ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

A BIOMECANICAL MODEL OF THE BACKWARD LONGSWING ON RINGS IN GYMNASTICS

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Keywords: biomechanical model, gymnastics, longswing

Abstract

For a process optimization it is necessary a scientific describe of this. Any subjective aspect calls off any effort in this way. The final limit of abstraction is the mathematic model of considered system and process. In the last years there are many attempts to achieve the mathematical model of human body in various motions. Our interdisciplinary efforts are directed to build up a valid biomechanical model of a gymnast in the backward longswing on rings.

1. INTRODUCTION

The figure 1 shows a scheme of the analyzed motion with its principal stages.

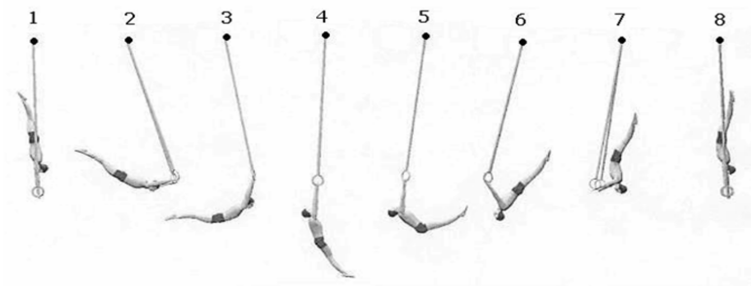

Fig. 1

On the mechanical point of view the system has four independent segments (fig. 2):

- the rings and cables = one-dimensional element with linear viscoelastic behavior (EL): $F(x,\dot{x}) = -B_1 x - C_1 \dot{x}$;

- the arms = one-dimensional element with nonlinear viscoelastic behavior (EN): $F(x,\dot{x}) = -A_1 x^2 - B_1 x - C_1 \dot{x}$

- the torso+head and legs (foots+shanks+thighs) = two one-dimensional element with rigid behavior (ER).

All the four elements are articulated on ideal rotation joints.

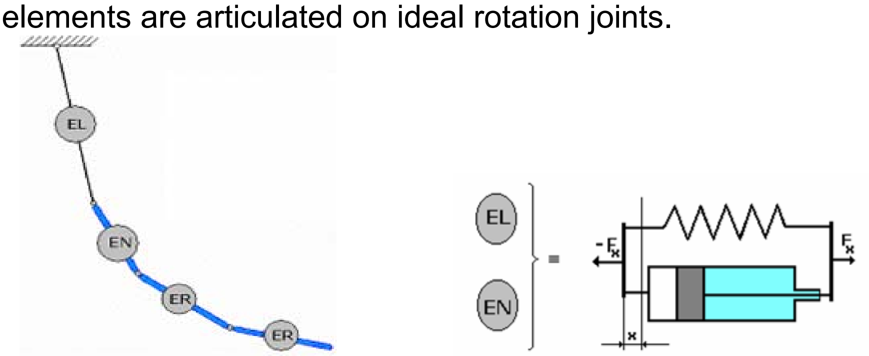

Fig. 2

In figure 3 it is presented the geometrical model of gymnast with analytical and mechanical notes adequate for the study.

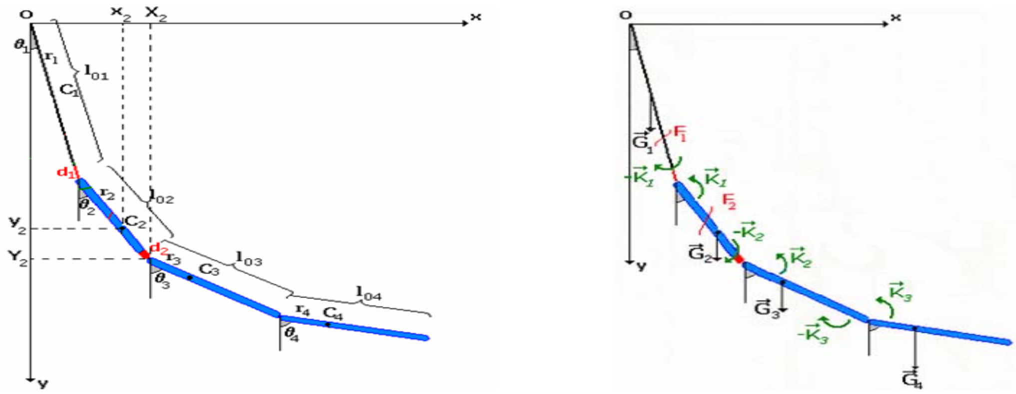

Fig. 3

2. MOVING EQUATIONS

These equations are deduced with the aid of Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right)-\frac{\partial E_c}{\partial q_k}=Q_k\,,\quad k=\overline{1,l} \tag{1}$$

The analytical expressions of the terms are deduced in succession.

- *kinetic energy:*

$$E_c=E_{c1}+E_{c2}+E_{c3}+E_{c4}=\sum_{i=1}^{4}E_{ci} \tag{2}$$

$$E_{ci}=E_{ci}^{transl}+E_{ci}^{rot}=\frac{1}{2}m_i v_i^{\,2}+\frac{1}{2}J_i\dot{\theta}_i^2=\frac{1}{2}m_i(\dot{x}_i^2+\dot{y}_i^2)+J_i\dot{\theta}_i^2 \tag{3}$$

In the final of calculus the kinetic energy are obtained as a symmetric square form:

$$E_c=\sum_{i=1}^{6}\sum_{j=1}^{6}a_{ij}\dot{q}_i\dot{q}_j=a_{ij}\dot{q}_i\dot{q}_j\,,\qquad a_{ij}=a_{ji}\,,\ i,j=\overline{1,6,\,i\neq j} \tag{4}$$

where:

$$\{q\}=[\theta_1\ \ \theta_2\ \ \theta_3\ \ \theta_4\ \ d_1\ \ d_2]^T \text{ - matrix 6x1 of generalized coordinates} \tag{5}$$

$$\{\dot{q}\}=[\dot{\theta}_1\ \ \dot{\theta}_2\ \ \dot{\theta}_3\ \ \dot{\theta}_4\ \ \dot{d}_1\ \ \dot{d}_2]^T \text{ - matrix 6x1 of generalized velocities} \tag{6}$$

$[a]$ - matrix 6x6 of square form coefficients:

$$
\begin{aligned}
&a_{11}=\frac{1}{2}\left[J_1+m_{11}(l_1+q_5)^2\right] && a_{16}=\frac{1}{2}m_{12}(l_1+q_5)\sin(q_2-q_1);\\
&a_{22}=\frac{1}{2}\left[J_2+m_{22}(l_2+q_6)^2\right] && a_{23}=\frac{1}{2}m_{13}l_3(l_2+q_6)\cos(q_3-q_2);\\
&a_{33}=\frac{1}{2}\left(J_3+m_{33}l_3^2\right); && a_{24}=\frac{1}{2}m_{14}l_4(l_2+q_6)\cos(q_4-q_2);\\
&a_{44}=\frac{1}{2}\left(J_4+m_{44}l_4^{\,2}\right); && a_{25}=-\frac{1}{2}m_{12}(l_2+q_6)\sin(q_2-q_1);\\
&a_{55}=\frac{1}{2}m_{11}; && a_{26}=0;\\
&a_{66}=\frac{1}{2}m_{22}; && a_{34}=\frac{1}{2}m_{14}l_3l_4\cos(q_4-q_3);\\
&a_{12}=\frac{1}{2}m_{12}(l_1+q_5)(l_2+q_6)\cos(q_2-q_1); && a_{35}=-\frac{1}{2}m_{13}l_3\sin(q_3-q_1);\\
&a_{13}=\frac{1}{2}m_{13}l_3(l_1+q_5)\cos(q_3-q_1); && a_{36}=-\frac{1}{2}m_{13}l_3\sin(q_3-q_2);\\
&a_{14}=\frac{1}{2}m_{14}l_4(l_1+q_5)\cos(q_4-q_1); && a_{45}=-\frac{1}{2}m_{14}l_4\sin(q_4-q_1);\\
&a_{15}=0; && a_{46}=-\frac{1}{2}m_{14}l_4\sin(q_4-q_2);\\
& && a_{56}=\frac{1}{2}m_{12}\cos(q_2-q_1).
\end{aligned}
\tag{7}
$$

with noting

$$
\begin{aligned}
m_{11} &= m_1 p_1^2 + m_2 + m_3 + m_4 \\
m_{22} &= m_2 p_2^2 + m_3 + m_4 \\
m_{33} &= m_3 p_3^2 + m_4 \\
m_{44} &= m_4 p_4^2 \\
m_{12} &= m_2 p_2 + m_3 + m_4 \\
m_{13} &= m_3 p_3 + m_4 \\
m_{14} &= m_4 p_4
\end{aligned}
\tag{8}
$$

- generalized forces:

$$Q_k = \sum_{n=1}^{N} \vec{F}_n \frac{\partial \vec{r}_n}{\partial q_k} = \sum_{n=1}^{p} \vec{F}_n \frac{\partial \vec{r}_n}{\partial q_k} + \sum_{n=p+1}^{N} \vec{F}_n \frac{\partial \vec{r}_n}{\partial q_k}, \quad k = \overline{1,6} \tag{9}$$

where

$\vec{F}_1, \vec{F}_2, ..., \vec{F}_p$ - potential force: $\vec{F}_n = -grad_n U_n, \quad n = \overline{1,p}$;

$\vec{F}_{p+1}, \vec{F}_{p+2}, ..., \vec{F}_N$ - unpotential forces

Noting

$$U = \sum_{n=1}^{p} U_n \tag{10}$$

generalized forces are given by the following expressions:

$$Q_k = -u_k + Q_k^{nepot}, \quad k = \overline{1,6}, \qquad Q_k^{nepot} = \sum_{n=p+1}^{N} \vec{F}_n \frac{\partial \vec{r}_n}{\partial q_k}, \quad u_k = \frac{\partial U}{\partial q_k} \qquad k = \overline{1,6} \tag{11}$$

- potential energy:

$$U = U^{gr} + U^{el} + U^{rot} \tag{12}$$

where

U^{gr} - potential of gravitational forces;

U^{el} - potential of elastic and viscous forces;

U^{rot} - potential of joint moments.

It is obtained:

$$
\begin{aligned}
U^{gr} &= -g[m_{10}(l_1 + q_5)\cos q_1 + m_{12}(l_2 + q_6)\cos q_2 + m_{13} l_3 \cos q_3 + m_{14} l_4 \cos q_4] \\
U^{el} &= U_1^{el} + U_2^{el} = +\frac{B_1}{2} d_1^2 + \frac{A_2}{3} d_2^3 + \frac{B_1}{2} d_2^2 = \frac{B_1}{2} q_5^2 + \frac{A_2}{3} q_6^3 + \frac{B_1}{2} q_6^2 \\
U^{rot} &= -K_1 q_1 - (K_2 - K_1) q_2 - (K_3 - K_2) q_3 - K_3 q_4
\end{aligned}
\tag{13}
$$

- unpotential generalized forces:

$$
\begin{aligned}
Q_1^{nepot} &\equiv Q_{\theta_1} = 0; \\
Q_2^{nepot} &\equiv Q_{\theta_2} = 0; \\
Q_3^{nepot} &\equiv Q_{\theta_3} = 0; \\
Q_4^{nepot} &\equiv Q_{\theta_4} = 0 \\
Q_5^{nepot} &\equiv Q_{d_1} = -C_1 \dot{d}_1 - C_2 \dot{d}_2 \cos(\theta_2 - \theta_1) = -C_1 \dot{q}_5 - C_2 \dot{q}_6 \cos(q_2 - q_1) \\
Q_6^{nepot} &\equiv Q_{d_2} = -C_2 \dot{d}_2 = -C_2 \dot{q}_6
\end{aligned}
\tag{14}
$$

The moving equations write now in matrix form below:

$$\boxed{[A]\{\ddot{q}\} + \{\dot{q}\}^T [B] \{\dot{q}\} + [C] = 0} \tag{15}$$

where matrices and final notations are:

$$\{q\} = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ d_1 \ d_2]^T, \quad \{\dot{q}\} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3 \ \dot{\theta}_4 \ \dot{d}_1 \ \dot{d}_2]^T, \quad \{\ddot{q}\} = [\ddot{\theta}_1 \ \ddot{\theta}_2 \ \ddot{\theta}_3 \ \ddot{\theta}_4 \ \ddot{d}_1 \ \ddot{d}_2]^T \tag{16}$$

$$[A] = 2[a], \quad A_{ki} = 2a_{ki}, \quad i, j = \overline{1,6} \tag{17}$$

$$B_{kij} = 2b_{kij} - b_{ijk}, \quad i, j, k = \overline{1,6}, \; b_{kij} = \frac{\partial a_{ki}}{\partial q_j}$$

$$
\begin{aligned}
b_{115} &= \frac{1}{2} m_{11}(l_1 + q_5) \\
b_{226} &= \frac{1}{2} m_{22}(l_2 + q_6) \\
b_{121} &= \frac{1}{2} m_{12}(l_1 + q_5)(l_2 + q_6)\sin(q_2 - q_1) \\
b_{122} &= -b_{121} \\
b_{125} &= \frac{1}{2} m_{12}(l_2 + q_6)\cos(q_2 - q_1) \\
b_{126} &= \frac{1}{2} m_{12}(l_1 + q_5)\cos(q_2 - q_1) \\
b_{141} &= \frac{1}{2} m_{14} l_4 (l_1 + q_5)\sin(q_4 - q_1) \\
b_{144} &= -b_{141} \\
b_{145} &= \frac{1}{2} m_{14} l_4 \cos(q_4 - q_1) \\
b_{131} &= \frac{1}{2} m_{13} l_3 (l_1 + q_5)\sin(q_3 - q_1) \\
b_{133} &= -b_{131} \\
b_{135} &= \frac{1}{2} m_{13} l_3 \cos(q_3 - q_1) \\
b_{161} &= -\frac{1}{2} m_{12}(l_1 + q_5)\cos(q_2 - q_1) \\
b_{162} &= -b_{161} \\
b_{165} &= \frac{1}{2} m_{12} \sin(q_2 - q_1) \\
b_{232} &= \frac{1}{2} m_{13} l_3 (l_2 + q_6)\sin(q_3 - q_2) \\
b_{233} &= -b_{232} \\
b_{236} &= \frac{1}{2} m_{13} l_3 \cos(q_3 - q_2) \\
b_{251} &= \frac{1}{2} m_{12}(l_2 + q_6)\cos(q_2 - q_1) \\
b_{252} &= -b_{251} \\
b_{256} &= -\frac{1}{2} m_{12} \sin(q_2 - q_1) \\
b_{343} &= \frac{1}{2} m_{14} l_3 l_4 \sin(q_4 - q_3) \\
b_{344} &= -b_{343} \\
b_{351} &= \frac{1}{2} m_{13} l_3 \cos(q_3 - q_1) \\
b_{353} &= -b_{351} \\
b_{242} &= \frac{1}{2} m_{14} l_4 (l_2 + q_6)\sin(q_4 - q_2) \\
b_{243} &= -b_{242} \\
b_{246} &= \frac{1}{2} m_{14} l_4 \cos(q_4 - q_2) \\
b_{362} &= \frac{1}{2} m_{13} l_3 \cos(q_3 - q_2) \\
b_{363} &= -b_{362} \\
b_{451} &= \frac{1}{2} m_{14} l_4 \cos(q_4 - q_1) \\
b_{454} &= -b_{451} \\
b_{462} &= \frac{1}{2} m_{14} l_4 \cos(q_4 - q_2) \\
b_{464} &= -b_{462} \\
b_{561} &= -\frac{1}{2} m_{12} \sin(q_2 - q_1) \\
b_{562} &= -b_{561}
\end{aligned} \tag{18}
$$

$$C_k = u_k - Q_k^{nepot}, \quad k = \overline{1,6} \tag{19}$$

3. NUMERICAL SIMULATION

The numerical values implicated in biomechanical model validation are real. These values agree with a gymnast (O. B.) participant in the many international contests:

- sex: male;
- age: 20 age;
- typology: Caucasian;
- weight: $m = 67{,}303\ kg$
- high: $h = 1{,}720\ m$

Based on these primary numerical values, Dempster's formulas, an own calculated program and the literature it estimated anthropometrical values implicated in equations (15):

Table 1

Segment	Anthropometrical parameter				
	length, l_i (m)	weight, m_i (kg)	center of weight position, r_i (m)	coefficient $p_i = \frac{r_i}{l_i}$	principal inertial moment, J_i (kgm^2)
2 - arms	0,695	3,365	0,355	0,511	26,964
3 - torso+head	0,543	38,901	0,127	0,234	162,355
4 - legs	1,177	25,037	0,410	0,348	21,160

Table 2

Equation parameter	Value	Source
A_2	24.855 *N/m*	*)
B_2	11.447 *N/m*	*)
C_2	1.901 *Ns/m*	*)
m_1 – cables with rings weight	35,500 *kg*	direct measures and calculus
l_1 – cable with rings length	3,200 *m*	direct measures and calculus
r_1 - center of weight position of cables with rings	2,972 *m*	direct measures and calculus
p_1	0,929	direct measures and calculus
J_1 – principal inertial moment of cables with rings	319,780 $kg\ m^2$	direct measures and calculus
B_1	24.372 *N/m*	international standard, *)
C_1	2.608 *Ns/m*	international standard, *)

*) *M. R. Yeadon, M. A. Brewin, Journal of Biomechanics, 36/2003, p. 545-552*

The numerical variations of joint moments were obtained from videocomputing method (Fig. 3a). Such variations are impossible to describe analytical; so these were approximate by Heaviside function (variations on steps - Fig. 3b):

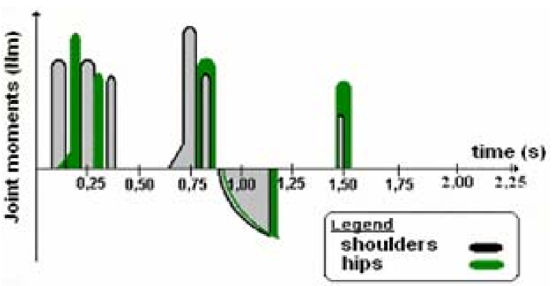

Fig. 3a

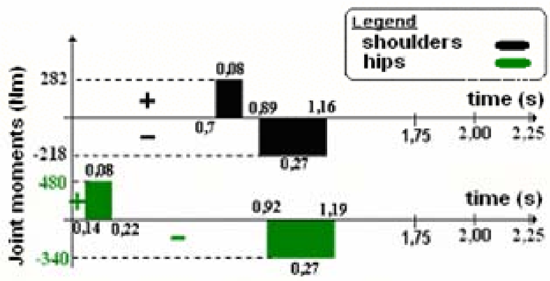

Fig. 3b

$$K_2 = \begin{cases} 0, & 0{,}00\ldots0{,}70\ s \\ 282, & 0{,}70\ldots0{,}78\ s \\ 0, & 0{,}78\ldots0{,}89\ s \\ -218, & 0{,}89\ldots1{,}16\ s \\ 0, & 1{,}16\ldots2{,}20\ s \end{cases} Nm \qquad K_3 = \begin{cases} 0, & 0{,}00\ldots0{,}14\ s \\ 480, & 0{,}14\ldots0{,}22\ s \\ 0, & 0{,}22\ldots0{,}92\ s \\ -340, & 0{,}92\ldots1{,}19\ s \\ 0, & 1{,}19\ldots2{,}20\ s \end{cases} Nm \tag{20}$$

Finally, with all these numerical values and variations, the system of equations (15) was integrated by Runge-Kutta fourth order method. The computing simulation was made by an own program. In the next figure we captured the variation of the motion parameters versus time.

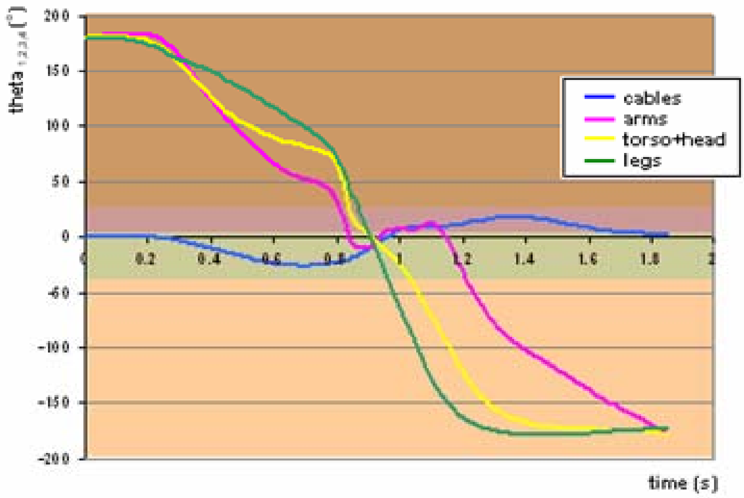

Fig. 4

4. CONCLUSIONS

Validation of the model consists in the comparison of the results of the simulation with the practical results. The research in this field has just begun in the last years and there are not many available data.

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