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KINEMATIC MODELING FOR A WALKING ROBOT'S LEG WITH PAIR OF REFERENCE SYSTEM METHOD

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Abstract: The concept "Pair of Reference System (PRS)", introduce by Kovacs Francisc Villiam, has application in the modelling of several technical phenomena based on relative motion of objects [4], [6]. The concept allows a unitary approach of modelling and development of generalized models. In this paper is formulating the kinematics equations of motion for a walking robot's leg, applying the Kovacs method, using the concept "Pair of Reference System (PRS)". In accordance with this convention is assigned the coordinate systems and develop the coordinate transformation matrixes to obtain the models degree 0 and 1 of the legs.

1. INTRODUCTION

Most creatures on earth use legs for locomotion on solid ground. Legs provide a unique tradeoff between efficient locomotion on level ground, and the ability to traverse uneven or difficult terrain. Other advantages of legged locomotion are numerous. Legged robots can travel with minor ground-robot contact as compared to wheeled or tracked vehicles, which require a continuous path of support. This is a major issue in the case of plantations, for example, where crop damage must be minimized. On the other hand, tracked vehicles can inflict serious damage to the supporting surface. On flat terrain, wheeled locomotion is faster and more efficient than legged locomotion but fails to function adequately in areas where the terrain is uneven. Legged locomotion has the advantage of reaching places that wheeled robots cannot. For legged robots to achieve practical utility, they must become faster, more robust, more efficient, more autonomous and less expensive than contemporary prototypes.

So, in the terrestriall displacement two locomotion system are used: based on rolling, with or without air cushion lifting technique, or based on walking. The rolling locomotion systems need an arranged way with strong, plane and smooth surface, which permit with great speed displacement. The walking locomotion system permit to displacement on avoidance terrain, but with a slow speed.

The mobile robots has development especially because is a great interest for applications in non - industrial environments, like explore the outer space and underwater, to manipulate nuclear substances and explosive material, respective applications in agricultural, building, medicine, education, rehabilitation for disable peoples and used like personal robots [2], [3].

2. MODELS OF GENERALIZED JOINTS (KINEMATICAL PAIRS) AND OF GENERALIZED OFFSETS

The concept "Pairs of Reference System" (PSR) was defined in [7]. It enables the unitary modeling of structure, kinematics, kinetostatics and kinetoelastostatics/dynamics of mechanical systems, inclusive mechanisms and robotic guiding device.

The elements N and N' in contact (each solidarized with a homonym reference systems) form a "generalized kinematical pair (joint)" (fig.1).

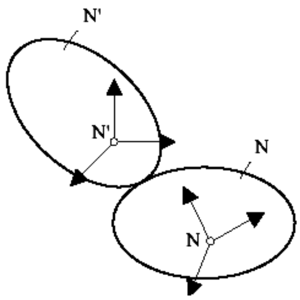

Fig.1 Generalized joint

The model degree 0 of the generalized joint is the transfer matrix from N' to N. Written in form (1):

$$
\begin{aligned}
{}^{N}\underline{T}^{V}_{N'} &= \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}^T = \\
&= \begin{bmatrix} q_{1c}+q_{1v} & q_{2c}+q_{2v} & q_{3c}+q_{3v} & q_{4c}+q_{4v} & q_{5c}+q_{5v} & q_{6c}+q_{6v} \end{bmatrix}^T,\ i=1\div 6
\end{aligned} \tag{1}
$$

where:

- $\frac{\partial q_{ci}}{\partial t} = 0$ and $q_{vi} = q_{vi}(t),\ (i = 1\div 6)$

The model degree 1 of the generalized joint is (in 1x6 matrix form) for velocities:

$$
{}^{N}\underline{\dot{T}}^{V}_{N'} = \begin{bmatrix} \dot{q}_{v1} & \dot{q}_{v2} & \dot{q}_{v3} & \dot{q}_{v4} & \dot{q}_{v5} & \dot{q}_{v6} \end{bmatrix}^T \tag{2}
$$

and for acceleration (in 1x6 matrix form):

$$
{}^{N}\underline{\ddot{T}}^{V}_{N'} = \begin{bmatrix} \ddot{q}_{v1} & \ddot{q}_{v2} & \ddot{q}_{v3} & \ddot{q}_{v4} & \ddot{q}_{v5} & \ddot{q}_{v6} \end{bmatrix}^T \tag{3}
$$

Giving particular values to the parameters: q_{ci} and q_{vi} (i = 1- 6) models of all possible joint (with different d.o.f and geometry) can be established.

For c_x, the model degree 0 of a rotational kinematical pair R_z, having N_x as axis is:

$$
({}^{N}\underline{T}^{V}_{N'})_{R_z} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \varphi_z \end{bmatrix}^T \tag{4}
$$

where:

- φ_z is the rotation angle around the N_x axis, at time t,

The model degree 1 of a spherical joint S is:

$$
({}^{N}\underline{\dot{T}}^{V}_{N'})_{S} = \begin{bmatrix} 0 & 0 & 0 & \varphi_x & \varphi_y & \varphi_z \end{bmatrix}^T \tag{5}
$$

In both cases, the origins N and N' of the two reference systems are coincident. The d.o.f. of a generalized joint can be computed using (12):

$$
L = \sum \frac{q_{v_j}}{q_{v_j}}\ ; \quad (j = 1\div 6),\ (q_{v_j} \neq q) \tag{6}
$$

The element N'M forms a "generalized offset" (Fig. 2a). A pair of reference systems N' and M is solidarized with the element.

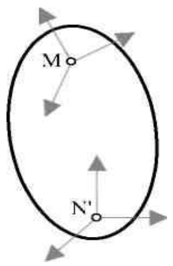

Fig.2a Generalized offset

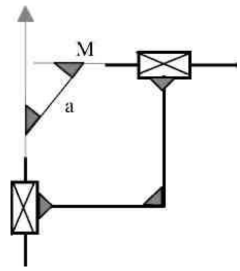

Fig.2b Component offset of a guiding device

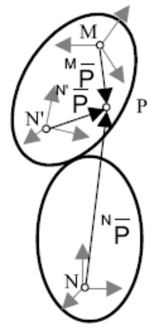

Fig.3 PRS NN' and N'M

The model degree 0 of the generalized offset is:

$$ {}^{N'}\underline{T}^{V}_{M} = \begin{bmatrix} q^{o}_{c_1} & q^{o}_{c_2} & q^{o}_{c_3} & q^{o}_{c_4} & q^{o}_{c_5} & q^{o}_{c_6} \end{bmatrix}^{T} \tag{7} $$

Accordingly to (4): $\dot{q}_{c_1} = \ddot{q}_{c_1} = 0$ and the model degree 1 of the generalized offset has no significance. The generalized offset can be particularized by giving certained values to the parameters $q^{o}_{c_i}$. For instance the degree 0 model of the offset show in fig.2b is:

$$ {}^{N'}\underline{T}^{V}_{M} = \begin{bmatrix} a & 0 & 0 & \frac{\pi}{2} & 0 & 0 \end{bmatrix}^{T} \tag{8} $$

In the case of the PRS, NN' (generalized joint) and N'M (generalized offset) (fig.3), between the position vectors of a point P relative to the three reference system exists the relation (7):

$$ \begin{aligned} {}^{N}\underline{p} &= {}^{N}\underline{T}_{M}\,{}^{M}\underline{p} = {}^{N}\underline{T}_{N'}\,{}^{N'}\underline{p} \\ {}^{N'}\overline{\underline{p}} &= {}^{N'}\underline{T}_{M}\,{}^{M}\underline{p} \\ {}^{N}\underline{p} &= {}^{N}\underline{T}_{N'}\,{}^{N'}\underline{T}_{M'}\,{}^{M}\underline{p} \end{aligned} \tag{9} $$

Using (7), the relation (9) can be deduced:

$$ {}^{N}\underline{T}_{M} = {}^{N}\underline{T}_{N'}\,{}^{N'}\underline{T}_{M} = \prod_{N}^{M} \underline{T} \tag{10} $$

An open kinematical chain is a string of generalized joints and generalized offsets linked together alternatively.

A generalized closed kinematical chain is a "parallel topology" structure: at least two elements are linked together by means of least 2 open kinematical chains.

In fact, legged mobile robots (those which are stepping, crawling and climbing) have their mechanical systems based on the mechanical system of parallel topology robots, the soil being the fixed platform.

3. MODELS OF MECHANICAL SYSTEM FOR A WALKING ROBOT'S LEG

The kinematical chain of anthropomorphic leg for a walking robot is show in fig.4 [9].

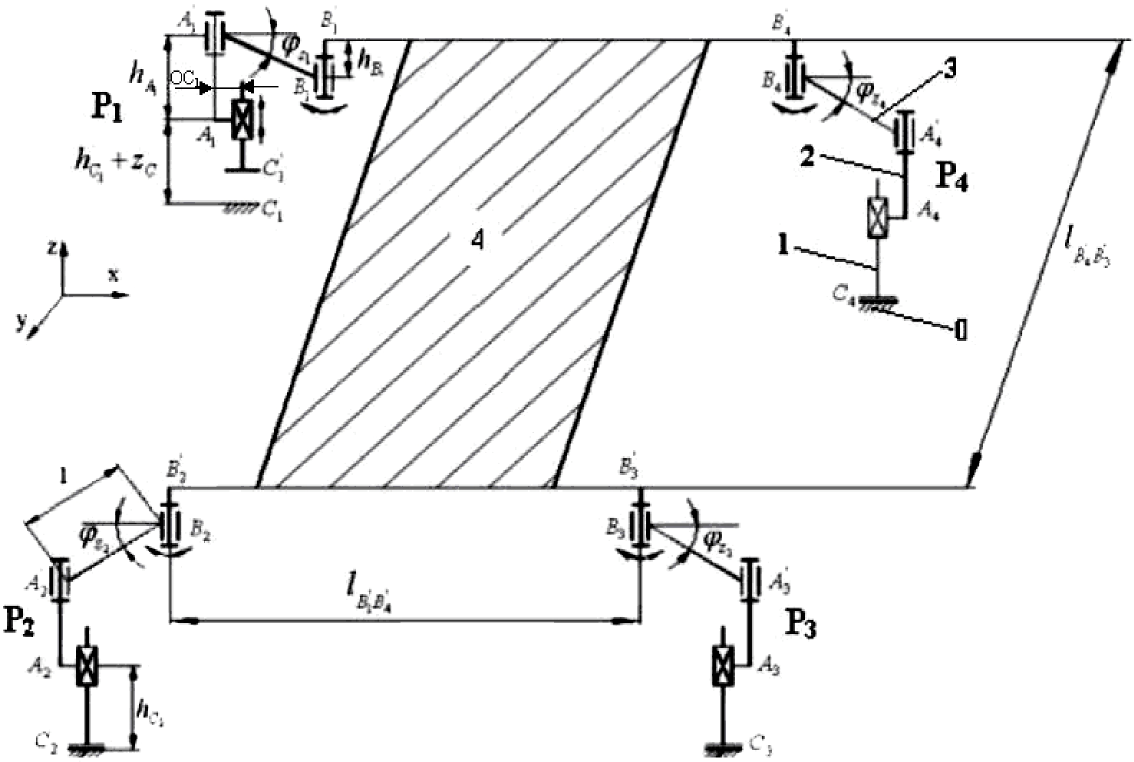

Fig.4 Kinematic diagram for a walking robot
A, A', B, B', C, C' are origins of reference systems, whose axis have not been drawn

The "leg" P_1 is performing a step, having a contact with soil 0. It can be considered as a mechanism, based on an open kinematical chain, having the chassis 4 of the stepping robot as "fixed" element and the rotational joint A_{P1} as actuated one.

The mechanism has mobility:

$$M = -\frac{q_{V_A}}{q_{V_A}} = 2 - \frac{\{q_{V_A}\}}{\{q_{V_A}\}} \qquad (11)$$

where:

- $\{q_{V_A}\}$ is a variable parameter imposed by an external source of motion

Consequently, it has desmodromy.

The "legs" P_2, P_3, P_4 performing support sequence are identical open kinematical chains (connexions) linking the chassis 4 to the soil. The joints of contact with soil are to classes 3, but they will be considered to classes 5 (it will be considered that mass and frictions not permit their motion relative to soil).

The axis of all robotically joints being parallel, the mechanism is a planar one (L_P=3) with the mobility:

$$M = 6 + \sum L_{OK_i} - L_{pc} = 6 - 6 = 0 \qquad (12)$$

where:

- $\sum L_{OK_i}$ is the sum of the DOF of the k_c connexions
- L_{pc} is the number of passive DOF introduced by the closing of loops of elements

So, in stepping phases, three kinematical joints of all kinematical joints of legs who are in support phases, have to be in rotational motion, id est they get an external source of motion. This motion can be with $\omega = 0$ (the legs are in support phases) or $\omega \neq 0$ (the chassis will be in a rotational motion when a leg is high up).

The structural model of the legs and those of the walking robot mechanical system is described by (11) and (12).
The parameters of the model degree 0 of the stepping leg are show in table1.

Table 1. The PRS parameters for the P_1

PSR \ Parameters	q_1	q_2	q_3	q_4	q_5	q_6
$C_1 - C'_1$	0	0	$h_{C_1} + z_c$	0	0	0
$C'_1 - A_1$	O_{C_1}	0	0	0	0	0
$A_1 - A'_1$	0	0	h_{A_1}	0	0	φ_{Z_1}
$A'_1 - B_1$	$l\cos\varphi_{Z_1}$	$l\sin\varphi_{Z_1}$	0	0	0	0
$B_1 - B'_1$	0	0	h_{B_1}	0	0	φ_{Z_1}

The model degree 0 of the stepping leg P_1 can be written (taking in account the notations from fig. 4 and the parameters from table 1) also expressed by:

$$\left({}^{4}\underset{-}{S}_{1}\right)_1 = \left({}^{B_1}\underset{-}{T}_{B'_1}\right)_1 \left({}^{A'_1}\underset{-}{T}_{B_1}\right)_1 \left({}^{A_1}\underset{-}{T}_{A'_1}\right)_1 \left({}^{C'_1}\underset{-}{T}_{A_1}\right)_1 \left({}^{C_1}\underset{-}{T}_{C'_1}\right)_1 = \prod_{B'_1}^{C_1} \underset{-}{T} \qquad (13)$$

where:

$$^{C_1}\underset{-}{T}_{C'_1} = \begin{bmatrix} 0 & 0 & h_{C_1} + z_C & 0 & 0 & 0 \end{bmatrix}$$

$$^{C'_1}\underset{-}{T}_{A_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$^{A_1}\underset{-}{T}_{A'_1} = \begin{bmatrix} OC_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$^{A'_1}\underset{-}{T}_{B_1} = \begin{bmatrix} l\cos\varphi_{Z_1} & l\sin\varphi_{Z_1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$^{B_1}\underset{-}{T}_{B'_1} = \begin{bmatrix} 0 & 0 & h_{B_1} & 0 & 0 & \varphi_{Z_1} \end{bmatrix}$$

The models degrees 1 and 2 of the stepping leg P_1 can be expressed by:

$$\left({}^{4}\underset{-}{\dot{S}}_{1}\right)_1 = \frac{\partial}{\partial t} \prod_{B'_1}^{C_1} \underset{-}{T} \qquad (14)$$

$$\left({}^{4}\underset{-}{\ddot{S}}_{1}\right)_1 = \frac{\partial^2}{\partial t^2} \prod_{B'_1}^{C_1} \underset{-}{T} \qquad (15)$$

The parameters of the model degree 0 for the three support legs P_2, P_3, P_4 is show in table 2.

Table 2. The PRS parameters for the support legs

PSR \ Parameters	q_1	q_2	q_3	q_4	q_5	q_6	Obs.
$C_2 - C'_2$	0	0	h_{C_2}	0	0	0	Leg P_2
$C'_2 - A_2$	0	0	0	0	0	0	
$A_2 - A'_2$	0	0	h_{A_2}	0	0	φ_{Z_2}	
$A'_2 - B_2$	$l\cos\varphi_{Z_2}$	$l\sin\varphi_{Z_2}$	0	0	0	0	
$B_2 - B'_2$	0	0	h_{B_2}	0	0	φ_{Z_2}	
$C_3 - C'_3$	0	0	h_{C_3}	0	0	0	Leg P_3
$C'_3 - A_3$	0	0	0	0	0	0	
$A_3 - A'_3$	0	0	h_{A_3}	0	0	φ_{Z_3}	
$A'_3 - B_3$	$l\cos\varphi_{Z_3}$	$l\sin\varphi_{Z_3}$	0	0	0	0	
$B_3 - B'_3$	0	0	h_{B_3}	0	0	φ_{Z_3}	
$C_4 - C'_4$	0	0	h_{C_4}	0	0	0	Leg P_4
$C'_4 - A_4$	0	0	0	0	0	0	
$A_4 - A'_4$	0	0	h_{A_4}	0	0	φ_{Z_4}	
$A'_4 - B_4$	$l\cos\varphi_{Z_4}$	$l\sin\varphi_{Z_4}$	0	0	0	0	
$B_4 - B'_4$	0	0	h_{B_4}	0	0	φ_{Z_4}	

4. CONCLUSION

The concept PRS allows the elaboration of a unitary solution for structural, degree 0 and 1 modelling of the mechanical system of all kind of mobile robots.
The above-described models allow a unitary programming of mobile robot's motion.

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