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# THE ABHORRENCE AT RISK FOR INSURANCES

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**Abstract:** The abhorrence's effects at risk are determined by two types of expenditures as part as the competitions to obtain a rent:

a) expenditures made to obtain a rent which increase the probability for a rent to be obtained;

b) expenditures for rent's increase which have the role to grow the rent's volume that will be cashed. We will demonstrate that the abhorrence at risk will lead at the expenditure's decrease of type b) and will have an unknown effect over the expenditures of type a).

In a co0mpetitive environment the agents that adopt a neutral position given the risk will spend a sum of money that will be equal with the rent's size for which they compete. This situation doesn't have place if the some agents are looking to obtain the rent through a strategical game where the relative expenditures determine for each agent the probability to win a fixed rent.

An agent's position given the risk is suggested in reality by a certain state of economy but this is rather a rule than an exception. Even the agents

are public companies, these can tackle a certain abhorrence at risk. Moreover, different social-economic factors as individual fortune or age are offen considered as being a factor that affects the grade of abhorrence at risk. So, it can be asked if the two lots of players will behave differently in sistematic way as part as the same competition to obtain the rent.

Besides the types of expenditures to obtain the rent which we spoke about, where the players pay a certain sum of money for their wish to get a fixed rent, we'll also consider the expenditures of resources for increase the rent's size. For example, besides the lobbies for a certain government contract, the players also can compete to increase the contract's size, wishing to obtain a bigger rent than the one that is usually obtained in this type of contract. We'll name this type of activity as the demeanour of rent's increase.

If the rent will be divided between many players, each of them fights to maximize his part. If this maximilization is determinative, we'll have demeanour of rent's maximilization; if this is made in a way that suppose a probability we'll have a demeanour of obtaining a rent.

We must observe that in the case of individual vested interests, not all the investors with abhorrence at risk will agree a risk increase but we don't know for sure that these will decrease their vested interests in the risky assets. So, we do not be surprised that a risk increase in a competition will have as effect that cannot be determined a priori. It's also true the vice versa: an investor who love the risk will always invest less in risky assets.

## 1. The demeanour of searching the rent:

We'll consider the next example: a company with abhorrence at risk who has the initial fortune of 20 u.m. and which preferences are described by the function of utility of the final fortune $u(y) = \ln y$. The company has the possibility to obtain the premium (the rent) of 50 u.m. with the probability $p$ = 0,5. For an expenditure of 9 u.m. the company can also increase the probability of obtaining a premium at $p$ = 0,7. It can be observed that a player who search a premium and is neutral given the risk, will want to spend 9 u.m., while the expected fortune will increase from 45 u.m. at 46 u.m.. The following calculus show that the player with abhorrence at risk previous described will not invest 9 u.m. to obtain the rent because this will decrease the utility from 3,622 at 3,597. So, the player with abhorrence at risk will invest less to obtain the rent than the player which is neutral given the risk.

To verify if the contrary hypothesis have place, we'll consider the initial fortune as the previous but the first one will be decrease at 5 u.m. with the probability of $p$ = 0,5. We'll consider the potential investment of 1,01 u.m. for the probability increase to obtain the rent at $p$ = 0,7. The neutral player who search a rent will not invest 1,01 u.m. because this investment will produce a decrease of expected fortune from 22,50 u.m. at 22,49 u.m. Anyway, the investor from 3,1073 u.m. at 3,1075 u.m. after the investment to obtain the rent.

Let's consider a fixed number of $n$ competitors (seekers of rent), each of them endowed with an initial fortune of $w > 0$, that compete for a given rent $b > 0$. Having $X \equiv (x_1, ..., x_n)$ the vector of expenditures made to obtain the rent for $n$ competitors. The probabilities to win the premium are showed by the vector $(p_1,..., p_n) \in S^{n-1}$, where $S^{n-1}$ represents the simplex an $\mathbf{R}^n$. The competition is characterized through a succesful function after the competition noted with $P : [0, w]^n \to S^{n-1}$, so $(x_1,...,x_n) \to (p_1,..., p_2)$. The element of $i$ degree of this function, $P(X) = [p_1(X), p_2(X),...p_n(X)]$, is the function of probability of the $I$ player. We suppose that $p_i$ has positive values when represents the effort of player and negative values when represents the competitors' effect, where $p_i^{'} = \frac{\partial^2 p_i}{\partial x_i} > 0$ and $\frac{\partial p_i}{\partial x_j} < 0$, for any $i \neq j$. Also, $p_i^{''} = \frac{\partial^2 p_i}{\partial x_i^2}$ and that $\frac{\partial^2 p_i(x, x,..., x)}{\partial x_i \partial x_j} \le 0$ for $i \neq j$. These conditions are valid succes functions more general of the type $p_i(X) = \frac{f(x_i)}{\sum f(x_i)}$ with $f' > 0$, $f'' < 0$ and $x_i = x_j$, $\forall\ i,j$.

Each player choose the level of monetary effort which maximizes his utility:

$$E_u = [y_i(X)] \equiv p_i(X)u(w - x_i + b) + [1 - p_i(X)]u(w - x_i) \tag{1}$$

The aleatory variable $y_i(X)$ represents the final profit of player which depends of the expenditures vector for obtaining the rent $X$.

A Nash equilibrium in pure strategy as part as the games for obtaining the rent is a vector of efforts of searching a rent $X^* \equiv \left(x_1^*, x_2^*,..., x_n^*\right)$ so, because the vector of other players efforts, $X_{-i}^* \equiv \left(x_1^*,..., x_{i-1}^*, x_{i+1}^*,..., x_n^*\right)$, $x_i^*$ maximizes the expected utility given by the relation (1), for any $i$ = 1,...,$n$. We suppose that all the players are likewise and that $E_u(y_i)$ has one maximum point $\forall\ i$, $\forall\ X_{-i}$. It's obtained an equilibrium of type Nash in pure strategies that satisfies the relation:

$$\left.\frac{\partial E_u}{\partial x_i}\right|_{x'} = p_i^{'}[u_1 - u_2] - E_{u'} \ge 0 \tag{2}$$

The equality has place when $x_i^{'} < w$ and $u_1 \equiv u(w - x_i + b)$, $u_2 \equiv u(w - x_i)$ and $E_{u'} \equiv p_i u_1^{'} + (1 - p)u_2^{'}$.

The condition of two degree given by the relation (2) defines the function of reaction of $i$ player, where the marginal utility of benefits equalizes the marginal utility of expenditures made to obtain the rent in optimum point. In the case where preferences are neutrally at risk, the ecuation (2) is reduced at an equality between the marginal benefits and marginal costs:

$$\left.\frac{\partial E_{y_i}}{\partial x_i}\right|_{x'} = p_i' b - 1 \qquad (3)$$

From the relation (2) and from the condition that $p_i'' < 0$ we obtain the following result:

**Proposition 1**: For each two functions of utility *u* and *v* has place the inequality $x_u^* > x_v^*$ only if $\frac{(u_1 - u_2)}{v_1 - v_2} > \frac{E_{u'}}{E_{v'}}$ where *u, u'* and *v'* are estimated in point $y(x_v^*)$. We abandoned the utilization of index i because we are interested in the symmetrical inequality $x_i^* = x_j^* \quad \forall i, j$. This proposition compares the expenditures to obtain a rent at equilibrium as part as two games to obtain symmetrical and different rent: one in which all the individuals have the same function of utility *u* and other in which all have the function of utility *v*. The feature of utility function of having a single maximum point on the study interval involves the fact that the relation (2) determines an unique value *x** in each game.

The condition $\frac{\partial^2 p_i}{\partial x_i \partial x_j} \le 0$ shows that the agents *i* and *j* expenditures are strategically replaced. Results that the marginal efficiency of one u.m. to obtain the rent by the player *i* is more little when *j* spends more. This condition leads us at: $\frac{\partial\left[\frac{\partial E_u(y_i)}{\partial x_i}\right]}{\partial x_j} \le 0$, in a symmetrical equilibrium of Nash type, we have *x** = $x_i$ = $x_j$ . Then,

$$\frac{\partial\left[\frac{\partial E_u(y_i)}{\partial x_i}\right]}{\partial x^*} = \frac{\partial^2 E_u(y_i)}{\partial x_i^2} + (n-1)\frac{\partial^2 E_u(y_i)}{\partial x_i \partial x_j} \qquad (4)$$

where the first part from left is negative because the hypothesis that says the conditions of two degree rest valid for $x_i$. In proposition 1, the second inequality involves the fact that $\frac{\partial E_u(y_i)}{\partial x_i} > 0$ in point $x_v^*$.

To understand the way how the individual behaveour affects the equilibrium, we take $x_j = x_v^*, \quad \forall j \ne i$ and we'll have in consideration the incitation of the individual *i*. We suppose that a change in the function of utility from *v* at *u* will lead to an increase of $x_i$. Because we have a symmetrical equilibrium exists a certain incitation of increase the expenditures in order to obtain the rent for all the players. If $\frac{\partial E_u(y_i)}{\partial x_i} > 0$ for player *i* when $x_j = x_i^*, \quad \forall j \ne i$ , the condition (4) guarantees that the inequality will keep itself when all other $x_j$ to obtain the rent. So, that the new level of equilibrium expenditures $x_u^*$ will be bigger than the old equilibrium level $x_v^*$. It can be observed that the restriction $x_i$ and $x_j$ to be strategically replaced is more powerful than the necessary one for obtaining the equality between the individual incitation to increase $x_i$ and much bigger value of equilibrium *x** as part as symmetrical game. Indeed, we need only the relation (4) to obtain this result.

For the case of comparison, between the preferences that are neutral given the risk, we will obtain:

**Corollary 1**: Having *v(y<sub></sub>i)* = $y_i$. Then $\begin{cases} x_u^* > x_v^* \\ x_u^* = x_v^* \\ x_u^* < x_v^* \end{cases}$ only if $\begin{cases} E_u' < \frac{u_1 - u_2}{b} \\ E_u' = \frac{u_1 - u_2}{b} \\ E_u' > \frac{u_1 - u_2}{b} \end{cases}$ where *u* and *u* are estimated at any $y\left(x_v^*\right)$.

The expression $\frac{u_1 - u_2}{b}$ represents the average progress of utility at one dollar from rent and is the slope line AB from the next figure. $E_u'$ represents the cost of utility for a marginal investment of one u.m. to obtain the rent and is a well-balanced mean of $u_1'$ and $u_2'$. With $\frac{u_1 - u_2}{b}$ fixed, any function of utility with abhorrence at risk will satisfy the condition $u_1' < \frac{u_1 - u_2}{b} < u_2'$, where *u* and *u'* are calculated in the point $x_v^*$ (the neutrality given the risk).

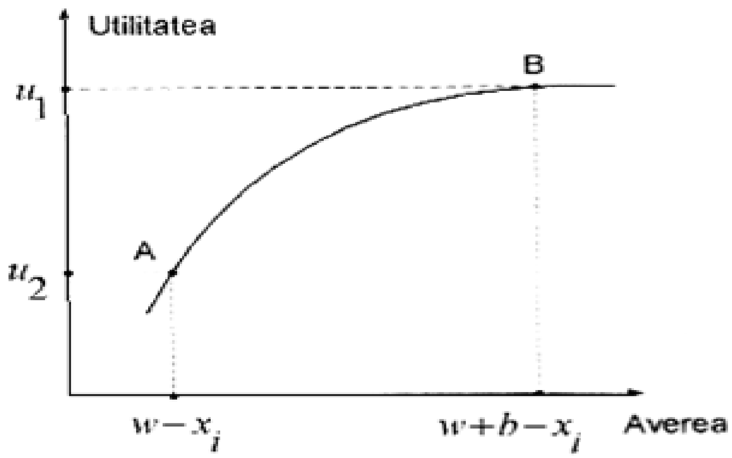

***Fig. 1: The marginal utility of costs and benefits for obtaining the rent***

Because $\frac{u_1 - u_2}{b}$ can be close by $u_1'$ or $u_2'$, $x_u^*$ can be more little or bigger than $x_v^*$. Because this reason it's possible the rent be more dissipated in the competition with players that have abhorrence at risk than the one where the neutral players given the risk.

For a general case, where *u* has a bigger abhorrence given the risk than *v*, it's possible that more rents to be spread in competitions with many players, because the second inequality from proposition 1 is valid even in this case.

To isolate the risk's effects we'll consider the case where the marginal investment to search the rent is neutral from the actuarial point of view. Supposing that the person *i* wants a little growth of expenditures to obtain the rent from $x_i$ to $x_i'$ and has opinions like those described by Nash concerning the contributions of the other agents $X_{-i}$. This thing will lead to a growth of $p_i$ at $\hat{p}_i$ level. If $\Delta x_i \equiv x_i' - x_i$ and $\Delta p_i \equiv \hat{p}_i - p_i$, the actuarial neutrality involves the $\Delta x_i = (\Delta p_i)b$. We make this supposition here only in an illustrative goal. Of course, the changes from the monetary value of expenditures for obtaining the rent will affect the player's behaviour. If we want to isolate the risk's effects we suppose that the expected value of profit is not affected.

The passing from the level $x_i$ of expenditures at level $x_i'$, as is described, is represented graphically in figure 2.a). It can be observed that a growth of *x* makes the support line of the fortune's distribution to move at left, while through the competition is given a big probability to the best profit and keep the expected fortune $E_y$.

The arrows from the figure 2 show the direction of the probability's change. The points circles thickened represent the fortune's levels which have positive probabilities after has place the change in rhombus points represent the levels of all unoccupied probabilities. In the section b) of figure 2 is moved all the probability from the value $w - x_i + b$ at $w - x_i' + b$ and a part from the probability $w - x_i$ at $w - x_i' + b$ because the average value $E_y$, of the fortune must stay unchanged. This represents a contraction for keeping the mean of fortune's repartition and a risk's decrease. In section c) of figure 2 we move the probability remained from $w - x_i$ and we'll move it at $w - x_i'$ and $w - x_i' - b$. This is a spreading of the probabilities so the fortune's mean stay unchanged and also a risk's increase:

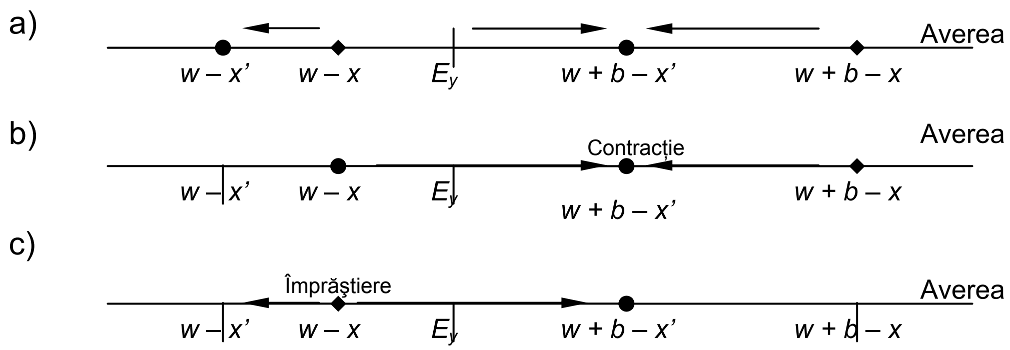


***Fig. 2: The risk effects' decompose in a mean's contraction and spreading***

Because the total change of the probability includes a contraction and also a spreading of fortune's distribution, in general, the risk remains unchanged. So, some players with abhorrence at risk will obtain a bigger utility if they invest $x_i$ in searching the rent when others will prefer to invest $x_i'$. It doesn't exist o correlation between the level of abhorrence at risk and the level of expenditures to obtain the rent.

The previous analysis involves the fact that the incitation of a person – of investing more or less – in function of his abhorrence changing given the risk cannot be a apriori predicted. If the comparative effect of change given the risk cannot be predicted for individual behaviour, not even when we have restrictions over the preferences such as the constant abhorrence given the risk, then it'll not be surprising if the effect of such changes over the equilibrium's level of expenditures as part as the games of obtaining the rent is also hard to predict.

In case which the persons expenditures to obtain the rent are not strategically replaced we can have the certitude that the equilibrium level of expenditures x* in the symmetrical game will be modified in the same direction as the individual incitation for changing the $x_i'$ when $X_{-i}^*$ is fixed, increasing the incertitude.

In case which the expenditures to search the rent are strategically replaced a comparison between the Nash equilibrium in pure strategies in a symmetrical game with neutral players at risk and with players with abhorrence given the risk is possible, if the number of players is big enough. If we fix *v(y) = y* as being the utility of neutral player given the risk and *u(y)* to be only concave, then, because $p^* \to \frac{1}{n}$ at equilibrium, we have $p^* \to 0$ when $n \to \infty$. So, at big values of *n*, we have $E_u'$ arbitrary close by $u_2'$. Thus, $E_u' > \frac{u_1 - u_2}{b}$ and $x_u^* < x_v^*$ from colloraly 1.

## 2. The growth behaviour of rent

We will consider that $p_i$ is exogen, but the individuals can increase their part of rent if this is received. For example, we'll consider one model with collective rents in which certain number of groups (coalitions between players) has a fight of the type "*the winner takes all for his group*". In this case the rent is shared between all the members of the group. Having $\beta_i(x_1,...,x_n)$ the part of rent corresponding to the player *i*, if his group wins the game to obtain the rent, where $x_i$ represents the expenditures of player *i* for the rent's growth. Is created the hypothesis that the function $\beta_i$ of rent's division is differentially is increasing and concave in direction of $x_i$, with $\beta_i' \equiv \frac{\partial \beta_i}{\partial x_i} > 0$ and $\beta_i'' \equiv \frac{\partial^2 \beta_i}{\partial x_i^2} < 0$. Moreover, we consider that $\frac{\partial \beta_i}{\partial x_i} < 0 \quad \forall\, i \neq j$ and $\beta_i$ is differentially everywhere.

In the previous scenario, we suppose that $(\beta_1,...,\beta_n)$ is part of unitary simplex on $\mathbf{R}^n$ that $\beta_i = 0$ when $x_i = 0$. Another scenario supposes that the rent's size can be growth, so $\beta_i \geq 1$ represents a scalar factor. Keeping the hypothesis where we have the same players in a symmetrical and pure strategy. The Nash equilibrium, the equilibrium levels of growth expenditures of the rent satisfies:

$$x_i' = \arg\max\{p \cdot u[w - x_i + \beta_i(X) \cdot b] + (1-p) \cdot u(w - x_i)\} \tag{5}$$

the solution of equality $X_{-i} = X_{-i}^* \equiv \left(x_1^*,...,x_{i-1}^*,x_{i+1}^*,...,x_n^*\right)$ with $p = p_i = \frac{1}{n}$.

The condition of first degree for each $i$ in the relation (5) is:

$$\left.\frac{\partial E_u}{\partial x_i}\right|_{x^*} = p \cdot u_1' \cdot \beta_i' \cdot b - E_{u'} \tag{6}$$

where $u_1' \equiv u'[w - x_i + \beta_i(X) \cdot b]$ and $E_{u'} \equiv p \cdot u' + (1-p) \cdot u' \cdot (w - x_i)$. For the neutral player given the risk and who wishes to increase his rent, we'll obtain the next condition of first degree:

$$\left.\frac{\partial y_i}{\partial x_i}\right|_{x^*} = \beta_i' \cdot p \cdot b - 1 = 0 \tag{7}$$

Unlike the model to obtain the rent, the conditions of second degree for (6) and (7) are satisfied because $E_u(y_i)$ is concave in $x_i$. Supposing that $\beta_i' \cdot p \cdot b > 1$ for $x_i = 0$ (so $x_i = 0$ in the case of neutrality given the risk), results from the standard fixed points certain arguments which uses the fact that $\beta_i \leq 1$ is everywhere and $\beta_i$ is concave in $x_i$ and $\beta_i \cdot (x_i, X_{-i})pb = x_i$ so the relation (7) is maintained. Therefore the neutral player given the risk who search to increase his rent invests less than the expected value of total rent, *pb*.

Also, we note that $x_i^* < pb \quad \forall X_{-i}$. So, the space of the efficient strategies is included in the cartesian product with size *n*: $[0, pb] \times [0, pb] \times ... \times [0, pb]$. Because this is a compact set and $E_u(y_i)$ is concave in $x_i$, we can say that exists a Nash equilibrium in pure strategy.

Unlike the activities to obtain the rent, the activities to rise the rent lead to rent's increase for obtaining the final fortune. This fact is easy to observ in growth rent expenditure's case. This investment leads at final support spreading of fortune decreasing the value $w - x_i$ and increasing the value $w - x_i + \beta(x) \cdot b$. Therefore such investment is a spreading of average value of fortune's repartition that represents a high level of risk.

As results, a player with a high level of abhorrence given the risk will be tempt to invest less in the rent's increase. Of course, the marginal expenditure for rent's increase will not keep untouched the average value of fortune. From the relation (6) results that $\beta_i' \cdot p \cdot b > 1$ at expenditure of equilibrium $x_u^*$ for a player with abhorrence given the risk, for example, the marginal expenditure with rent's increase leads at expected fortune growth.

The final effect at high level of abhorrence given the risk over the growth expenditures of rent is similary to the final effect in models of individual insurance. If is considered that $x_i$ and $x_j$ are strategically replaced, thus

$$\frac{\partial^2 E_u(y_i)}{\partial x_i \partial x_j} \leq 0 \quad \forall\, i \neq j \tag{8}$$

Results from the relation (8) and from the condition of second degree for $x_i$ that:

$$\frac{\partial}{\partial x^*}\left[\frac{\partial E_u(y_i)}{\partial x_i}\right]\Bigg|_{x^*} = \frac{\partial^2 E_u(y_i)}{\partial x_i^2} + (1-n)\frac{\partial^2 E_u(y_i)}{\partial x_i \partial x_j} < 0 \tag{9}$$

Thus, a change of the utility from *u* at *v* which makes the value of $x_i$ to growth, supposing that $X_{-i}^*$ stays fixed will also make that the equilibrium value $x^*$ to growth. The individual behaviour as an answer at changing inside the utility's level is the some from qualitative point of view like a modification at equilibrium expenditures level. So, in strategical replacement's case wi'll obtain the next result:

**Proposition 2**: For any two functions of utility *u* and *v*, $x_u^* > x_v^*$ only if $\frac{u_1'}{v_1'} > \frac{E_{u'}}{E_{v'}}$ where *u'* and *v'* are estimated at the value $y(x_v^*)$.

As example at proposition 2, we can observe that for *u* concave and *v(y)* = *y*, the second inequality from the preposition 2 is inverted. Thus, $x_v^* > x_u^*$; the equilibrium expenditures for rent's growth in a game with neutral players given the risk are bigger than those as part as a play with participants with abhorrence at risk. This fact can be expressed more general like this:

**Corrolary 2**: We'll consider two functions of utility *u* and *v* thus *u* is with abhorrence at risk bigger than *v*. Then, $x_v^* > x_u^*$.

Also proposition 2 and the corrolary are valid any time when $x_i$ and $x_j$ are strategically replaced. If $x_i$ and $x_j$ are strategically complementary, then the inequality (9) can't be quaranteed. Even if the individuals who have a high level of abhorrence at risk will personally invest less for rent's growth the equilibrium expenditure doesn't need an expenditure less than *x**in the condition of a high level of abhorrence at risk. For example, we suppose we have only the condition that $\frac{\partial^2 \beta_i}{\partial x_i \partial x_j} \leq 0$, is not sufficient to generate a strategical replacements. As result, a game of rent's growth with players that have a high level of abhorrence given the risk can lead to obtain a high level of equilibrium expenditures with more little abhorrence given the risk.

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