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ECCENTRICITY INFLUENCE UPON THE DYNAMIC VELOCITIES AND ACCELERATIONS OF A CRANK - PISTON MECHANISM

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Abstract: The crank - piston mechanism has a very wide use in technique, when the rotation motion must be transformed in translation or inversely. This mechanism is often used as an eccentric mechanism. The eccentricity of this mechanism may come into sight in small values as a consequence of manufacturing imprecision or also of wear. The purpose of this paper is to emphasize the influence of eccentricity upon the dynamic kinematical characteristics of this mechanism. Also the paper demonstrates the influence of mass, of forces and of crank balancing upon velocities and accelerations of the considered mechanism.

1. NOTATIONS

Bellow in the paper, the following notations are used: M_{red1} - equivalent moment of the mechanism, reduced at element (1); $I_i(\varphi_1)$ - equivalent moment of inertia of the mechanism, reduced at element (1); I_{i0} - equivalent moment of inertia of the mechanism, reduced at element (1), for $\varphi_1 = 0$; e - eccentricity of the mechanism; l_1 - crank length; l_2 - rod length; φ_1,φ_2 - (see Fig. 1); I_{11} - moment of inertia of the balanced crank; I_{12} - moment of inertia of the unbalanced crank; m_{12} - mass of the unbalanced crank; m_2 - mass of the rod; I_{22} - moment of inertia of the rod; m_3 - mass of the piston; ω_{21}, ω_{2r1} - kinematical and dynamic angular velocity of the rod, respectively; ε_{20}, ε_{2ri} - kinematical and dynamic acceleration of the rod, respectively; v_{M21}, v_{M31} - kinematical velocity of the mass centers M_2 and M_3, respectively; v_{M2ri}, v_{M3ri} - dynamic velocity of the mass centers M_2 and M_3, respectively; a_{M20}, a_{M30} - kinematical acceleration of the mass centers M_2 and M_3, respectively; a_{M2ri}, a_{M3ri} - dynamic acceleration of the mass centers M_2 and M_3, respectively.

2. GENERAL CONSIDERATIONS

As examples of using the crank - piston mechanism in technique, when the rotation motion must be transformed in translation or inversely, at least there can be mentioned: the engines, compressors, pumps with piston mechanisms, the mechanisms of feeding systems from automatic lines, the mechanisms of machines with percussion (forging machines, punching machines, machines for stony materials processing, machines for drilling of hard rocks), the mechanism of needle from sewing machines, [1], [8], [13], [14], [15], [19]. The eccentricity of the mechanism may come into sight in small values as a consequence of manufacturing imprecision or also of wear [4], ..., [7], [16].

In traditional design calculations kinematical analysis of a crank - piston mechanism is made considering that the angular velocity of crank ω_1 is considered to be constant and consequently crank acceleration ε_1 is considered to be null [1], [8], [14], [15], [19], [20].

Taking into account, when designing, the mass, the moments of inertia as well as the forces acting on mechanism links, and unbalancing of crank, kinematical parameters are changed, fundamentally.

In this paper, two structural mechanism schemes (Fig. 1) were considered. For the computer simulation four cases were considered:

1 - crank balanced, rod mass neglected ($m_2 = 0$);
2 - crank balanced, mass rod $m_2 \neq 0$;
3 - crank unbalanced, rod mass neglected ($m_2 = 0$);
4 - crank unbalanced, rod mass $m_2 \neq 0$.

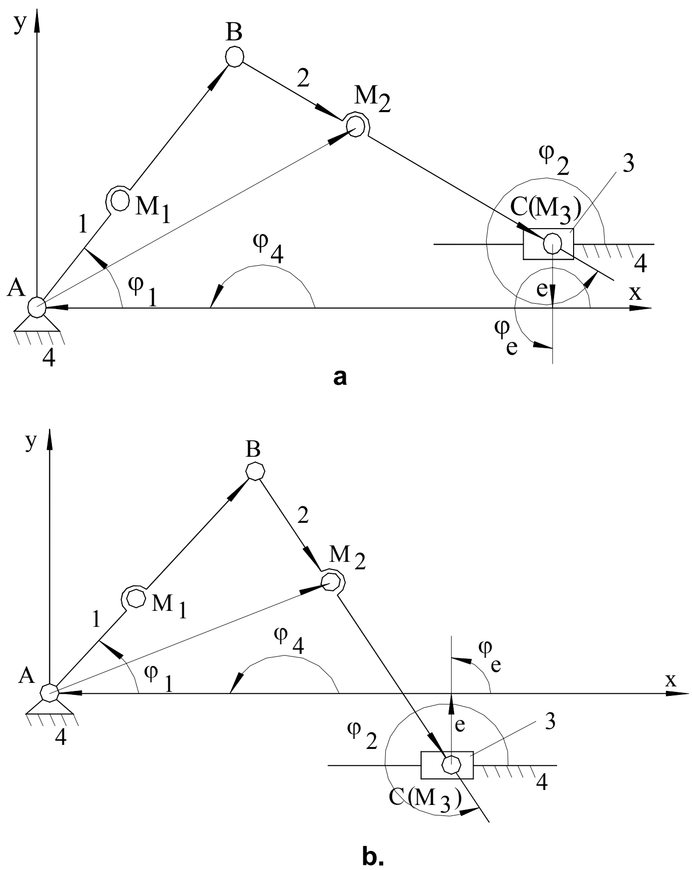


Fig. 1. Scheme of the crank mechanism: a) positive eccentricity, b) negative eccentricity

3. DYNAMIC MATHEMATICAL MODELING OF MECHANISMS

In according with Fig. 1, for Fig.1a:

$$\varphi_{2p} = a\sin\left(\frac{e - l_1 \cdot \sin\varphi_1}{l_2}\right) \tag{1}$$

and for Fig. 1b:

$$\varphi_{2n} = a\sin\left(-\frac{e + l_1 \cdot \sin\varphi_1}{l_2}\right) \tag{2}$$

For e = 0:

$$\varphi_2 = a\sin\left(-\frac{l_1 \cdot \sin\varphi_1}{l_2}\right) \tag{3}$$

$$l_4 = l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 \tag{4}$$

Starting from Lagrange equation of second species, [1], ... , [3], [8], ..., [18] there have been established the calculus relations for angular real (dynamic) velocities of leading element (1):

$$\omega_{1ri} = \sqrt{\frac{I_{i0}}{I_i(\varphi_1)} \cdot \omega_1^2 + \frac{2}{I_i(\varphi)_1} \cdot \int_0^{\varphi_1} M_{red1}(\varphi_1) \cdot d\varphi_1} \tag{5}$$

for angular accelerations of leading element (1), using in calculations ω_1 = constant:

$$\varepsilon_{1i} = \frac{M_{red1} - a_i \cdot \omega_1^2}{I_i(\varphi_1)} \tag{6}$$

and for real (dynamic) angular accelerations of leading element (1):

$$\varepsilon_{1ri} = \frac{Mred_1 - a_i \cdot \omega_{1ri}^2}{I_i(\varphi_1)} \tag{7}$$

with real (dynamic) angular velocities ω_{1ri}.

In relations (1), ..., (7), i=1,...,4, corresponding to the four instances already announced,

$$a_1 = m_3 \cdot l_{41} \cdot l_{411} \tag{8}$$

$$a_2 = I_{22} \cdot \varphi_{21} \cdot \varphi_{211} + m_2 \cdot A \quad \cdot A_1 + m_3 \cdot l_{41} \cdot l_{411} \tag{9}$$

$$a_3 = a_1, a_4 = a_2 \tag{10}$$

where:

$$A = \sqrt{l_1^2 + \frac{l_2^2}{4} \cdot \varphi_{21}^2 + l_1 \cdot l_2 \cdot \varphi_{21} \cdot \cos(\varphi_2 - \varphi_1)} \tag{11}$$

The equivalent moments of inertia of the mechanism, reduced at element (1) are:

$$I_1(\varphi_1) = I_{11} + m_3 \cdot l_{41}^2 \tag{12}$$

$$I_2(\varphi) = I_{11} + I_{22} \cdot \varphi_{21}^2 + m_2 \cdot A^2 + m_3 \cdot l_{41}^2 \tag{13}$$

$$I_3(\varphi_1) = I_{12} + m_{12} \cdot \frac{l_1^2}{4} + m_3 \cdot l_{41}^2 \tag{14}$$

$$I_4(\varphi_1) = I_{12} + m_{12} \cdot \frac{l_1^2}{4} + I_{22} \cdot \varphi_{21}^2 + m_2 \cdot A^2 + m_3 \cdot l_{41}^2 \tag{15}$$

$$\varphi_{21} = \frac{d\varphi_2}{d\varphi_1}, \varphi_{211} = \frac{d\varphi_{21}}{d\varphi_1}, l_{41} = \frac{dl_4}{d\varphi_1}, l_{411} = \frac{dl_{41}}{d\varphi_1}, A_1 = \frac{dA}{d\varphi_1} \tag{16}$$

For the rod (2):

$$\omega_{21} = \omega_1 \cdot \varphi_{21},\ \omega_{21p,n} = \omega_1 \cdot \varphi_{21p,n}, \varepsilon_{20} = \omega_1^2 \cdot \varphi_{211},\ \varepsilon_{20p,n} = \omega_1^2 \cdot \varphi_{211p,n} \tag{17}$$

$$\varepsilon_{2ri} = \omega_{1ri}^2 \cdot \varphi_{211} + \varepsilon_{1ri} \cdot \varphi_{21},\ \varepsilon_{2rip,n} = \omega_{1rip,n}^2 \cdot \varphi_{211p,n} + \varepsilon_{1rip,n} \cdot \varphi_{21p,n} \tag{18}$$

$$v_{M21} = \omega_1 \cdot A,\ v_{M21p,n} = \omega_1 \cdot A_{p,n},\ v_{M2ri} = \omega_{1ri} \cdot A,\ v_{M2rip,n} = \omega_{1rip,n} \cdot A_{p,n} \tag{19}$$

$$a_{M2xop,n} = -\omega_1^2 \cdot (l_1 \cdot \cos\varphi_1 + \frac{l_2}{2} \cdot (\varphi_{211p,n} \cdot \sin\varphi_{2p,n} + \varphi_{21p,n}^2 \cdot \cos\varphi_{2p,n})) \tag{20}$$

$$a_{M2yop,n} = \omega_1^2 \cdot (-l_1 \cdot \sin\varphi_1 + \frac{l_2}{2} \cdot (\varphi_{211p,n} \cdot \cos\varphi_{2p,n} - \varphi_{21p,n}^2 \cdot \sin\varphi_{2p,n})) \tag{21}$$

$$a_{M20} = \sqrt{a_{M2x0p,n}^2 + a_{M2yop,n}^2} \tag{22}$$

$$\begin{aligned} a_{M2xrip,n} = &-(\varepsilon_{1rip,n}(l_1 \sin\varphi_1 + \frac{l_2}{2}\varphi_{21p,n} \sin\varphi_{2p,n}) + \\ &+ \omega_{1rip,n}^2(l_1 \cos\varphi_1 + \frac{l_2}{2}(\varphi_{211p,n} \sin\varphi_{2p,n} + \varphi_{21p,n}^2 \cos\varphi_{2p,n}))) \end{aligned} \tag{23}$$

$$a_{M2yrip,n} = \varepsilon_{1rip,n}(l_1\cos\varphi_1 + \frac{l_2}{2}\varphi_{21p,n}\cos\varphi_2 + \\ + \omega^2_{1rip,n}(-l_1\sin\varphi_1 + \frac{l_2}{2}(\varphi_{211p,n}\cos\varphi_{2p,n} - \varphi^2_{21p,n}\sin\varphi_{2p,n})) \quad (24)$$

$$a_{M2rip,n} = \sqrt{a^2_{M2xrip,n} + a^2_{M2yrip,n}} \quad (25)$$

For the piston (3):

$$v_{M31} = \omega_1 l_{41}, v_{M3rip,n} = \omega_{1rip,n} l_{41p,n}, a_{M30} = \omega_1^2 l_{411}, a_{M3rip,n} = \varepsilon_{1rip,n} l_{41p,n} + \omega^2_{1rip,n} l_{411p,n} \quad (26)$$

4. NUMERICAL EXAMPLE

For a computer simulation there were used the following numerical data: l_1=0.1 m; l_2 = 0.4 m; I_{11} = 0.004 Nms^2; m_{12} = 0.55 kg; I_{12} = 0.0005 Nms^2; m_2 = 2.2 kg; I_{22}=0.03 Nms^2; m_3 = 2 kg; ω_1 = 100 s^{-1}; $\varphi_1 = 0,10\cdot\frac{\pi}{180}..2\pi$; F_3 (φ_1) = if ($\varphi_1 \leq \pi$, 38, -5); M_1 = 3.9 Nm; e=0.05 m; M_{red1} (φ_1) = M_1+$F_3(\varphi_1)$ l_{41}; ε_1 = 0; i = 1,...4, for the four cases before announced.

5. OBTAINED RESULTS AND COMMENTARY

In Figs. 2, ... , 15, there are represented the results obtained through a computer simulation.

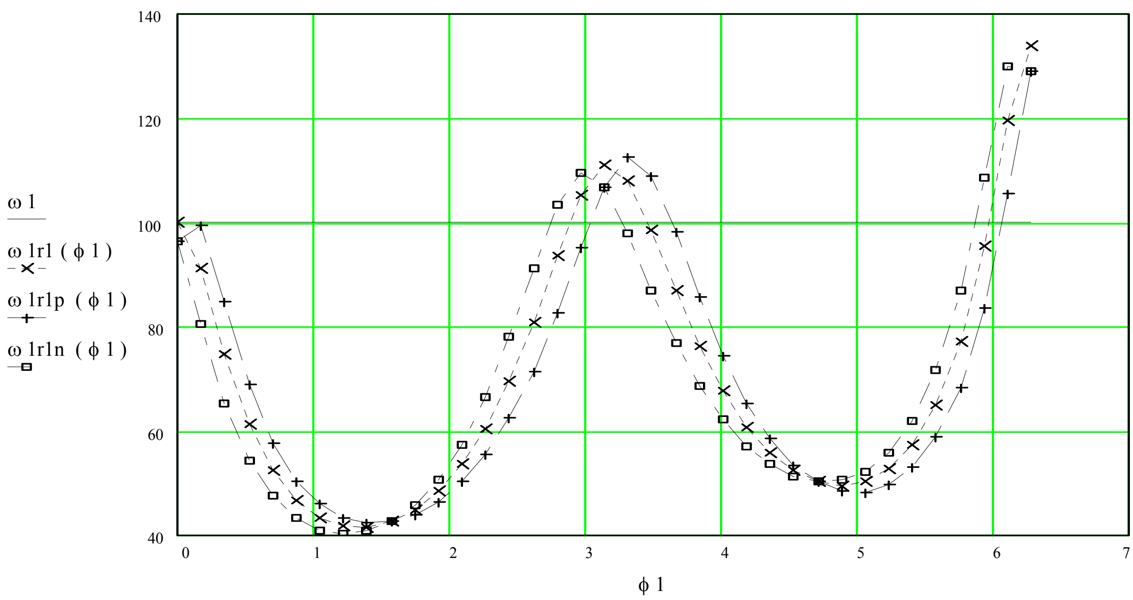


Fig. 2. ***ω_1, ω_{1r1}, ω_{1r1p}, ω_{1r1n} depending on φ_1***

The velocities ω_{1r1}, ω_{1r1p}, ω_{1r1n},. ω_{1r2}, ω_{1r2p}, ω_{1r2n}, ω_{1r3}, ω_{1r3p}, ω_{1r3n}, reach bigger values in comparison with ω_1=100s $^{-1}$. The velocities ω_{1r3}, ω_{1r3p}, ω_{1r3n}, reach the biggest values and vary between 30 and 130s^{-1}. Eccentricity influence is of 20s^{-1} for positive and 40s^{-1} for negative. Eccentricity moves curves to the left with 9^0 for positive and with 18^0 for negative (Figs. 2, 3, 4). The angular velocities ω_{2r1}, ω_{2r1p}, ω_{2r1n} vary between -34 and 27s^{-1}. Horizontal change of place of curves is insignificant (Fig. 7). The velocities v_{M2r1}, v_{M2r1p}, v_{M2r1n} have smaller values (4.2....6.7 ms^{-1}) in comparison with velocity v_{M21} (5...10.1 ms^{-1}).

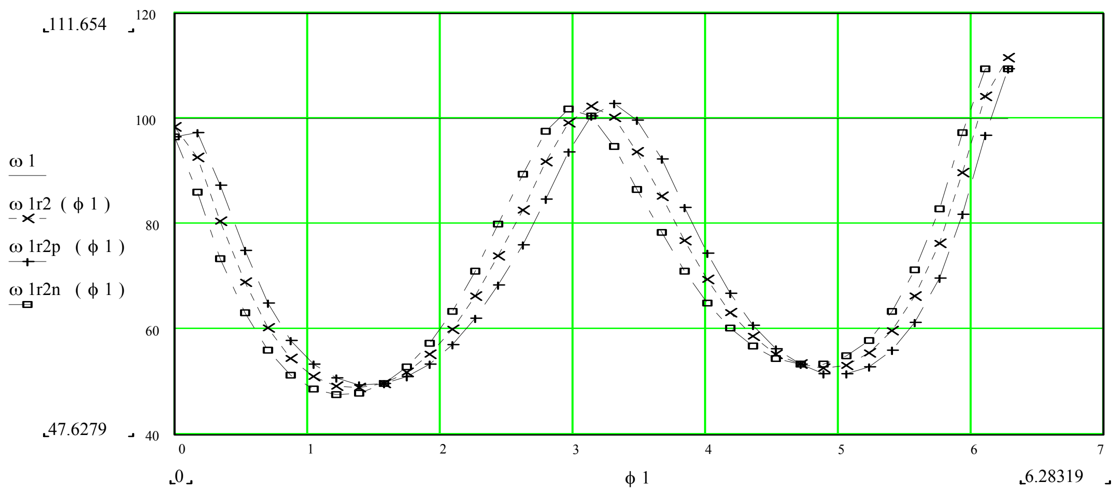


Fig. 3. ω_1, ω_{1r2}, ω_{1r2p}, ω_{1r2n} **depending of** φ_1

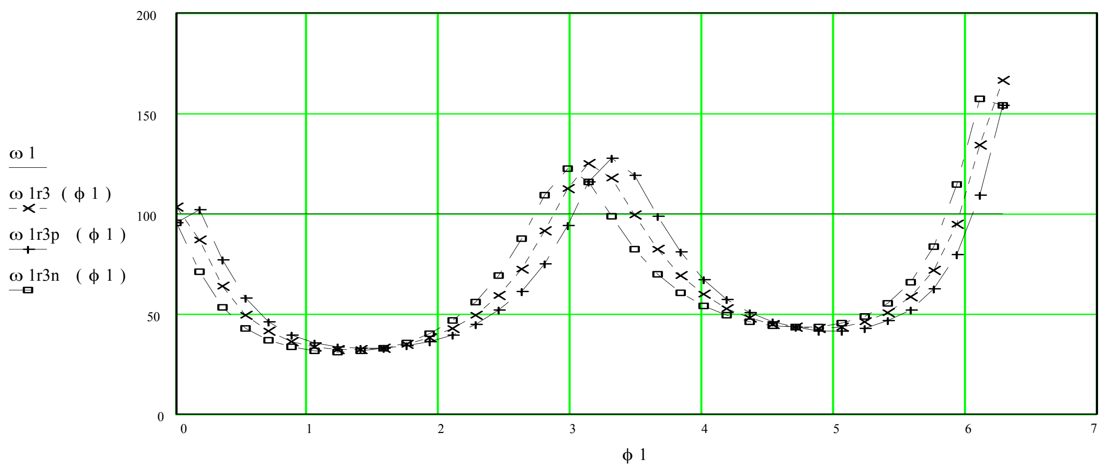


Fig. 4. ω_1, ω_{1r3}, ω_{1r3p}, ω_{1r3n} **depending of** φ_1

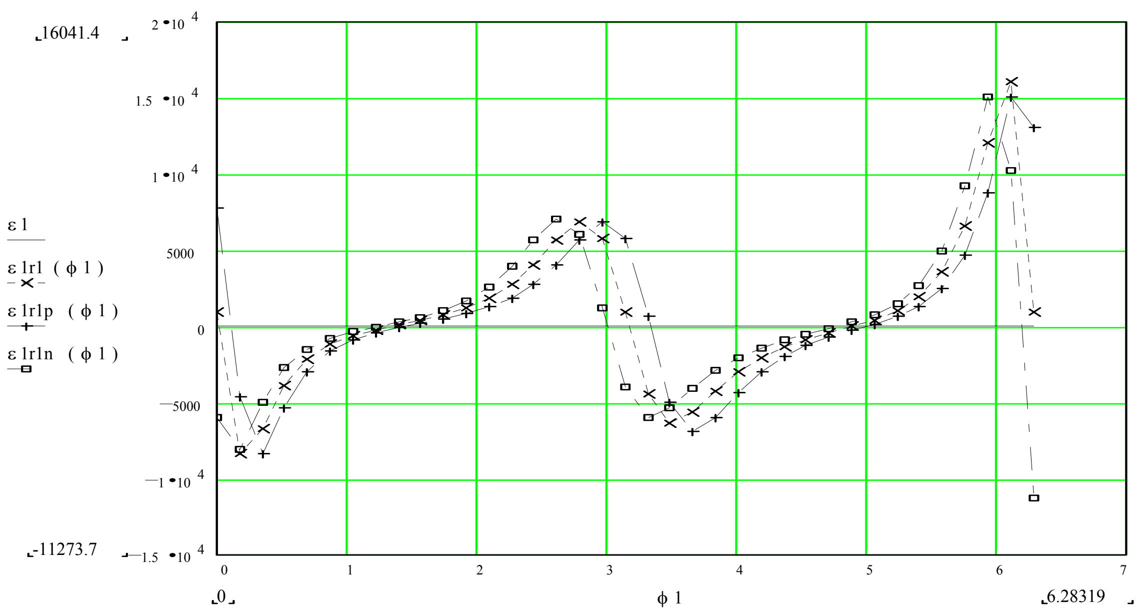


Fig. 5. ε_1, ε_{1r1}, ε_{1r1p}, ε_{1r1n} **depending of** φ_1

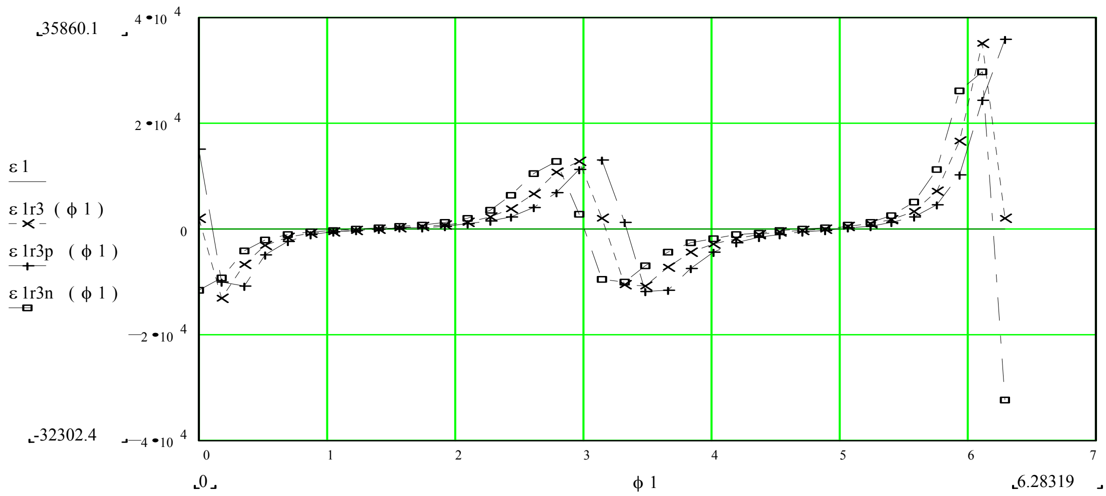


Fig. 6. ε_1, ε_{1r3}, ε_{1r3p}, ε_{1r3n} ***depending of*** φ_1

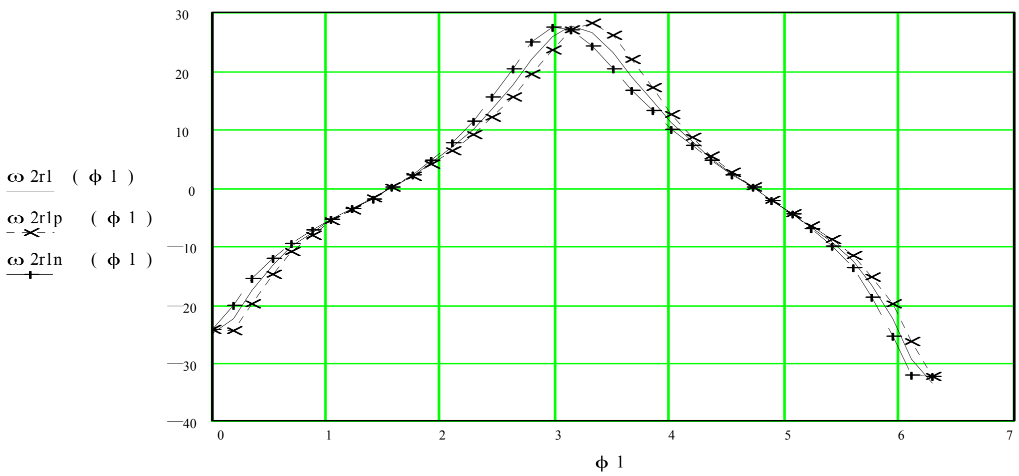


Fig. 7. ω_{2r1}, ω_{2r1p}, ω_{2r1n} ***depending of*** φ_1

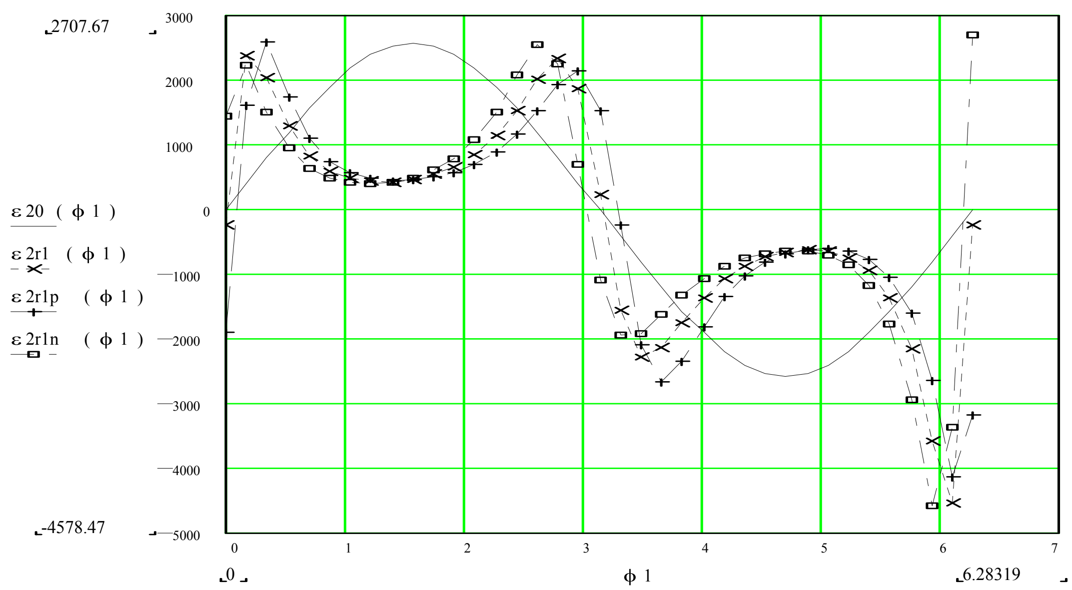


Fig. 8. ε_{20}, ε_{2r1}, ε_{2r1p}, ε_{2r1n} ***depending of*** φ_1

Horizontal change of places of curves is of 27^0 for positive eccentricity and of 55^0 for negative (Figs. 9, 10).

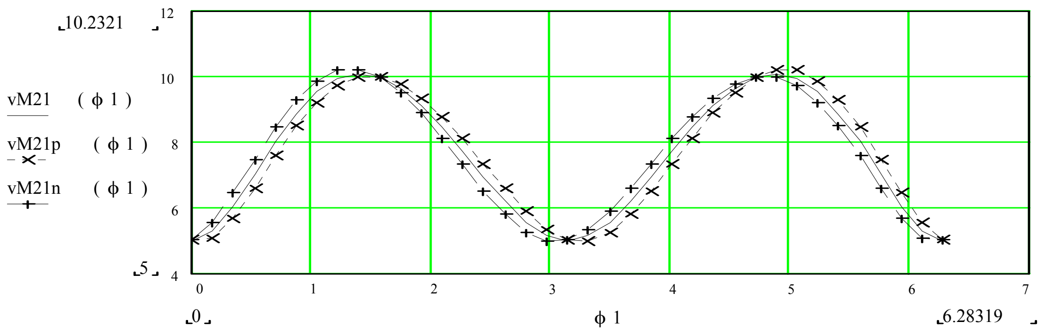


Fig.9. ***v_{M21},v_{M21p}, v_{M21n} depending of φ_1***

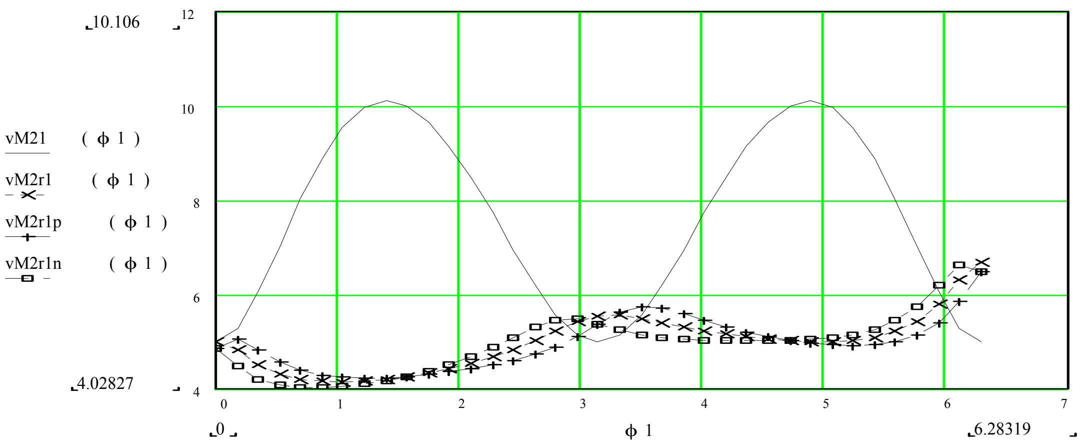


Fig. 10. ***v_{M2r1}, v_{M2r1p}, v_{M2r1n} depending of φ_1***

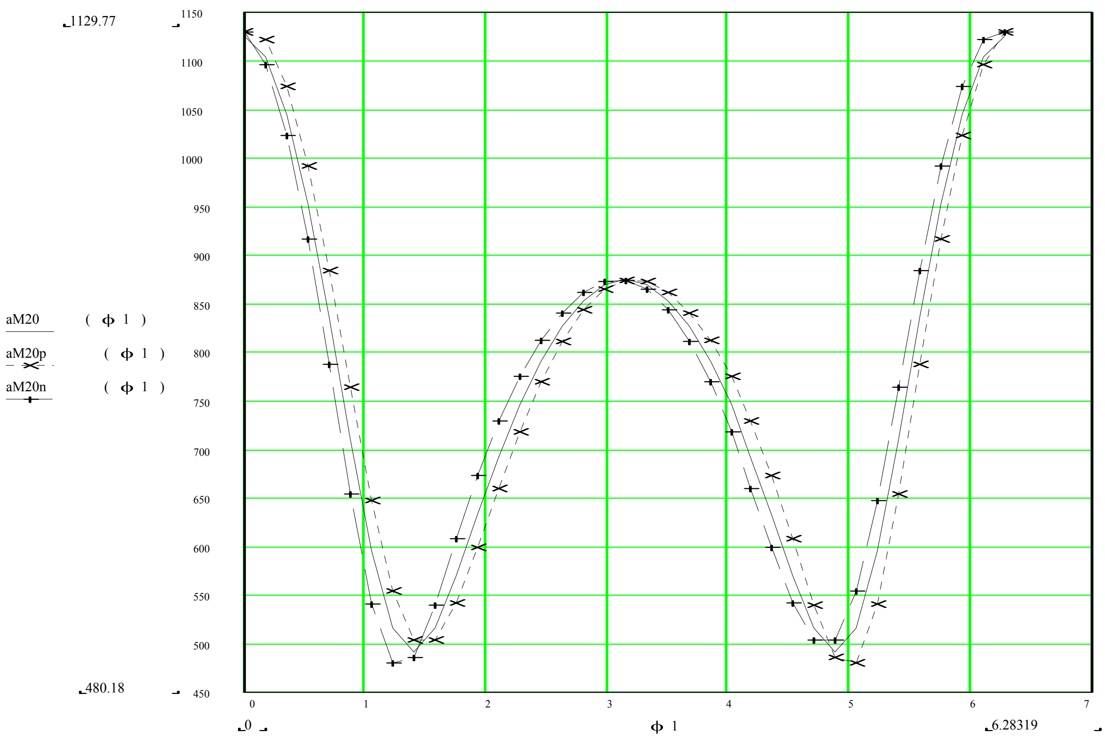
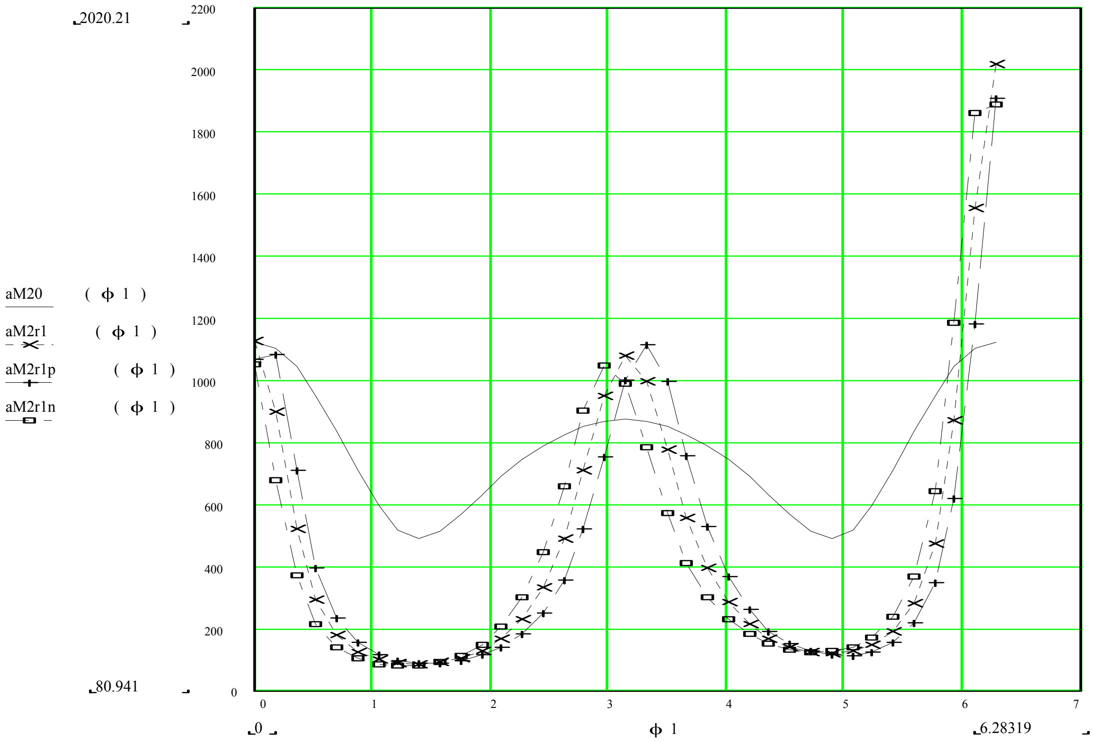


Fig. 11. ***a_{M20}, a_{M20p}, a_{M20n} depending of φ_1***

Piston velocities v_{M3r1}, v_{M3r1p}, v_{M3r1n} vary between -4.3 and 5.7ms^{-1} and horizontal change of place of curves is of 13^0 to the right for positive eccentricity and 13^0 to the left for negative (Fig. 13).



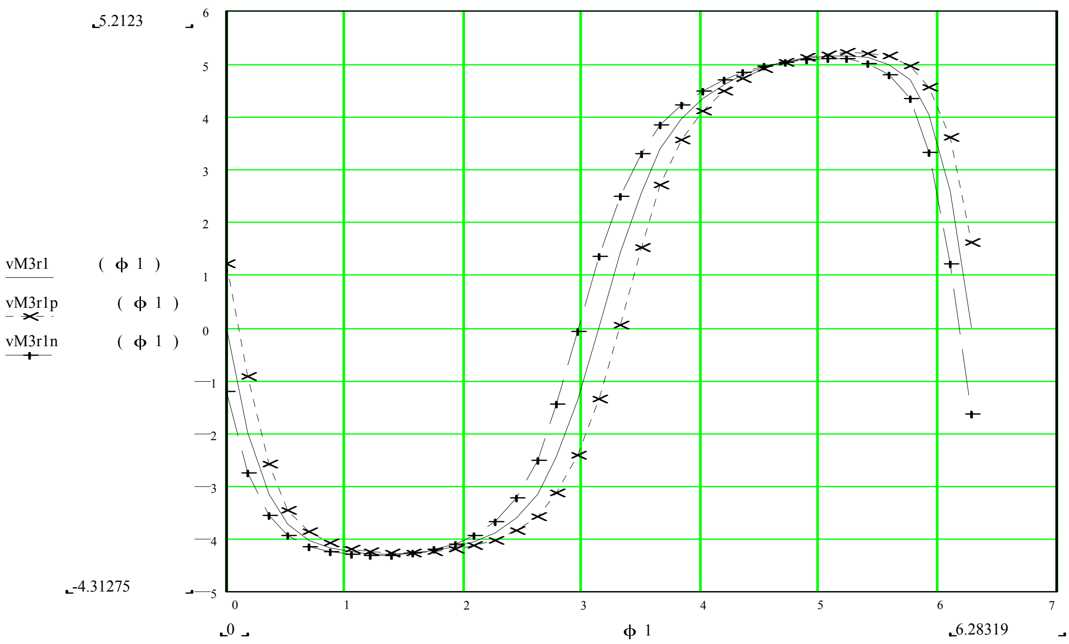


Fig. 12. ***a_{M20}, a_{M2r1}, a_{M2r1p}, a_{M2r1n} depending of φ_1***

Fig. 13. ***v_{M3r1}, v_{M3r1p}, v_{M3r1n} depending of φ_1***

Angular accelerations ε_{1r1}, ε_{1r1p}, ε_{1r1p} vary between -11270 and 16040 s^{-2} but angular accelerations ε_{1r3}, ε_{1r3p}, ε_{1r3n} vary between -37302 and 35860 s^{-2} in comparison to

ε_1=0. Horizontal change of place of curves is of 12^0 to the right for positive eccentricity and of 12^0 to the left for negative (Figs. 5, 6).

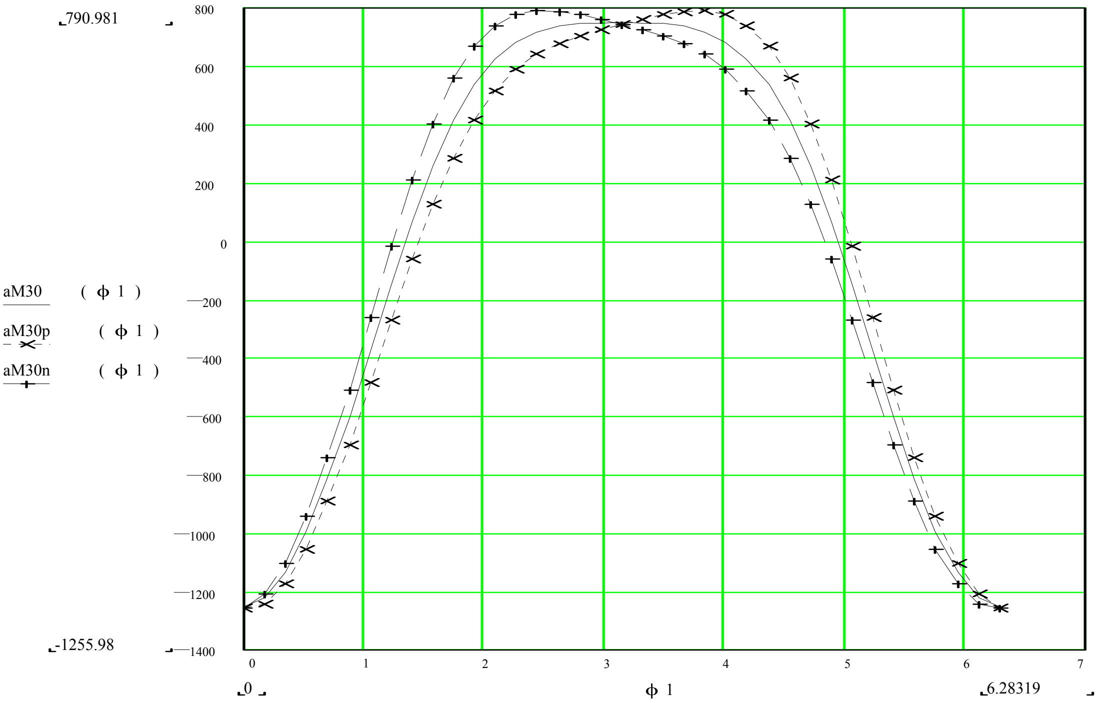


Fig. 14. ***a_{M30}, a_{M30p}, a_{M30n} depending of φ_1***

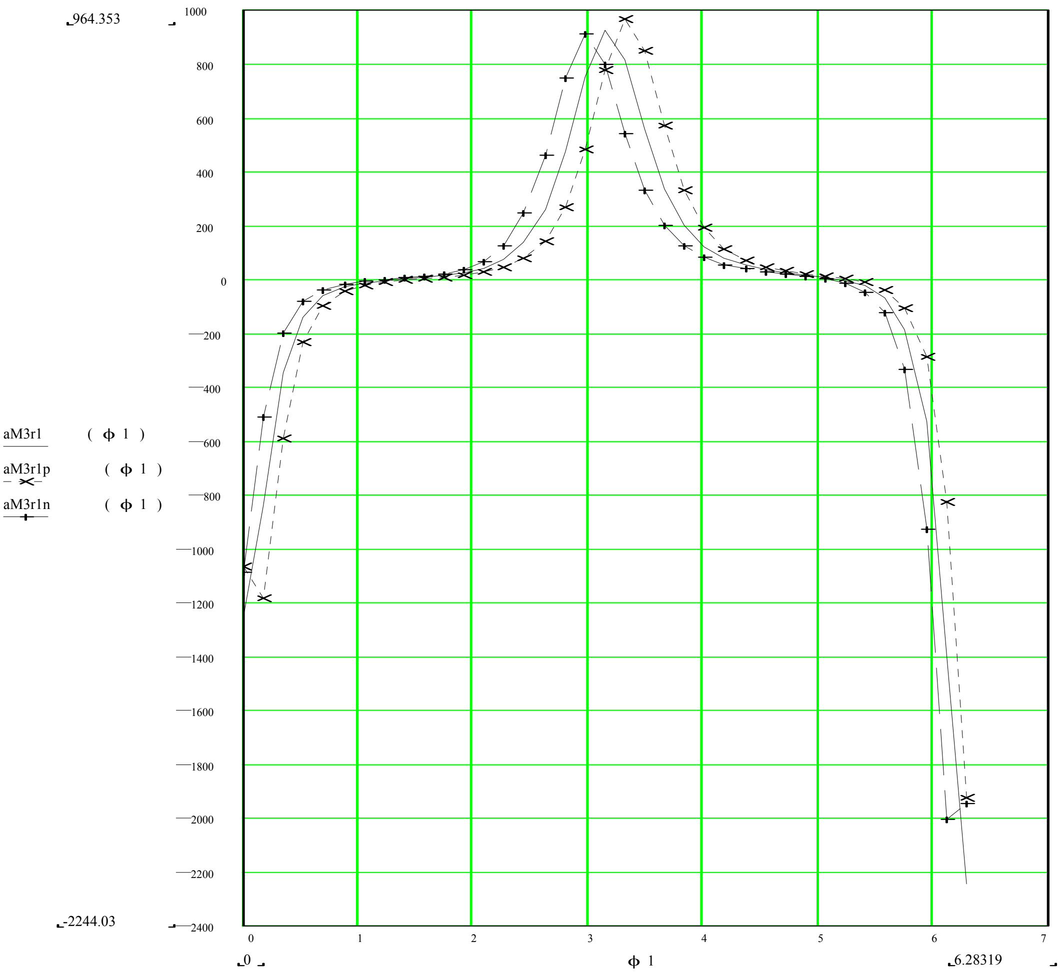


Fig. 15. a_{M3r1}, a_{M3r1p}, a_{M3r1n} ***depending of*** φ_1

Angular accelerations ε_{2r1}, ε_{2r1p}, ε_{2r1n} vary between -4578 and 2708 s^{-2} in comparison with ε_{20} that varies between -2600 and 2600 s^{-2}. Horizontal change of place of curves is of 13^0 to the right for positive eccentricity and of 13^0 to the left for negative (Fig.8). Accelerations a_{M20p} and a_{M20n} vary between 480 and 1130 s^{-2} but accelerations a_{M2r1}, a_{M2r1p}, a_{M2r1n}, vary between 80 and 2020 ms^{-2} in comparison with a_{M20} that varies between 500 and 1100 ms^{-2}. Horizontal change of place of curves is insignificant (Fig.11, Fig.12).

Piston accelerations a_{M30p}, a_{M30n} vary between -1255 and 790 ms^{-2} but accelerations a_{M3r1}, a_{M3r1p}, a_{M3r1n} vary between -2244 and 964 ms^{-2} in comparison with a_{M30} that varies between -1255 and 760 ms^{-2}.

6. CONCLUSIONS

Eccentricity influence is small relatively for eccentricity value of 0.05m for a length of crank of 0.1m. Technological eccentricities of values comparable with dimensions tolerances have an insignificant influence upon dynamic velocities and accelerations. Dynamic velocities and accelerations have much bigger values in comparison with kinematical and they are strongly variable.

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