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MODELLING OF DYNAMIC MANUFACTURING PROCESS

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ABSTRACT: This paper has presented some models for study of dynamic manufacturing process, in special for milling cutting process. These models of machining process analysed systems with linear cutting forces and nonlinear cutting forces, and milling model by using semi-discretization method with delay time. All these models are necessary in analysis of stability of dynamic machining process.

1. INTRODUCTION

The study of dynamic manufacturing process is focused many researchers which had goal improving dynamic performance of machine tools and rise quality of workpieces and productivity of machining process.

It's well-known that during cutting process can occurred chatter with had a direct effect about accuracy of machining, wear tool, quality of surface finished workpiece, etc., being followed by strong noises. All these perturbations are influenced by parameter dynamic cutting as: depth of cut, thickness of cut, main spindle speed, rank and relief angle, and some effect parameters such as cutting force, instantaneous depth of cut and temperature from cutting zone.

The dynamic machining system (DMS) [1,2] is composed in general from two main parts: elastic system of cutting process (SE)-formed by workpiece-cutter assembly and cutting process system (PA). A system of DMS with one freedom-type Kelvin-Voight associated model is presented in fig.1, where the cutting process is described by static stiffness-K_a of cutting force:

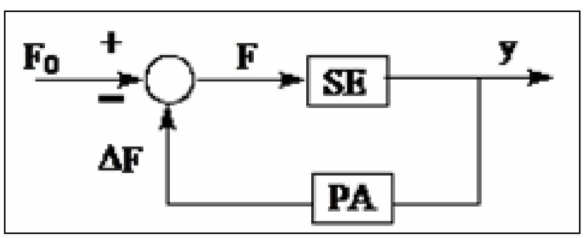

Fig.1. Diagram of dynamic machining system.

$$F = F_0 - \Delta F \tag{1}$$

$$\Delta F = K_a\left(1 - \mu e^{-pT_a}\right)y \tag{2}$$

$$\mathrm{m}\ddot{\mathrm{y}} + \mathrm{c}\dot{\mathrm{y}} + \left[\mathrm{K} + \mathrm{K_a}\left(1 - \mu \mathrm{e}^{-\mathrm{pT_a}}\right)\right]\mathrm{y} = \mathrm{F_o} \tag{3}$$

Where: T_a-is delay time, μ-overlapping factor, ΔF-dynamic force, F_0-initial force, *m*-mass of system, *c*-dumping constant, *K*-stiffness, *y*-dynamic displacement.

In general a complex DMS is composed beside these two main components-*SE* and *PA*, another two auxiliary parts as: *SA*- dynamic acting system for kinematics chains of machine tools, and *PF*- dynamic system of friction process.

2. MODELS OF DMS

For study of DMS is required to build of mathematical model [4,5,6] of system which is followed the behaviour of machining tools and cutting process, that determining the stability of system by critical depth of cut and chatter vibration. The choice of DMS model is influenced by type of machining process, machine tools, cutting parameters and tools.

The milling cutting process has a greater percent from mechanical cutting process given by its widely used and complex process, being indispensable for high machining process and hard materials cutting.

A complex analysis of milling process for linear cutting force and non-linear cutting force for determining stability of milling process [4,6] has at base Nyquist Stability Criteria, Time Domain Simulation and Experimental Studies, Asymptotic Stability Border, Generate Stability Lobes Diagrams for Linear and Nonlinear Machining Forces.

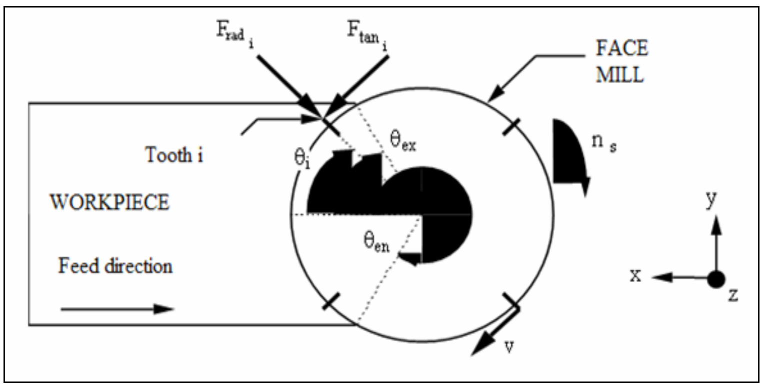

Fig.1. Model of milling process.

The milling cutting forces [4] on three axes can be written:

$$F_x = \sum_{i=1}^{N_t} \left[-F_{rad_i} \cos(\theta_i) + F_{\tan_i} \sin(\theta_i) \right] \sigma(\theta_i) \tag{4}$$

$$F_y = \sum_{i=1}^{N_t} \left[-F_{rad_i} \sin(\theta_i) - F_{\tan_i} \cos(\theta_i) \right] \sigma(\theta_i) \tag{5}$$

$$F_z = \sum_{i=1}^{N_t} \left[-F_{lon_i} \right] \sigma(\theta_i) \tag{6}$$

, where: $\sigma(\theta_i)$-is a function which determining the contact between workpiece-tool:

$$\sigma(\theta_i) = \begin{cases} 1 & dacă \quad \theta_i \le \theta_{ex} \\ 1 & dacă \quad \theta_{en} \le \theta_i \\ 0 & dacă \quad \theta_{en} \ge \theta_i \ge \theta_{ex} \end{cases} \tag{7}$$

The model of milling process for dynamic zone of model (the moving of *z*-axe is ignoring getting by its stiffness versus *x* and *y* axes) can be written:

$$\begin{bmatrix} \Delta F_x \\ \Delta F_y \end{bmatrix} = dA \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = d \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{8}$$

, where A-is matrix of directional coefficients forces, which can be expressed as:

$$A_{11} = \sum_{i=1}^{N_t} \left[-K_r \cos(\psi_r)\cos^2(\theta_i) + K_t \cos(\theta_i)\sin(\theta_i) \right] \sigma(\theta_i) \tag{9}$$

$$A_{12} = \sum_{i=1}^{N_t} \left[-K_r \cos(\psi_r)\sin(\theta_i)\cos(\theta_i) + K_t \sin^2(\theta_i) \right] \sigma(\theta_i) \tag{10}$$

$$A_{21} = \sum_{i=1}^{N_t} \left[-K_r \cos(\psi_r)\sin(\theta_i)\cos(\theta_i) - K_t \cos^2(\theta_i) \right] \sigma(\theta_i) \tag{11}$$

$$A_{22} = \sum_{i=1}^{N_t} \left[-K_r \cos(\psi_r)\sin^2(\theta_i) - K_t \sin(\theta_i)\cos(\theta_i) \right] \sigma(\theta_i) \tag{12}$$

, where: K_r, K_t-are cutting force factors, which these are been determined by cutting process conditions.

For system with <u>linear cutting forces </u>[4] the equation of face milling process in Fourier form is became:

$$\begin{bmatrix} \Delta \overline{F}_x \\ \Delta \overline{F}_y \end{bmatrix} e^{j\omega_c t} = \frac{dN_t}{2\pi}\left(1 - e^{-j\omega_c T_t}\right) G^0(j\omega_c) \begin{bmatrix} \Delta \overline{F}_x \\ \Delta \overline{F}_y \end{bmatrix} e^{j\omega_c t} \tag{13}$$

The matrix of transfer function-G^0 on initial approximation (N_t-is number of tooth cutter) at chatter frequency-ω_c can be written:

$$G^0(j\omega_c) = \frac{2\pi}{N_t} A^0 \left[G_t(j\omega_c) + G_r(j\omega_c) \right] \tag{14}$$

By mathematical calculus of characteristic equation (14) can be determined the eigenvalues of system:

$$\Lambda = \Lambda_R + j\Lambda_I = -\frac{dN_t}{2\pi}\left(1 - e^{-j\omega_c T_t}\right) \tag{15}$$

The critical depth of cut is determined by formula:

$$d_{\lim} = -\frac{\pi \Lambda_R}{N_t}\left(1 + \kappa^2\right) \tag{16}$$

, where: coefficient-κ represented:

$$\kappa = \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\omega_c T_t)}{1 - \cos(\omega_c T_t)} \tag{17}$$

For system with <u>nonlinear cutting forces</u> [4,5] these forces can be expressed as:

$$F_t = K_t s_{de}^{1+\alpha_C} d^{1+\beta_C} \left(\frac{v}{1000} \right)^{\gamma_C} + \eta_t d\left(s_d - s_{de}\right) \tag{18}$$

$$F_r = K_r s_{d\,e}^{\;1+\alpha_T} d^{1+\beta_T}\left(\frac{v}{1000}\right)^{\gamma_T} + \eta_r d\left(s_d - s_{d\,e}\right) \tag{19}$$

, where is done the notation: s_{dt}-feed on teeth at enter, s_{de}-feed on teeth at out, ($s_{d\,e} = s_{d\,t}\cos(\theta_i)$), *and parameters*-η*:*

$$\eta_t = \left[(1+\alpha_C)K_t s_{d\,e}^{\;\alpha_C} d^{\beta_C}\left(\frac{v}{1000}\right)^{\gamma_C}\right] \tag{20}$$

$$\eta_r = \left[(1+\alpha_T)K_r s_{d\,e}^{\;\alpha_T} d^{\beta_T}\left(\frac{v}{1000}\right)^{\gamma_T}\right] \tag{21}$$

, where: *α, β* and *γ*- are exponents of cutting forces in dependence with feed, depth of cut, respectively spindle speed. The equation of milling process in Fourier form at initial condition for nonlinear cutting force can be written:

$$\begin{bmatrix}\Delta F_x\\ \Delta F_y\end{bmatrix} = dB^0\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} = d\begin{bmatrix}B_{11}^0 & B_{12}^0\\ B_{21}^0 & B_{22}^0\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} \tag{22}$$

$$B_{11}^0 = -\eta_r\int_{\theta_{en}}^{\theta_{ex}}\left\{\cos^{(2+\alpha_T)}(\theta_i)\right\}d\theta + \eta_t\int_{\theta_{en}}^{\theta_{ex}}\left\{\cos^{(1+\alpha_C)}(\theta_i)\sin(\theta_i)\right\}d\theta \tag{23}$$

$$B_{12}^0 = -\eta_r\int_{\theta_{en}}^{\theta_{ex}}\left\{\sin(\theta_i)\cos^{(1+\alpha_T)}(\theta_i)\right\}d\theta + \eta_t\int_{\theta_{en}}^{\theta_{ex}}\left\{\sin^2(\theta_i)\cos^{\alpha_C}(\theta_i)\right\}d\theta \tag{24}$$

$$B_{21}^0 = -\eta_r\int_{\theta_{en}}^{\theta_{ex}}\left\{\sin(\theta_i)\cos^{(1+\alpha_T)}(\theta_i)\right\}d\theta - \eta_t\int_{\theta_{en}}^{\theta_{ex}}\left\{\cos^{(2+\alpha_C)}(\theta_i)\right\}d\theta \tag{25}$$

$$B_{22}^0 = -\eta_r\int_{\theta_{en}}^{\theta_{ex}}\left\{\sin^2(\theta_i)\cos^{\alpha_T}(\theta_i)\right\}d\theta - \eta_t\int_{\theta_{en}}^{\theta_{ex}}\left\{\sin(\theta_i)\cos^{(1+\alpha_C)}(\theta_i)\right\}d\theta \tag{26}$$

Similar with model of linear milling cutting force, the critical depth of cut at chatter frequency for nonlinear force can be determined with:

$$d_{\lim} = -\frac{\pi\Lambda_R(K_t, K_r)}{N_t}\left[1+\kappa^2(K_t, K_r)\right] \tag{27}$$

For analysis of milling process, in both cases, is determined critical depth of cut for every spindle speed of milling machine which due to obtaining stability diagram of milling process.

Another method for determination stability of DMS by using semi-discretization method for milling model with time delay [3,6] is modelling for milling process with a single edge tool by an autonomous delay-differential equation (fig.2).

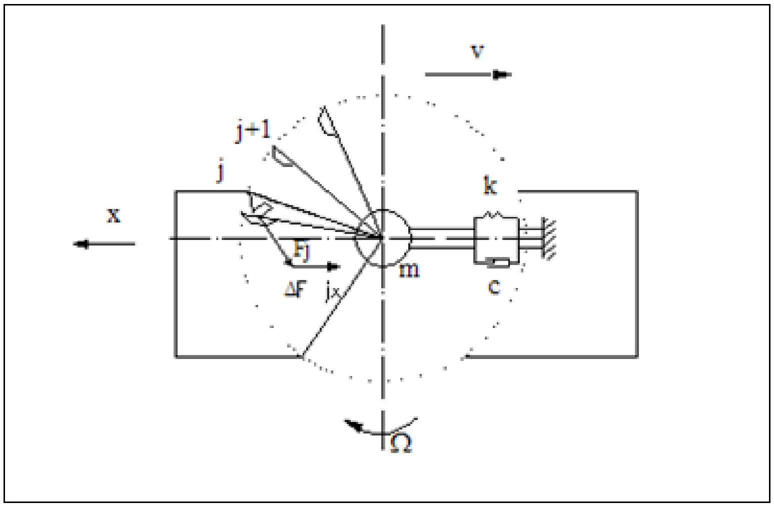

Fig.2. Mechanical model of milling process.

The semi-discretization [3] has represented the partial discretization of an autonomous delay-differential equation only at time delay- τ_o. By considering a second T-periodic delay differential equation can be written:

$$\ddot{x}(t)+b_o(t)\dot{x}(t)+c_o(t)x(t)=b_1(t)\dot{x}(t-\tau_o)+c_1(t)x(t-\tau_o) \quad (28)$$

By dividing time period-*T* into *k*- intervals of length: $\Delta t = \tau_o/(n+1/2)$, where n-approximation number, and decreasing interval- Δt due to rising approximation number-*n* and error decreasing. So, the eq.(28) can be approximated by a autonomous ordinary differential equation [3]:

$$\ddot{x}(t)+b_{oi}\dot{x}(t)+c_{oi}x(t)=f_{i-n} \quad (29)$$

, where constant excitation from right side is:

$$f_{i-1}=b_{1i}\dot{x}_{i-n}+c_{1i}x_{i-n}\approx b_{1i}\dot{x}(t-\tau_o)+c_{1i}x(t-\tau_o) \quad (30)$$

, and coefficients-b_{oi}, c_{oi}, b_{1i} and c_{1i}-are constant approximations (average values) of time depending of time dependent coefficients- $b_o(t)$, $c_o(t)$, $b_1(t)$ and $c_1(t)$ in a ith interval. The discretization is corresponding to a linear approximation of piecewise of constant time delay (fig.3).

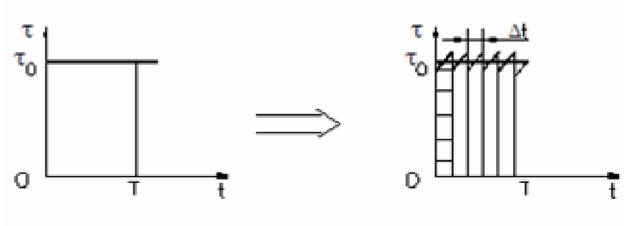

Fig.3. Discretization time delay.

The discretization function is:

$$y_{i+1} = A_i y_i \tag{31}$$

$$y_i = col(\dot{x}_i x_i f_i ... f_{i-n}) \tag{32}$$

$$A_i = \begin{pmatrix} A_{1i} & A_{2i} & 0 & 0 & \cdots & 0 & A_{3i} \\ A_{4i} & A_{5i} & 0 & 0 & \cdots & 0 & A_{6i} \\ b_{1i} & c_{1i} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \tag{33}$$

The transition matrix-*Π* has ensured connection between states of system at time -t_o and $t_k = t_o + T$, which is actually, as is similar with a finite dimensional approximation of monodromy operator, which due to:

$$\Pi = A_{k-1} A_{k-2} \cdots A_1 A_0 \tag{34}$$

The interpretation of asymptotic stability is assured by all eigenvalues of transition matrix-*Π*, which must to be less that one in modulus.

3. CONCLUSIONS

For analysis dynamic machining process is necessary determining a mathematical model of system process, which attended the behavior of dynamic system by cutting forces near chatter frequency for determining critical depth of cut for all spindle speeds of machine and plotted the stability diagram of DMS.

For validation of mathematical model is required to continued with experimental tests or simulation, between all these results must to be more less difference values.

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