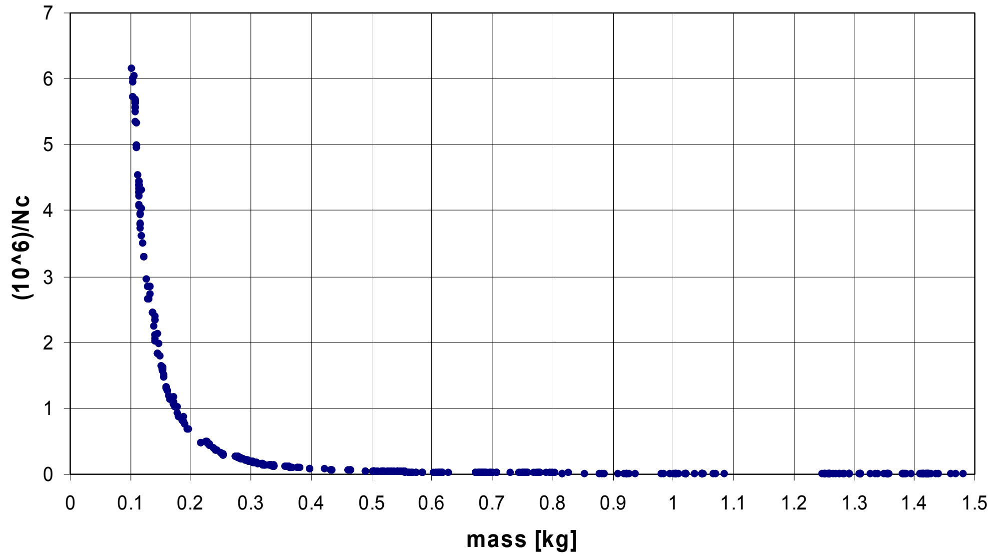

Figure 2. Pareto front ($10^6/N_c$ versus spring mass)

4. CONCLUSIONS

Obviously, it is up to the designer to choose, from the optimal Pareto set, the design solution. In order to facilitate the process, in figure 3 we present the sub sets of the Pareto front according to different sets of values of the wire diameter. As one can see, the knee part of the graph is the most interesting part as a result of the deal between the mass and the number of working cycles.

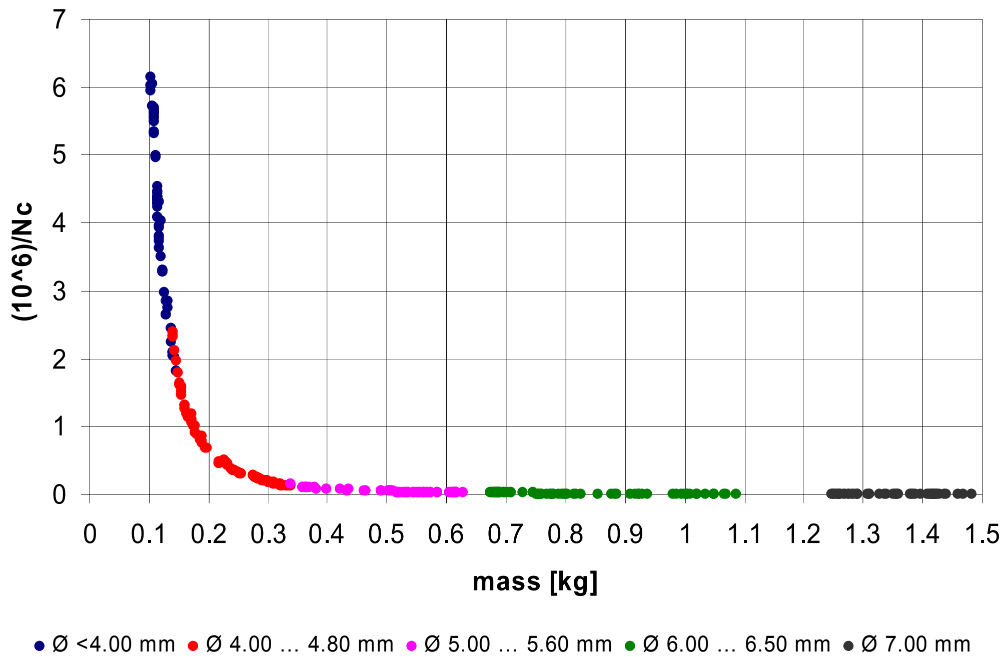

Figure 3. $10^6/N_c$ versus spring mass for different values of the wire diameter

Figure 4 is a zoom in on the interesting part of the Pareto front. In this case every color represents a single type of wire diameter, and the most recommended zone is marked by a red circle and some solutions are presented in the lower part of the table 3. It is worth noting here that a solely value of the wire diameter is obtained.

Table 3. Optimal Pareto set: Extreme and recommended solutions

Wire diameter d [mm]	Spring index i	Coefficient of the distance between coils k_Δ	Spring mass m [kg]	Number of cycles N_c [millions]	Notes
3.60	6.241	0.156	0.103	0.163	Minimum mass
7.00	5.990	0.342	1.482	568.931	Maximum number of cycles
4.80	5.987	0.163	0.338	8.452	Recommended solutions
4.80	6.045	0.242	0.333	7.910	
4.80	6.014	0.146	0.336	8.195	
4.80	6.045	0.245	0.333	7.910	

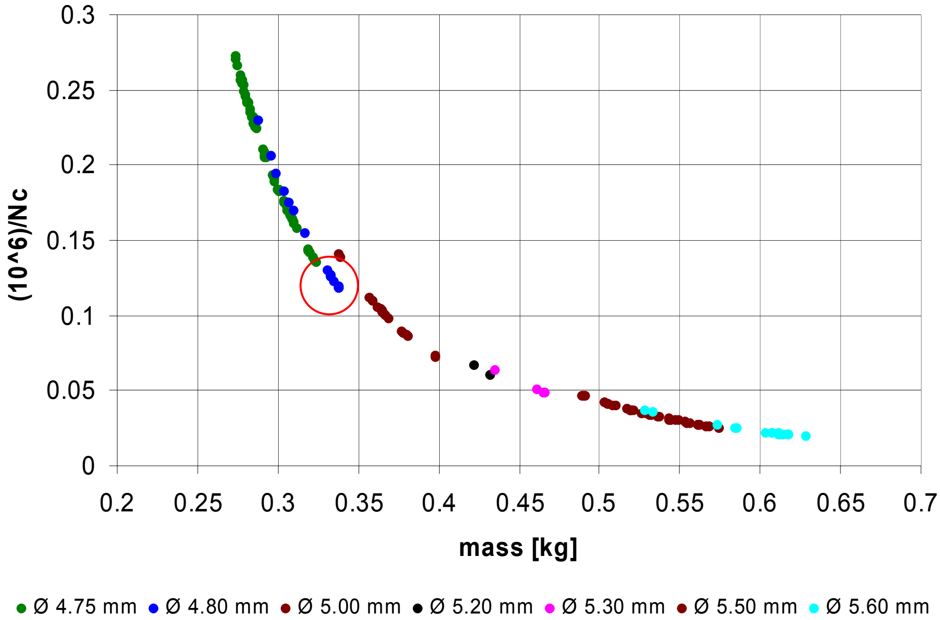

Figure 4. Recommended design solution

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