

THE INFLUENCE OF SOME OF THE CYLINDRICAL GEAR PARAMETERS ON THE CRITICAL ROTATION

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Abstract: In the case of cylindrical gears, ISO calculus method gives two possibilities for determination of the K_v interior dynamic factor, depending on gear importance and calculus accuracy. For an accurate calculus, the interior dynamic factor is determined depending on critical rotation of pinion. This paper presents the method for determination of the critical rotation and the influence of several geometrical and kinematical parameters over the value of the critical rotation at the pinion of an exterior cylindrical gear.

1. INTRODUCTION

One of the hypotheses adopted for both the contact and bending stress calculus models was that the teeth interaction normal force is statically applied on the teeth. Since this hypothesis cannot be applied for real gears ISO recommends the introduction of several correction factors which take account of the differences between the real gear and the calculus model. The dynamic action of the F_n normal force is due to two causes: external dynamic action, dependent of the motor and driven machines, and internal dynamic action, dependent of manufacturing and fitting errors and also of the elastic deformations of shaft, casing etc.

The K_v dynamic factor is defined as a ration between the total load of the teeth (internal dynamic effects and transmitted tangential load) and the tangential load of the teeth and it is influenced by both design and machining parameters.

From the design parameters, most important are: peripheral velocity on the reference cylinder; teeth load considering the K_A application factor; masses and mass inertia moments of the pinion and gear and the masses coupled to the gear; tooth bending stiffness and, especially, the stiffness of simultaneously gear teeth and the variation of this stiffness during the gearing process; stiffness of the shafts, bearings and casing; Damping properties of the transmission the gear is part of but also of the lubricant between the teeth; critical rotation and the internal gear vibrations. Of the machining parameters, the most important are: gear ratio deviations due to the f_{pb} base pitch and f_f profile tolerances; balancing of the gears and other parts; bearing adjustments and loads.

2. THEORETICAL BASIS

The ISO/DIN calculus method presents two calculus methods: method *B* – for precise calculus and method *C* – for a calculus with sufficient precision for many of the real life cases. Both methods are based on the simplifying hypothesis [] that the teeth pair is replaced by an elastic system shown in fig. 1, made of only one mass obtained by combining the masses of the pinion and the gear, reduced to the gearing line, with a mean gearing teeth stiffness c_v , a mean damping k , given by the lubricant and the base pitch and profile tolerances of the pinion (f_1) and of the wheel (f_2).

Calculus method *B* is based on the following hypotheses:

- Each gear is considered to be independent, meaning that, in a multiple step transmission, the influence of the other gear steps is neglected; this consideration is applicable where the shafts supporting both driven gear and pinion (for example, the intermediary shaft of a reducer) have relatively low stiffness.
- The flexional vibrations of the shaft – gear system are neglected because usually the bending stiffness of the shaft is very big and the specific frequency of the vibration is greater than the working rotation of the shaft.
- The damping in bearings and couplings is neglected, since they are already taken into account by the K_A application factor.

This paper presents the influence of some parameters of the gear on its critical rotation. Some of these considered parameters are: pinion number of teeth; the gear ratio; the helix angle; the addendum modification coefficient etc.

The critical rotation for a cylindrical gear can be determined with the following formula []:

$$n_{cr1} = n_{E1} = \frac{30 \cdot 10^3}{\pi Z_1} \sqrt{\frac{c_\gamma}{m_{red}}}, \quad (1)$$

where m_{red} represents the gearing toothed wheels mass, reduced on the gearing line, per unit of length, in kg/mm; c_γ – mean value of total tooth stiffness per face width unit, in N/(mm.µm).

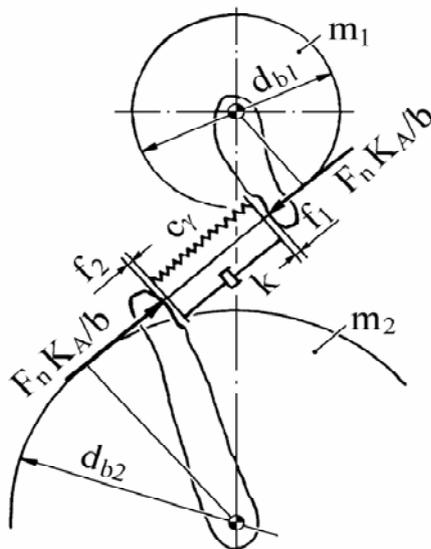


Fig. 1

where c' is the stiffness of a gear teeth pair, per length unit, which is determined with the formula:

$$c_\gamma = c' (0,75 \varepsilon_\alpha + 0,25), \quad (2)$$

where: c'_{th} represents the theoretic stiffness of a gear teeth pair; $C_M=0,8$ – correction factor which takes account of the differences between theoretical calculus, made for a specific load of $F_t/b=300$ N/mm, and the result of the measurements; C_R – additional factor referring to the toothed wheels construction; C_B – additional factor referring to the reference profile.

The c'_{th} theoretic stiffness can be determined with

$$c'_{th} = 1/q', \quad (4)$$

and the q' factor with

$$q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + C_4 x_{n1} + C_5 \frac{x_{n1}}{z_{n1}} + C_6 x_{n2} + C_7 \frac{x_{n2}}{z_{n2}} + C_8 x_{n1}^2 + C_9 x_{n2}^2, \quad (5)$$

the values of the $C_1...C_9$ coefficients are given in Table 1.

Table 1

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0,04723	0,15551	0,25791	-0,00635	-0,11654	-0,00193	-0,24188	0,00529	0,00182

The C_R gear construction factor takes in account the flexibility of the gear disk and rim. If the gear is manufactured in massive construction, $C_R=1.0$. For a hub – disk – rim construction, the C_R factor is determined with (fig. 2)

$$C_R = 1 + \frac{\ln(b_s/b)}{5e^{s_R/(5m_n)}}, \quad (6)$$

with the following restrictions:

- if $b_s/b < 0.2$, then on considers $b_s/b = 0.2$;
- if $b_s/b > 1.2$, then on considers $b_s/b = 1.0$;
- if $s_R/m_n < 1.0$, then on considers $s_R/m_n = 1.0$.

The C_B reference profile factor considers the differences between the used reference profile and the standard reference profile ($\alpha_n=20^\circ$, $h_{an}^*=1.0$ and $c_n^*=0.2$), the last being considered for the theoretic stiffness calculus, and is determined with

$$C_B = [1 + 0.5(1.2 - h_{an}^* - c_n^*)][1 - 0.02(20^\circ - \alpha_n)] \quad (7)$$

If the specific load is $K_A F_t / b < 100$ N/mm, then the stiffness of the teeth pair decreases with the decrease of the load. In this case the (3) equation becomes

$$c' = c'_{th} C_M C_R C_B [(K_A F_t / b) / 100]^{0.25} \cos \beta. \quad (8)$$

The m_{red} reduced mass for an exterior cylindrical gear is

determined with

$$m_{red} = \frac{\pi}{8} \left(\frac{d_{m1}}{d_{b1}} \right)^2 \frac{d_{m1}^2}{\frac{1}{(1-q_1^4) \rho_1} + \frac{1}{(1-q_2^4) u^2 \rho_2}}, \quad (9)$$

where $d_{m1,2} = \frac{d_{a1,2} + d_{f1,2}}{2}$ is the average teeth diameter for the pinion, respectively for the driven gear; $q_{1,2} = d_{i1,2} / d_{m1,2}$, with d_a , d_f , d_i according to fig. 2; $1 - q_1^4 = 1$, if the pinion is in

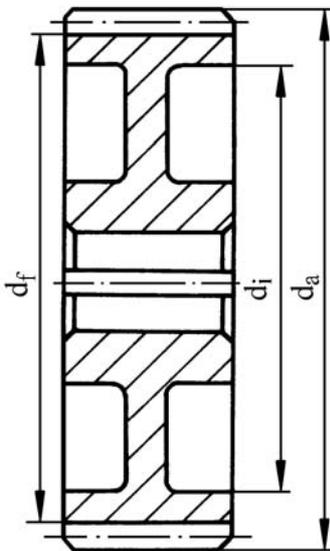


Fig. 2

the same piece with the shaft or in massive construction; $1 - q_2^4 = 1$, if the gear is in massive construction; $\rho_{1,2} = 7,83 \cdot 10^{-6} \text{ kg/mm}^3$, steel density.

3. THE INFLUENCE OF THE GEAR PARAMETERS ON THE CRITICAL ROTATION

To analyze the influence of the parameters on the critical rotation, a software application was developed and its main menu is presented in fig. 3. The main required initial data is shown in fig. 4, where one can choose the parameters/influences on the critical rotation which will be studied, but also the fixed values for some other parameters.

Fig. 5 displays the way to choose the construction of the toothed wheel so that the application may calculate the gear reduced mass. A fraction of the results obtained by running the program is presented in fig. 6.

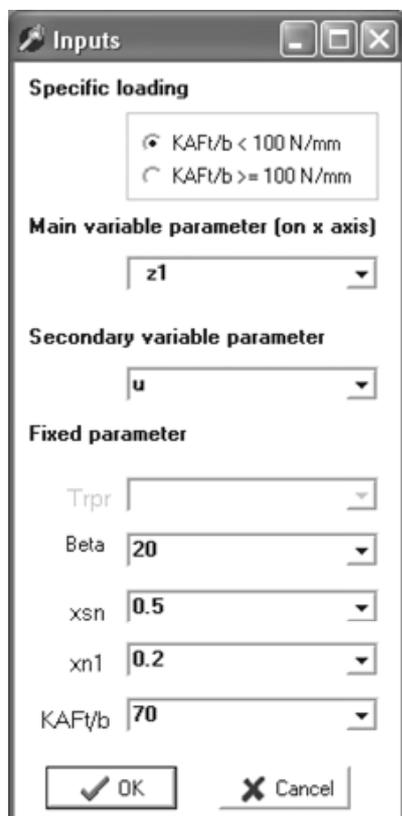


Fig. 4



Fig. 3

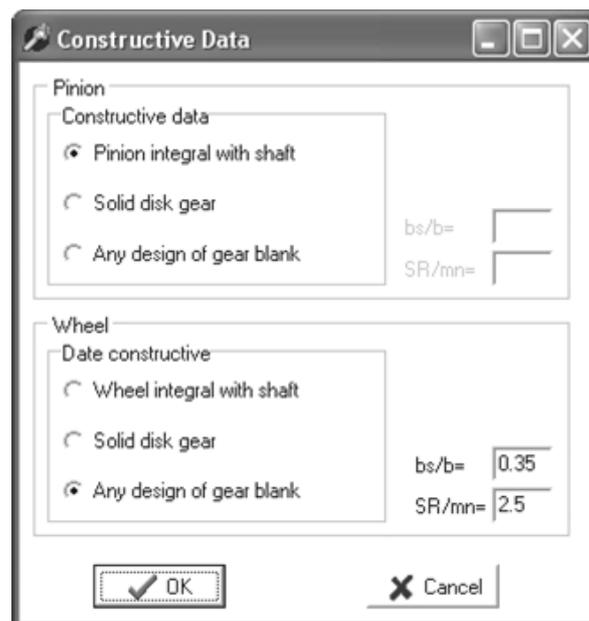


Fig. 5

The program offers the choice of the parameter which will be on the abscissa but also of the parameter for which the curves will be drawn. For example, in fig. 6 the abscissa variable parameter is the z_1 pinion number of teeth and the curves are drawn for different values of the u gear ratio.

To study the influence of diverse parameters on the critical rotation, the results obtained by running the developed and shortly introduced (in fig. 3...6) application were graphically displayed in the following.

$z_1=$	15	20	25	30	40	50
		beta=20	x _{sn} =0.5	x _{n1} =0.2	K _A F _t /b=70	
u=1	33738.0	20862.0	14404.0	10661.0	6653.3	4629.9
u=2	29412.0	17959.0	12244.0	8956.7	5479.6	3752.6
u=3	28449.0	17176.0	11596.0	8411.4	5074.2	3436.0
u=4	28072.0	16823.0	11287.0	8142.7	4867.0	3270.7
u=6	27770.0	16500.0	10989.0	7876.6	4656.0	3099.5
u=8	27650.0	16349.0	10844.0	7744.6	4548.8	3011.5

Fig. 6

Fig. 7 presents the variation of the n_{E1} critical rotation for an exterior cylindrical spur gearing, with negative addendum modification coefficient ($x_{sn}=-0.5$) and a reduced specific load ($K_A F_t / b = 50$ N/mm), for three values of the u gear ratio. To respect the conditions imposed by ISO 6336-1 [], more exactly: $x_{n1} \geq x_{n2}$; $-0.5 \leq x_{n1} + x_{n2} \leq 2$ and $x_{n2} \geq -0.5$, for the pinion there was considered a value for the addendum modification coefficient of $x_{n1}=0$.

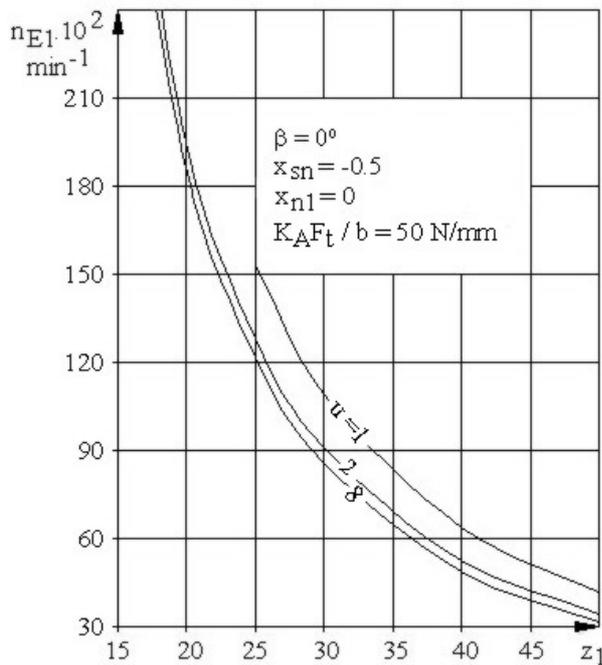


Fig. 7

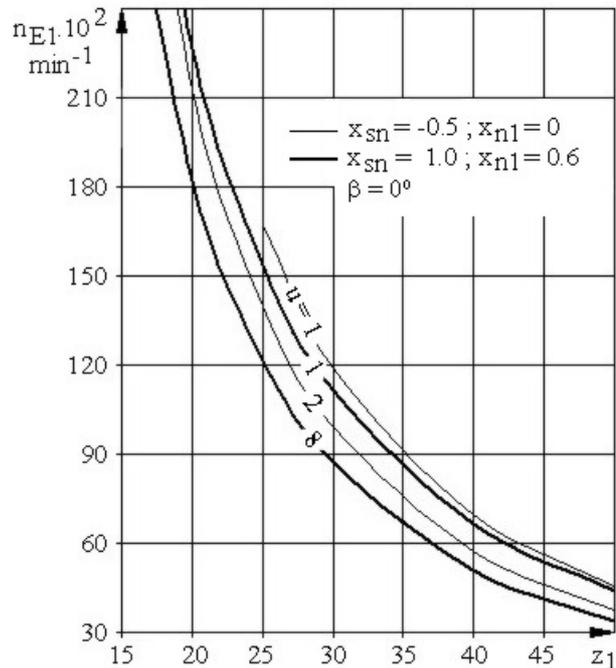


Fig. 8

For the same parameters of the gear, fig. 8 displays the variation of the n_{E1} critical rotation for a gear with specific load $K_A F_t / b > 100$ N/mm. On the diagrams, the curves which refer to a gearing with negative addendum modification are drawn using a thin line ($x_{sn}=-0.5$), and the curves that refer to a gearing with positive addendum modification are drawn with a fat line.

For the two diagrams, the following conclusions can be drawn:

- the critical rotation decreases very much with the increase of the pinion teeth number;
- with the increase of the total addendum modification coefficient of the gearing and the addendum modification coefficient of the pinion, the critical rotation decreases, but in a relatively small quantity, especially for small teeth numbers of the pinion; for big teeth numbers, the decrease is extremely small (see fig. 8);
- for small gear ratios, the critical rotation decreases with the increase of the gear ratio in a noticeable manner, while for big gear ratios the respective curves are overlaid or the differences between them are very small;
- the increase of the gearing specific load leads to a certain decrease of the critical rotation.

Fig. 9 presents the variation of the n_{E1} critical rotation for a exterior cylindrical gearing with negative addendum modification ($x_{sn}=-0,5$), but with a high specific load ($K_A F_t / b > 100$), for comparison with the gearing shown in fig. 7, and fig. 10 shows the same variation for a gearing with positive addendum modification ($x_{sn}=1.0$).

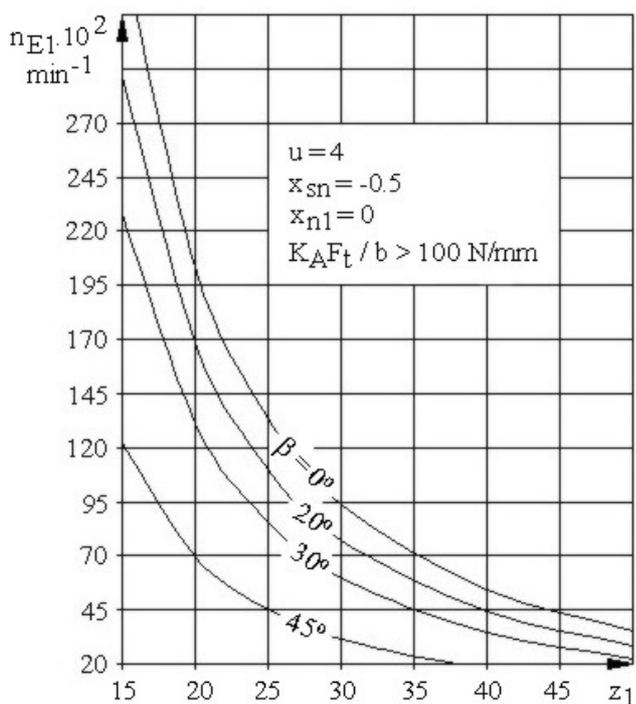


Fig. 9

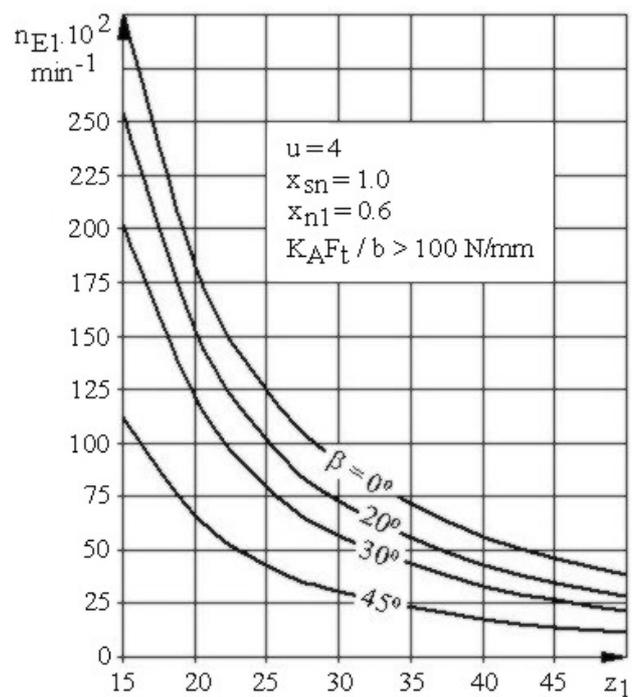


Fig. 10

The analysis of the two diagrams, compared also with the ones from fig. 7 and fig. 8, leads to the following conclusions:

- the critical rotation decreases in relatively high proportion with the increase of the helix angle, independent of the addendum modification;
- the higher specific load ($K_A F_t / b > 100 \text{ N/mm}$, comparing with the situation where $K_A F_t / b = 50 \text{ N/mm}$), leads to a small increase of the critical rotation of the pinion, no matter if the addendum modification is positive or negative;

- a comparison between the diagrams presented in fig. 9 and fig. 10 shows that the critical rotation for the gear with positive addendum modification is a little smaller than for the gear with negative addendum modification.

Fig. 11 and fig. 12 present the influence of the gear hub – disk – rim construction type on the critical rotation of the exterior cylindrical gearing pinion, compared with the previous cases where the pinion was considered to be manufactured integrated with the shaft and the driven gear in massive construction.

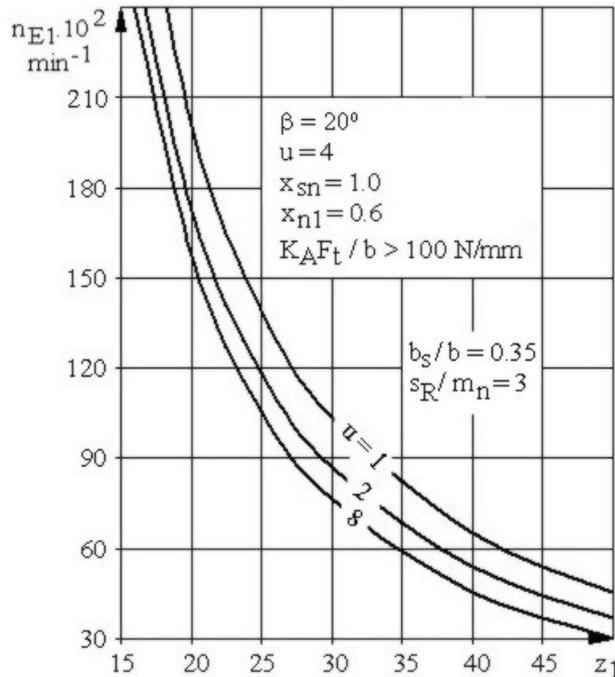


Fig. 11

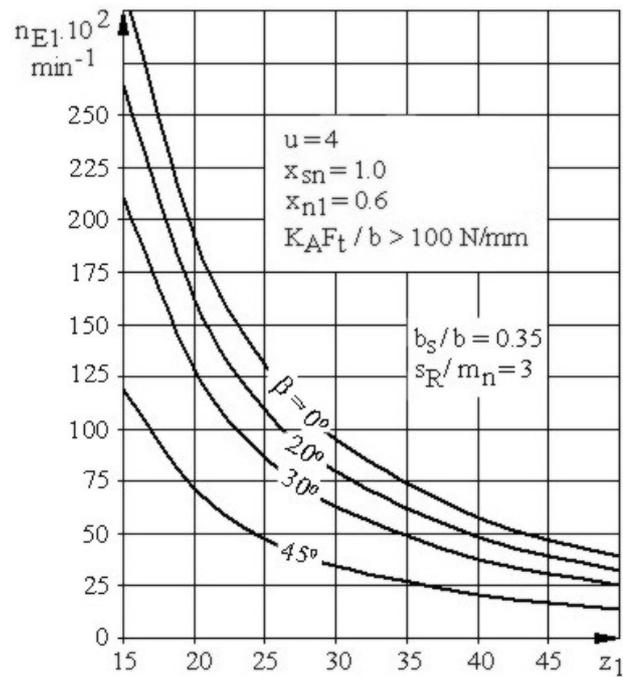


Fig. 12

The shown situation are about a gear with positive addendum modification ($x_{sn}=1.0$) and a pinion with the addendum modification coefficient $x_{n1}=0.6$. The curves are drawn for the u gearing ratio (see fig. 11) and the β helix angle (see fig. 12). Mean values were considered for the b_s/b and s_R/m_n ratios.

The study of the diagrams from fig. 11 and fig. 12 allows drawing the following conclusions:

- The value of the pinion critical rotation decreases with the increase of the gear ratio, when compared with the case where the driven gear was made in massive construction (fig. 10);
- For the same value of the gear ratio, $u=4$, the pinion critical rotation decreases with the increase of the β helix angle, especially for big angles and big pinion teeth numbers;

4. GENERAL CONCLUSIONS

The analysis done and presented in this paper leads to underlining the following general conclusions:

- The biggest influence on the critical rotation of the pinion of a exterior cylindrical gearing belongs to the z_1 number of teeth, the β helix angle and the gear ratio; the increase of these parameters leads to the decrease of the critical rotation;
- A small influence over the critical rotation is exercised by the x_{sn} total addendum modification coefficient, the x_{n1} pinion addendum modification coefficient and the construction of the driven gear, whether massive or as hub – disk – rim, represented by the b_s/b ratio between the hub width and gear width and the s_R/m_n ratio between the rim thickness and normal module;
- The $K_A F_t/b$ specific load has an average influence inducing a decrease of the pinion critical rotation with its increase.

Bibliography

- [1] Moldovean, Gh., Velicu, D., Velicu, R. and others. Angrenaje cilindrice și conice, Calcul și construcție. (Cylindrical and bevel gears. Calculus and constructions.) Lux Libris, Brașov, 2001.
- [2] Moldovean, Gh., Velicu, D., Chișu, E. and others. Angrenaje cilindrice și conice, Metodici de proiectare. (Cylindrical and bevel gears. Design Methods.) Lux Libris, Brașov, 2001.
- [3] Sprengers, J. La normalisation ISO et le calcul des engrenages. In European Journal of mechanical engineering, 42(2), p. 83-87.
- [4] Stokes, A. Gear Handbook. Design and Calculation. Butterworth – Heinemann, 1992.
- [5] Velicu, R., Moldovean, Gh. Angrenaje cilindrice. Reductoare cilindrice. (Cylindrical gears. Cylindrical speed reducers.) Editura Universității Transilvania din Brașov, 2002.
- [6] DIN 3990 Teil 1. Tragfähigkeitsberechnung von Stirnrädern. Einführung und allgemeine Einflussfaktoren. Beuth Verlag, 1987.
- [7] ISO 6336-1: 1996 (E). Calculation of load capacity of spur and helical gears. Part 1: Basic principles, introduction and general influence factors.