ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

USING TRIANGULAR FUZZY NUMBERS FOR MEASURING QUALITY OF SERVICE FROM THE CLIENT' S PERSPECTIVE IN THE HOTEL INDUSTRY

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hotel, service quality, aggregation, triangular fuzzy number

The answers to the questionnaires applied in the tourist industry contain a certain level of un-determination. In the paper we are trying to overcome this impediment by using the "triangular fuzzy number". We transform into triangular fuzzy numbers the tourists' answers, we operate with them, we obtain as results triangular fuzzy numbers also, and when needed – we defuzzify the result. The results obtained can make the managers aware of the position of the unit they manage and can signal to them the activities that they should improve in order to gain a better position in the clients' opinion.

INTRODUCTION

We are processing the data obtained as a result of the questionnaire implementation to the tourists accommodated at three of the hotels in the Neptun resort. We are using a linguistic scale with five stages for each of the 14 important criteria taken into consideration. For the answers obtained contain a certain level of un-determination, the configuration with the help of fuzzy numbers is proved suitable. We are using the "triangular fuzzy numbers" (as the simplest fuzzy numbers) to be able to operate with the tourists' answers. In order to make a synthesis of the results for each criterion and each hotel, we aggregate the fuzzy numbers using the arithmetic average operator in its fuzzified form. We obtain a synthesis indicator for each hotel aggregating the results already obtained for the criteria considered separately. This aggregation is weighted by the tourists' answers to another question of the questionnaire regarding the importance of the criteria considered, independently of the unit where they are accommodated now. The result obtained is a triangular fuzzy number. We obtain a hierarchy of the hotels observed, defuzzifying the results and comparing the real numbers found.

In the second section there are presented in a simple form the notions necessary for the paper regarding the triangular fuzzy numbers, as well as some recent works where the fuzzy mathematics has been used to configure in the filed of service quality. The third section contains the work hypotheses and the primary processing of the results obtained. The fourth section is the main section of the paper. Here are obtained the results regarding the quality of services provided in these three hotels, either in the form of triangular fuzzy numbers or in the form of classical numbers, using simple aggregation operators. These allow us to order the hotels, both for each criterion separately, as well as synthetically, taking into account the totality of criteria considered.

TRIANGULAR FUZZY NUMBERS

Fuzzy numbers allow us to model the linguistic expressions appeared in different scientific areas mainly because of dependence on human judgement, finite resolution of measuring instruments or finite representation of numbers in computers.

The service quality is a composite of various criteria, among them many are difficult to measure exactly such that the linguistic terms like *poor, fair, good, very good* or *somewhat important, important, very important, extremely important* are used. Under these

circumstances, fuzzy set theory (fuzzy numbers, particularly) was already proved to be an adequate frame to overcome some linguistic problems ;[see J. M. Benitez, J. C. Martin, C. Roman, 2007; S.-H. Tsaur, T.-Y. Chang, C.-H. Yen, 2002 and C. H. Yeh, Y.-L. Kuo, 2003].

Let X be a nonempty set. A fuzzy set A in X, is a set $A=\{(x,\mu_A(x)): x\in X\}$ in X is a set, where $\mu_A : X \to [0,1]$ is called the membership function of A and $\mu_A(x)$ indicates the value of truth of the statement that the element x belongs to set A [see L. Zadeh, 1965].

Triangular fuzzy numbers are particular fuzzy sets in **R** [see D. Dubois, H. Prade, 1978] completely characterized by three real numbers t_1,t_2,t_3 such that $t_1<t_2<t_3$ with the membership function

$$\mu_T(x)=\begin{cases}\dfrac{x-t_1}{t_2-t_1}, & if \quad t_1\le x\le t_2\\ \dfrac{x-t_3}{t_2-t_3}, & if \quad t_2\le x\le t_3\\ 0, & otherwise.\end{cases}$$

We denote by $T=(t_1,t_2,t_3)$ a triangular fuzzy number as above.

The addition of two triangular fuzzy numbers $T=(t_1,t_2,t_3)$ and $T'=(t_1',t_2',t_3')$ is defined b

$$T\oplus T'=(t_1+t'_1,t_2+t'_2,t_3+t'_3)$$

and the scalar multiplication of T by real number $\lambda>0$ is defined by

$$\lambda\cdot T=(\lambda t_1,\lambda t_2,\lambda t_3).$$

A well-known aggregation operator (in fact the fuzzified form of the weighted arithmetic mean) of the triangular fuzzy numbers $T_1,...,T_n$ is defined by

$$MAP^f_\omega(T_1,...,T_n)=\omega_1\cdot T_1\oplus...\oplus\omega_n\cdot T_n, \qquad (1)$$

where $\omega_1,...,\omega_n\in[0,1]$ with $\sum_{i=1}^{n}\omega_i=1$, the addition and scalar multiplication defined above.

Often it is need to convert a triangular fuzzy number into a crisp real number, that is to locate the best non-fuzzy value to the initial fuzzy value. There exist several available methods, in this paper we use the expected value [D. Dubois, H. Prade, 1987; S. Heilpern, 1992]. If $T=(t_1,t_2,t_3)$ then its expected value is introduced by

$$EV(T)=\frac{t_1+2t_2+t_3}{4}. \qquad (2)$$

Many authors have proposed different methods for ranking fuzzy numbers, particularly triangular fuzzy numbers [see G. Bortolan, R. Degani, 1985] for the review of methods). In the present paper we use, for its simplicity especially, the order denoted by $\prec$ and defined by [R. R. Yager, 1981]

$$T\prec T'\Leftrightarrow EV(T)\le EV(T').$$

WORK HYPOTHESES AND PRIMARY RESULTS

- We have used a questionnaire applied during 28 July – 4 August 2005 to a number of 56 tourists accommodated [see O. Ban, A. Tomescu, 2005] [1] at three hotels in the Neptun resort [see O. Ban, A. Tomescu, 2005];
- We have taken into consideration the answers to the question: "4. Which criteria regarding the quality of services (from the list of those 14 in Table 1) are important for you? Please mark them from 1 to 10, 10 points for the most important. The scores can be repeated "to take into account in this way the tourists' opinions regarding the quality of services. We obtained the weights ω_i in Table 2, which were calculated in the article [O. Ban, 2007].
- To question "5. How do you appreciate these aspects at this hotel? (from the list of those 14 in Table 1) Give points from 1 to 10, 1 point for the worst complying with the criterion. We have replaced the scores obtained with linguistic scales to be able to illustrate the use of triangular fuzzy numbers in processing the answers, these containing a certain level of un-determination. The answers to question 5 are synthesised in Table 1. The numbers written in the table represent the number of tourists accommodated in one of the three hotels (R= Romanța, T= Terra, C= Clăbucet) who gave the answers in the upper part of the table. We add that the level written in the table are "extremely important", "very important", "important", "to a certain extent", "to a little extent" are just generic, these could be modified varying with the criterion considered, keeping the number of stages (5) and the descending order (for example, from "extremely important" to " to a certain extent").

Table1Tourists' answers to questionnaire
H.Romanța (R)-21 tourists; H.Terra (T)-16 tourists; H.Clăbucet (C)-19 tourists.

Quality criteria	Extremely important			Very important			Important			To a certain extent			To a little extent		
	R	**T**	**C**	**R**	**T**	**C**	**R**	**T**	**C**	**R**	**T**	**C**	**R**	**T**	**C**
1. Interior and exterior aspect	6	5	6	6	4	5	3	3	3	5	2	4	1	2	1
2. Modernity of the building	5	4	4	4	5	4	3	3	3	5	3	3	4	1	4
3. Modern equipment facilities	8	4	7	4	5	3	2	5	2	3	1	3	4	0	4
4. Ambient	15	9	12	3	3	3	2	2	2	1	2	1	0	0	0
5. Cleaning state	21	15	19	0	1	0	0	0	0	0	0	0	0	0	0
6. Using computer-based systems for registering tourists and taking	7	4	7	3	5	4	4	3	2	6	2	5	1	2	1
7. Well trained personnel	9	6	9	5	3	4	7	2	5	0	2	1	0	3	0
8. Personnel's availability to serve	17	13	17	3	2	2	1	1	0	0	0	0	0	0	0
9. Promptness in serving clients	19	12	17	2	3	2	0	1	0	0	0	0	0	0	0
10. Communication skills	15	10	15	3	3	1	2	2	2	1	1	1	0	0	0
11. Kindness	18	14	13	2	2	4	1	0	2	0	0	0	0	0	0
12. Enthusiasm and good-humored	8	5	7	6	4	5	3	2	3	2	3	2	2	2	2
13. Capacity of dealing and solving problems	17	6	15	1	3	1	1	2	1	2	4	2	0	1	0
14. Capacity of task fulfilling	13	5	13	2	5	1	5	3	4	1	2	1	0	1	0

SYNTHESIS RESULTS

To operate the answers provided by the tourists and to achieve the objective had in view, that to order the hotels varying with each criterion and then reported to the totality of criteria, we are assigning for each possible stage a triangular fuzzy number. We are making a standard choice which is represented in Figure 2. This means that to "extremely important" answer corresponds the triangular fuzzy number u_5=(7.5, 10,10), to the "very important" answer corresponds the triangular fuzzy number u_4= (5.0, 7.5, 10), to the "important" answer the triangular fuzzy number u_3=(2.5,5.0,7.5), to the "a certain extent " answer the triangular fuzzy number u_2=(0,2.5,5.0),and to the "little extent" answer the triangular fuzzy number u_1=(0,0,2.5).

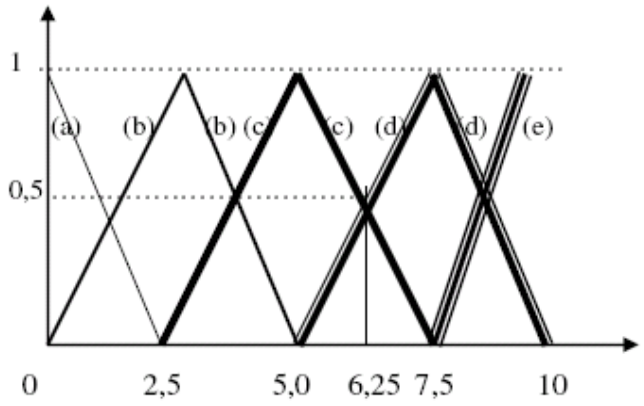

Figure 2 The triangular fuzzy number for the characteristics: "to a little extent" ((a), thin line), "to a certain extent" ((b), callous line), "important"((c), thick line), "very important" ((d), double line) and "extremely de important" ((e), triple line)

For instance, (d) in Figure 2 should be interpreted as follows: a tourist who answers "very important" to one of the questions gives 7,5 with degree 1, gives 6,5 with degree 0,5 and gives 5 with degree 0. Or we can say that the degree of membership of the mark 7,5 to the "very important" answer is 1, that of the mark 6,5 is 0,5, and that of the mark 5 is 0.

We evaluate each hotel from the point of view of criterion k, $k \in \{1,...,14\}$, from the list of those 14 considered, calculating the arithmetic average of the fuzzy numbers obtained as answers provided by the tourists, that is using for criterion k the formula (formula (1) with the weights considered equal with 1/n)

$$\frac{1}{n}(n_1 \cdot u_1 \oplus n_2 \cdot u_2 \oplus n_3 \cdot u_3 \oplus n_4 \cdot u_4 \oplus n_5 \cdot u_5),$$

where $n_i, i \in \{1,2,3,4,5\}$ notes the number of tourists providing an answer corresponding the triangular fuzzy number u_i to the question regarding the k quality criterion, and $n = n_1 + n_2 + n_3 + n_4 + n_5$.

For example, the evaluation of the Romanta hotel from the point of view of *Interior and exterior aspect* criterion is calculated as follows:

$$\frac{1}{21}(6 \cdot u_5 \oplus 6 \cdot u_4 \oplus 3 \cdot u_3 \oplus 5 \cdot u_2 \oplus 1 \cdot u_1)$$

$$= \frac{1}{21}((45,60,60) \oplus (30,45,60) \oplus (7.5,15,22.5) \oplus (0,12.5,25) \oplus (0,0,2.5))$$

$$= \frac{1}{21}(82.5,132.5,170) = (3.93,6.31,8.10).$$

Similarly the evaluation of the Terra hotel by the tourist is calculated from the point of view of *Enthusiasm and good-humored* criterion:

$$\frac{1}{16}(5\cdot u_5 \oplus 4\cdot u_4 \oplus 2\cdot u_3 \oplus 3\cdot u_2 \oplus 2\cdot u_1)$$

$$=\frac{1}{16}((37.5,50,50)\oplus(20,30,40)\oplus(5,10,15)\oplus(0,7.5,15)\oplus(0,0,5))$$

$$=\frac{1}{16}(62.5,97.5,125)=(3.91,6.09,7.81).$$

Similarly, the other data obtained from the tourists for each criterion considered and for each hotel are synthesised. The results are presented in Table 2.

Table 2 Synthesis of answers under the form of triangular fuzzy numbers and classical form

Criteria and their importance		Romanța H.		Terra H.		Clăbucet H.	
		fuzzy	real	fuzzy	real	fuzzy	real
1.	0.065	(3.93,6.31,8.10)	6.16	(4.06,6.25,7.97)	6.13	(4.08,6.45,8.16)	6.28
2.	0.058	(3.95,5.12,7.02)	5.30	(3.91,6.25,8.13)	6.14	(3.29,5.26,7.24)	5.26
3.	0.059	(4.05,6.07,7.62)	5.95	(4.22,6.72,8.59)	6.56	(3.82,5.79,7.37)	5.69
4.	0.063	(6.31,8.81,9.52)	8.36	(5.47,7.97,9.06)	7.62	(5.79,8.16,9.08)	7.30
5.	0.084	(7.50,10,10)	9.37	(7.03,9.37,9.37)	8.79	(7.50,10,10)	9.37
6.	0.076	(3.69,6.07,7.74)	5.89	(3.91,6.09,7.97)	6.02	(4.08,6.45,8.03)	6.25
7.	0.064	(5.24,7.74,9.17)	7.72	(4.06,6.09,7.66)	5.97	(5.26,7.76,9.08)	7.46
8.	0.073	(6.90,9.40,9.88)	8.89	(6.87,9.38,9.84)	8.87	(7.24,9.74,10)	9.18
9.	0.084	(7.26,9.76,10)	9.19	(6.72,9.22,9.84)	8.75	(7.24,9.74,10)	9.18
10	0.081	(6.31,8.81,9.52)	8.36	(5.94,8.44,9.37)	8.04	(6.45,8.95,9.47)	8.45
11	0.085	(6.19,8.69,9.52)	8.27	(7.19,9.69,10)	9.14	(6.45,8.95,9.73)	8.52
12	0.073	(4.64,6.90,8.45)	6.72	(3.91,6.09,7.81)	5.98	(4.47,6.71,8.29)	6.55
13	0.066	(6.43,8.93,9.40)	8.42	(4.06,6.41,7.97)	6.21	(6.32,8.82,9.34)	8.32
14	0.069	(5.71,8.21,9.17)	7.82	(4.38,6.72,8.44)	6.56	(5.92,8.42,9.21)	7.99

To be able to compare the situation of the hotels relatively to a certain criterion, we defuzzified the fuzzy numbers obtained using the expectance value formula (2). In the case of criterion *1.Interior and exterior aspect* for the Romanta hotel we obtain:

$$\frac{3.93+2\cdot 6.31+8.10}{4}=6.16,$$

and in the case of criterion *12.Enthusiasm and good-humored* for the Terra hotel:

$$\frac{3.91+2\cdot 6.09+7.81}{4}=5.98.$$

Similarly we obtain the other numbers contained in Table 2.

We notice that we cannot classify the three hotels globally because for different criteria we obtain different hierarchies. For example, for criterion *2.Modernity of the building* the order (ascending) is Terra, Romanta, Clabucet, for criterion *4.Ambient* the order (ascending) is Romanta, Terra, Clabucet, and for criterion *10.Communication skills* the order (ascending) is Clabucet, Romanta, Terra. Moreover, for a correct hierarchy we should take into account the importance given by the tourists to different criteria, that is the weights of the criteria presented in the second column in Table 2. To measure synthetically the quality of services in each of the three hotels, from the tourists' perspective, when the questionnaire was applied and based on the quality criteria suggested, indicator noted $S_{Romanta}, S_{Terra}, S_{Clabucet}$, we will calculate the fuzzy weighted arithmetic average of the results obtained for each hotel.

For Romanta hotel we obtain:

$$\begin{aligned}
S_{Romanta} &= 0.065\cdot(3.93,6.31,8.10)\oplus 0.058\cdot(3.95,5.12,7.02)\oplus 0.059\cdot(4.05,6.07,7.62)\\
&\oplus 0.063\cdot(6.31,8.81,9.52)\oplus 0.084\cdot(7.50,10,10)\oplus 0.076\cdot(3.69,6.07,7.74)\\
&\oplus 0.064\cdot(5.24,7.74,9.17)\oplus 0.073\cdot(6.90,9.40,9.88)\oplus 0.084\cdot(7.26,9.76,10)\\
&\oplus 0.081\cdot(6.31,8.81,9.52)\oplus 0.085\cdot(6.19,8.69,9.52)\oplus 0.073\cdot(4.64,6.90,8.45)\\
&\oplus 0.066\cdot(6.43,8.93,9.40)\oplus 0.069\cdot(5.71,8.21,9.17)\\
&=(5.67472,8.03482,9.00966),
\end{aligned}$$

and similarly for Terra and Clabucet hotels:

$$\begin{aligned}
S_{Terra} &= (5.24568,7.60484,8.78258)\\
S_{Clabucet} &= (5.68218,8.07467,9.00981)
\end{aligned}$$

If we want to compare the results according to the order introduced in the section *Fuzzy triangular number*, we must calculate the expectancy values using the formula (2). We obtain:

$$\begin{aligned}
EV(S_{Romanta}) &= 7.688505\\
EV(S_{Terra}) &= 7.309485\\
EV(S_{Clabucet}) &= 7.710332
\end{aligned}$$

and also the order (descending) of complying with the quality criteria

Clabucet, Romanta, Terra,

With the mention that the difference between Clabucet and Romanta hotels is extremely small (below 0.022 on a scale with 10 units or, in percentage terms, below 2.2%).

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