ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

THE STRUCTURAL SYNTHESIS OF PARALLEL ROBOTS

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Keywords: robotics, serial robot, parallel robot, open-loop kinematic chain, structural synthesis

Abstract — This paper presents the structural synthesis of parallel robots. The mechanism of the parallel robots is composed by 3 or 6 legs that connect the moving platform to the fixed base. Each leg is an open-loop kinematic chain. Using the structural synthesis, we study the parallel robots with three or six degrees of mobility operating in a planar or 3D workspace and having the number of open-loop kinematic chains, that connect the mobile platform with the fixed base, equal to the number of degree of mobility.

1. INTRODUCTION

From designing and functional point of view, the advantages of the parallel robots structures, compared to conventional serial robots structures, are:

⇒ outstanding place (position and orientation) accuracy;
⇒ a much convenient ratio of object mass to robot mass, in case of parallel robots compared to serial robots;
⇒ execute small and very small movements with a high accuracy, a great advantages especially in automated assembly stations;
⇒ a higher stiffness of the parallel robots compared to serial robots, which normally use an open-loop kinematic chain;
⇒ high operating speed;
⇒ prompt dynamic replay compared to serial robots, that have a slow reaction;

Having all the drives fixed to the robot basis, outside of its workspace, they form an ideal parallel robot structure for tasks of handling in automation assembly station.

The parallel robots have some disadvantages:

⇒ the limited workspace;
⇒ the higher cost of joints manufacture;
⇒ the higher cost of kinematic calculations in real time;

Since 1965, when D. Stewart has developed six degrees of mobility flight manipulator [9], the parallel robots and their applications have been studied with a great interest [1], [2], [5]. In Romania many authors have been treated parallel robots in very important articles [3], [4], [7]. This paper presents the structural synthesis of parallel robots. Using the structural synthesis, we study the parallel robots with three or six degrees of mobility operating in a planar or 3D workspace and having the number of open-loop kinematic chains, that connect the mobile platform with the fixed base, equal to the number of degree of mobility.

2. STRUCTURAL SYNTHESIS OF PARALLEL ROBOTS

From the structural point of view, any parallel robot can be decomposed into three basic components: the moving platform, the fixed base and the connecting legs. The connecting legs can be viewed as black boxes consisting of links and joints connecting the two platforms, the moving and the fixed base. Each leg is an open-loop kinematic chain and their number is usually equal to the number of degrees of mobility of the moving platform. This means that one single actuator drives each open-loop kinematic chain. In this way, all actuators can be mounted on or nearby the fixed base and as a result the parallel robots have the advantages of a low inertia, a high stiffness and a high dynamics in order to be able to cover a large payload.

According to the freedom criteria, the Gruebler-Kutzbach relation [8] calculates, the number of degrees of mobility, M for a mechanism of b family:

$$M = b(n-1) - \sum_{i=1}^{c}(b - f_i) = b(n - c - 1) + \sum_{i=1}^{c} f_i \tag{1}$$

where:

- f_i is the number of degrees of freedom for, i, joint;
- b is the number of general freedom of the kinematic chain links.

The total number of degrees of freedom of all joints of the open-loop kinematic chains that connect the mobile platform to the fixed base is:

$$\sum_{i=1}^{c} f_i = M + bN \tag{2}$$

For closed-loop mechanisms the number of independent loops, N, is reflected by the equation [8]:

$$N = c - n + 1 \tag{3}$$

where:

- n is the number of links;
- c is the number of joints.

Because each connecting leg between mobile platform and fixed base is an open-loop kinematic chain, and their number, k, is equal to the number of degrees of mobility, M, the following equation is obtained [5]:

$$k = M = N + 1 \tag{4}$$

Eliminating n and c from equations (2) and (3), a criteria is obtained, depending of the number of degrees of mobility, M, and the number of independent loops, N, that permit to calculate the total number of degrees of freedom of the all joints of open-loop kinematic chains between mobile platform and fixed base [5], [8]:

$$\sum_{i=1}^{c} f_i = M + bN \tag{5}$$

If each open-loop kinematic chain has the connectivity C_j, defined [5] as the total number of degrees of freedom associated with the every open-loop kinematic chain, k, they are imposing the following equation:

$$\sum_{j=1}^{k} C_j = \sum_{i=1}^{c} f_i \tag{6}$$

The connectivity C_j of each leg should not be less than number of degrees of mobility, M, of the moving platform, not is greater than number of general freedom of the kinematic chain links, b that is:

$$M \le C_j \le b \qquad \text{for} \qquad j = \overline{1,k} \tag{7}$$

Equations (4), (6) and (7) characterize completely the parallel robots from a structural point of view.

A classification of the mostly used parallel robots is:

b) parallel robots with a planar workspace (2D), having M=3;
c) parallel robots with a spatial workspace (3D), that has two types:
   - with M=3 – "active wrist" [6], that can be used as orientation mechanisms of a serial robot;
   - with M=6 – "left hand" [6], that execute small and very small amplitude moving, they can be used as end effectors of serial robot;.

The spatial parallel robots can be used in connection with serial robot, in order to obtain robots with hybrid structure.

3. PARALLEL ROBOTS WITH A PLANAR WORKSPACE

The parallel robots with a planar workspace (2D), with M=3, have according to the relation (4):

$$k=3 \tag{8}$$

open-loop kinematic chain between the mobile platform and the fixed base.

Consequently for the parallel robots with three degree of mobility, M=3, with a planar workspace, b=3, according to the relations (4) and (5), the total number of degrees of freedom of the all open-loop kinematic chains joints is:

$$\sum_{i=1}^{c} f_i = 9 \tag{9}$$

In conclusion, according to the relations (6), (7) and (8), the connectivity of the all open-loop kinematic chains between mobile platform and fixed base, in the case of planar workspace parallel robots is:

$$\sum_{j=1}^{k} C_j = 9 \tag{10}$$

or:

$$C_1 + C_2 + C_3 = 9 \tag{11}$$

considering that:

$$3 \le C_j \le 3 \qquad \text{for} \qquad j = \overline{1,3} \tag{12}$$

The joints number of an open-loop kinematic chain, that has the total number of degrees of freedom equal to the required connectivity C_j, influences the number of links of the respective chain. In this way, we obtain the maximum number of links when all the joints have one degree of freedom. Obviously this will lead to a large number of feasible solutions.

Solving equation (11) for positive integers of C_j and considering the constraint imposed by (12), we will consider only the planar workspace parallel robots with three identical open-loop kinematic chain configurations (from designing point of view). Thus the (3, 3, 3) connectivity becomes the only feasible solution. Only the solutions with one driving joint, revolute or prismatic joint, are kept, and the number of prismatic joints on every open-loop kinematic chain, are limited to two.

Further we assume only the most important joints:

a) rotation joints (R) and translation joints (T) with $f_i = 1$;
b) cylindrical joints (C) and Cardan or Universal joints (U) with $f_i = 2$;
c) spherical joints (S) with $f_i = 3$;

By combining the joints in this way, we obtain varied structures of open-loop kinematic chains, to ensure the connecting legs between the moving platform and the fixed base of the planar workspace parallel robots. The structures of the open-loop kinematic chains of the planar workspace (2D) parallel robots are presented in Table 1. The joint arrangement is presented from fixed base to moving platform.

TABLE 1

Joints with f_i			Structural variants code
1	2	3	
3	0	0	RRR, RRT, RTR, TRR

4. PARALLEL ROBOTS WITH A SPATIAL WORKSPACE

The parallel robots with a spatial workspace (3D), with M=3, respectively M=6, has according to the relation (4):

$$k=3 \tag{13}$$

and respectively

$$k=6 \tag{14}$$

open-loop kinematic chain between the mobile platform and the fixed base.

Consequently, the spatial workspace parallel robots with M=3, respectively M=6, with a spatial workspace (3D), b=6, according to the relations (4) and (5), the total number of degrees of freedom of the all open-loop kinematic chains joints is:

$$\sum_{i=1}^{c} f_i = 15 \tag{15}$$

respectively

$$\sum_{i=1}^{c} f_i = 36 \tag{16}$$

In conclusion, according to the relations (6), (7) and (8), the connectivity of the all open-loop kinematic chains between mobile platform and fixed base, in case of spatial workspace parallel robots is:

$$\sum_{j=1}^{k} C_j = 15 \tag{17}$$

respectively

$$\sum_{j=1}^{k} C_j = 36 \tag{18}$$

or:

$$C_1 + C_2 + C_3 = 15 \tag{19}$$

respectively

$$C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 36 \tag{20}$$

considering that:

$$3 \le C_j \le 6 \qquad \text{for} \qquad j = \overline{1,3} \tag{21}$$

respectively

$$6 \le C_j \le 6 \qquad \text{for} \qquad j = \overline{1,6} \tag{22}$$

Solving equation (19) for positive integers of C_j and considering the constraint imposed by (21), we will consider only the spatial workspace parallel robots with M=3, and having three identical open-loop kinematic chain configurations (from designing point of view).

Thus the (5, 5, 5) connectivity becomes the only feasible solution. Only the solutions with one driving joint, revolute or prismatic joint, are kept, and the number of prismatic joints on every open-loop kinematic chain, are limited to two.

By combining the joints in this way we obtain varied structures of open-loop kinematic chains, to ensure the connecting legs between the moving platform and the fixed base of the spatial workspace parallel robots with M=3. The structures of the open-loop kinematic chains of the spatial workspace (3D) parallel robots, with M=3 are presented in Table 2. The joint arrangement is presented from fixed base to moving platform.

TABLE 2

Joints with f_i			Structural variants code
1	2	3	
5	0	0	RRRRR, TRRRR, RTRRR, RRTRR, RRRTR, RRRRT, RRRTT, RRTTR, RTTRR, TTRRR, TRTRR, TRRTR, TRRRT, RTRTR, RTRRT, RRTRT
3	1	0	RRRU, RRTU, RTRU, TRRU, RTTU, TRTU, TTRU, RRUR, RRUT, RTUR, TRUR, RTUT, TRUT, TTUR, RURR, RURT, RUTR, TURR, RUTT, TURT, TUTR, URRR, URRT, URTR, UTRR, URTT, UTRT, UTTR*
2	0	1	RRS, RTS, TRS, RSR, RST, TSR, SRR, SRT, STR, TTS, TST, STT
1	2	0	RUU, TUU, URU, UTU**

Foot note:

* Universal joint (U) can be replace with Cylindrical joint (C) resulting more 28 variants;

** Universal joint (U) can be replace with Cylindrical joint (C) resulting more 12 variants.

Solving equation (20) for positive integers of C_j and considering the constraint imposed by (22), we will consider only the spatial workspace parallel robots with M=6, and having three identical open-loop kinematic chain configurations (from designing point of view).

Thus the (6, 6, 6) connectivity becomes the only feasible solution. Only the solutions with one driving joint, revolute or prismatic joint, are kept, and the number of prismatic joints on every open-loop kinematic chain, are limited to two.

By combining the joints in this way we obtain varied structures of open-loop kinematic chains, to ensure the connecting legs between the moving platform and the fixed base of the spatial workspace parallel robots with M=6. The structures of the open-loop kinematic chains of the spatial workspace (3D) parallel robots, with M=6 are presented in Table 3. The joint arrangement is presented from fixed base to moving platform.

TABLE 3

Joints with f_i			Structural variants code	Joints with f_i			Structural variants code
1	2	3		1	2	3	
6	0	0	RRRRRR, TRRRRR, RTRRRR, RRTRRR, RRRTRR, RRRRTR, RRRRRT, RRRRTT, RRRTTR, RRTTRR, RTTRRR, TTRRRR, TRTRRR, TRRTRR, TRRRTR, TRRRRT, RTRTRR, RTRRTR, RTRRRT, RRTRTR, RRTRRT, RRRTRT	3	0	1	RRRS, RRTS, RTRS, TRRS, RRSR, RRST, RTSR, TRSR, RSRR, RSRT, RSTR, TSRR, SRRR, SRRT, SRTR, STRR
4	1	0	RRRRU, RRRTU, RRTRU, RTRRU, TRRRU, RRTTU, RTRTU, RTTRU, TRTRU, TTRRU, RRRUR, RRRUT, RRTUR, RTRUR, TRRUR, RRTUT, RTRUT, RTTUR, TRTUR, TTRUR, RRURR, RRURT, RRUTR, RTURR, TRURR, RRUTT, RTURT, RTUTR, TRUTR, TTURR, RURRR, RURRT, RURTR, RUTRR, TURRR, RURTT, RUTRT, RUTTR, TURTR, TUTRR, URRRR, URRRT, URRTR, URTRR, UTRRR, URRTT, URTRT, URTTR, UTRTR, UTTRR*	2	2	0	RRUU, RURU, URRU, URUR, UURR, TTUU, TUTU, UTTU, UTUT, UUTT**
				1	1	1	RUS, RSU, TUS, TSU, URS, USR, UTS, UST***

Foot note:
* Universal joint (U) can be replace with Cylindrical joint (C) resulting more 50 variants;
** Universal joint (U) can be replace with Cylindrical joint (C) resulting more 30 variants.
*** Universal joint (U) can be replace with Cylindrical joint (C) resulting more 8 variants.

5. CONCLUSIONS

Using the structural synthesis, we study the parallel robots with three or six degrees of mobility operating in a planar or 3D workspace and having the number of open-loop kinematic chains, that connect the mobile platform with the fixed base, equal to the number of degree of mobility.

The structural synthesis of parallel robots allows identifying parallel robots structures with three degrees of mobility, having the mobile platform only in translation moving. These parallel robots can be used as trajectory generator mechanisms. Jointing this mechanism with a classic orientation mechanism we obtain robots with hybrid structure.

The study also permits to identify other parallel robots structures with three degrees of mobility, having the mobile platform only in rotational moving. These parallel robots can be used as orientation mechanisms. Jointing this mechanism with a classic Cartesian serial manipulator we obtain robots with hybrid structure.

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