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THE INFLUENCE OF THE PRESSURE ANGLES ON THE TRANSMISSION ERROR OF THE ASYMMETRIC GEARS

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Abstract: The software for designing, modelling and analysing the asymmetric spur gears, developed by the paper authors, use as initial data the numbers of the teeth and the distance beetwen the axes. For the same initial data the designing engineer choose the following designing variables: the two pressure angles for the direct profile and for the inverted profile, the generation with one or two racks, one of the gear rack angles, the second resulting from the coefficient of assymmetry. So one can obtain different functional parameters. In this paper is analysed the influence of the pressure angles on the transmission error of the asymmetric gear.

1 Introduction

Asymmetric gears, formed of gear wheels with asymmetric involute profile teeth, add further advantages to the already known advantages of the involute gears. By choosing a convenient asymmetry coefficient there can obtained an increase in the load capacity, a decrease in weight, a reduction in vibrations, an increase in efficiency and durability [4], [5], [6], [7].

The paper is only a part of an extended study [2],[3] on spur gears with asymmetric teeth. This study resulted in the conception of “Software for designing, modelling and analysing the geometrical and functional parameters of gears formed of asymmetric teeth spur gears”. The programms developed by the paper authors [1], [3] have been used for to determine the resulted which are presented.

The study is based on the following considerations about the nomenclature, simbols and subscript:

- It is used the subscript “d” for the elements of the profile with the biger pressure angle (α_{wd}), which is caled **“**direct profile **”.**

- It is used the subscript “i” for the elements of the profile with the smaller pressure angle (α_{wi}), which is caled “inverted profile”.

- For to distinguish the elements reffering to pinion and the gear it is used the subscript 1 for the pinion and 2 for the gear.

From a functional point of view, there are analyzed:

- “The direct asymmetric gear” – for which the “direct" profile is the active one. This case is characterised by an asymmetry coefficient “k” biger than 1.

- “The inverted asymmetric gear” – for which the “inverted” profile is the active one. The asymmetry coefficient of the gear is smaller than 1.

For a given center distance and numbers of teeth, which one carry in, as initial data in the designing application, one also carry in, as designing variable, the two different pressure angles α_{wd} and α_{wi} for the asymmetric profiles and also decide which is the active one. So it is established the coefficient of asymmetry k which is defined as the ratio beetwen the diameter of the base circle of the involute of the inactive profile and the diameter of the base circle of the involute of the active profile. The choosing of these values has a big influence on the performances of the designed transmission.

The study is performed in the same time for the direct asymmetric gear and for the inverted asymmetric gear. This is necesary because these gears, with a certain geometry established in the first stage of the design, depending on the profile used as an active profile, represent two different gears, with different forces of incidence to the profile of the tooth and also different values of the elasticity of the teeth in meshing.

2. The definition of the transmission error

The transmission error, noted with A_e, is equal with the supllementary displacement of the point of contact, on the line of action, due to the elastic deformations of the teeth.

The transmission error for the j point of contact can be obtained by the multiplication of the elasticity of the pair of teeth with the load, booth caculated for the j point of contact:

$$A_{ej} = F_{nj(1)} \cdot V_{j(1)} = F_{nj(2)} \cdot V_{j(2)}. \quad (7)$$

Figure 1 show the initial data, the coefficient of asymmetry and the diagram of transmission error variation, as a function of the point of contact position, for one asymmetric gear.

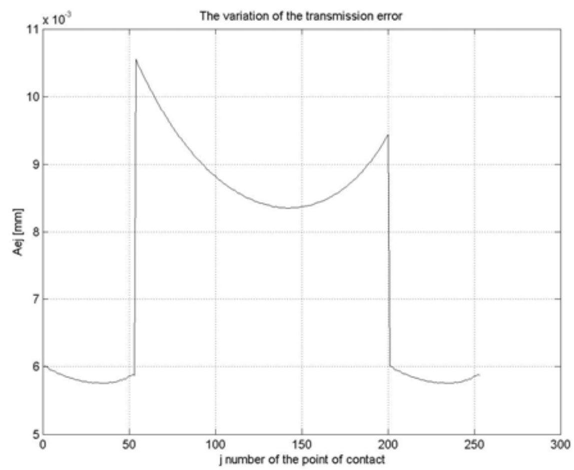

Fig. 1 The variation of the transmission error during a meshing cycle

Direct asymmetric gear $\alpha_{wd} = 40^o, \alpha_{wi} = 20^o,$ *k>1,* $z_1 = 16, z_2 = 57, a = 120$ mm

The developed designing application (in Matlab) make posible to determine the load of each pair of teeth in meshing and the elasticity of the pair of teeth for all points of contact during the meshing cycle. That is the tyme between the first contact point and the last one, on the tooth profile The number of points of contact which are analysed during the time of a meshing period can be choose. For the diagrams presented in this paper have been calculated the values for the mentioned parameters using 200 points of contact for the meshing period, coresponding with a distance on the line of action equal with the pitch on the base circle. Figure 1 show that the maximum transmission error results in the first contact point of the segment of the line of action with one pair of teeth in contact

3. The definition of the degree of asymmetry

To carry out the analyse of the functional parameters, for the booth variants in wich can be used an asymmetric gear, as related how accentuated is the asymmetry of the tooth, in this work, it is used a parameter named degree of asymmetry:

$$As = \frac{\cos\left(\min(\alpha_{wd}, \alpha_{wi})\right)}{\cos\left(\max(\alpha_{wd}, \alpha_{wi})\right)} - 1. \quad (1.1)$$

This parameter has the same value for the direct and inverted asymmetric gears with the same geometrycal parameters but with different asymmetry coefficient. Using this parameter one can compare, on the same diagram, the performances obtained with the asymmetric gear used as asymmetric gear with k>1 (direct asymmetric gear) and with the asymmetric gear used as asymmetric gear with k<1 (inverted asymmetric gear).

4. The variation of the transmission error in relation with the degree of asymmetry

The initial data used for designing the gears with different coefficient of asymmetry are: $z_1=16$, $z_2=57$, $a=120mm$, $\alpha_{wi}=20^o$, $P=18kW$, $n=1000rot/min$. Modification of the degree of asymmetry it is obtained by using the following values for the pressure angle for the direct profile $\alpha_{wd}=[20^o, 25^o, 30^o, 35^o, 40^o]$. The angle α_{cd} of the direct profile of the gear rack it is considered equal with α_{wd}. In this way in the diagram is represented the value of the studyed parameter for the symmetric gear generated with the gear rack with the profile angle $\alpha_{cd}=20^o$. So one can compare the asymmetric gears with the ordinary gear In the table 1 it is presented the correspondence between the degree of asymmetry and the coefficient of asymetry for the ten analysed gears (figure 2).

Table 1 Corespondence between the degree of asymetry and the coefficient of asymetry

Gears	1 / 6	2 / 7	3 / 8	4 / 9	5 / 10
α_{wa}	20^o	25^o	30^o	35^o	40^o
α_{wi}	20^o	20^o	20^o	20^o	20^o
Direct asymmetric gears $k \geq 1$ (1-5)	1	1,0368	1,0850	1,1471	1,2266
Inverted asymmetric gears $k \leq 1$ (6-10)	1	0,9645	0,9216	0,8717	0,8152
As	0	0,0368	0,0850	0.1471	0,2266

At the direct asymmetric gears (k>1), the maximum normal force to the involute of the activ profile, for the time when one pair of teeth are in contact, increase when increase the coefficient of asymmetry by choosening a biger value for the pressure angle α_{wd}. This situation is caused by the fact that the transmited power and the rotation speed are constant, so the torque is constant, but the radius of the base circle of the involute of the active profile decrease by increasing the coefficient of asymmetry. The calculated values for the maximum normal forces for the analysed gears are presented in the table 2.

Table 2 The maximum normal force to the tooth profile

Angrenaj	1	2	3	4	5
k	1,0000	1,0368	1,0851	1,1472	1,2267
Fn [kN]	6,9552	7,2114	7,5469	7,9787	8,5319
Variation%	0	3,6836	8,5064	14,7153	22,6682

In the case of the inverted asymmetric gears (k<1), on the activ profile "i" the pressure angle value is the same for all five analysed gears. ($\alpha_{wi}=20^o$), and the radius for the base circle of the involute activ profile is also the same. In consequence at constant transmited power the result is constant maximum normal force. This force, for all five inverted asymmetric gears, is equal with the force coresponding to the symmetric gear ($\alpha_{wd}=20^o, \alpha_{wi}=20^o$).

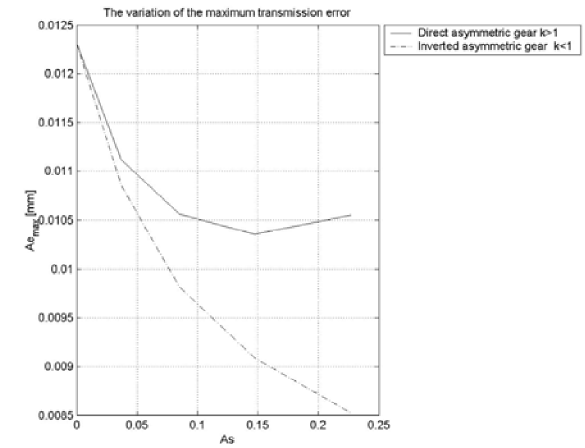

Fig. 2 The variation of maximum transmission error in relation with As

The transmission error decrease by increasing the degree of asymmetry for the direct asymmetric gears and also for the inverted asymmetric gears (figure 2). The values of the transmission error for all asymmetric gears are smaller in comparison with the symmetric gear. The variation is more significant for the gear with coefficient of asymmetry smaller than 1 because in these cases the elasticity of the pair of teeth decrease and the maximum normal force is constant.

5 The variation of the transmission error in relation with the pressure angles at constant coefficient of asymmetry

Designing gears with the same degree of asymmetry one can study how the functional parameters, transmission error beeing one of those, modify by modifying the pressure angles, choosen as designing variables. This can be carry out in the same time for booth variant of using of the asymmetric gear (direct and inverted; figure 3). For to emphasize the advantages of the asymmetric gears, one can calculate and compare the parameters of the direct asymmetric gears with the parameter of the symmetric gears with the same pressure angle on the active profile (figure 4) and with the ordinary symmetric gear ($\alpha_{cd} = 20^{o}$). This time the angle of the gear rack profile is different from the direct profile pressure angle $\alpha_{cd} \neq \alpha_{wd}$.

Table 3 Pressure angles, coefficients of asymmetry for the analysed gears

Gear	$z_1 = 16, z_2 = 57, a = 120$,								
	k	α_{wd} [grade]				α_{wi} [grade]			
1-4/5-8	1,12 /0,89	31	33	35	37	16.25	20.06	23.44	26.55
9-12	1	31	33	35	37	31	33	35	37
13	1; $\alpha_{dc} = 20^{o}$	23				23			

In the table 3 are presented the values for the pressure angles and coefficients of asymmetry for the analysed gears. The gears numbered with 1 – 4 are direct asymmetric gears and those numbered with 5 – 8 are inverted gears with the same geometry. The gears numbered with 9 – 12 are symmetric gears designed with the pressure angles equal to the pressure angles for the direct profile of the asymmetric gears. The gear numbered with 13 is symmetric gear, generated with the gear rack with the profile angle $\alpha_{dc} = 20^{o}$.

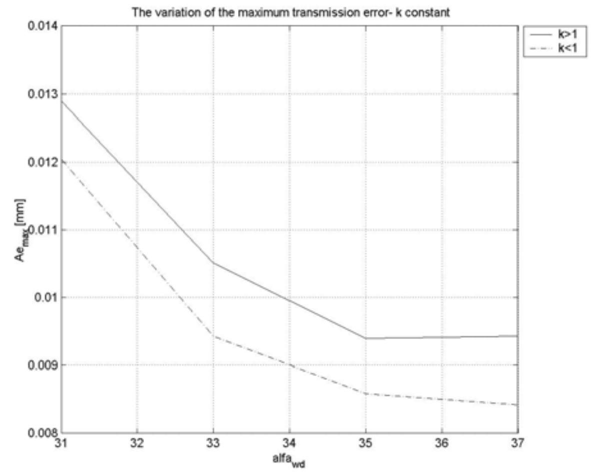

Fig.3 The variation of the maximum transmission error for the asymmetric gears 1-4 /5-8

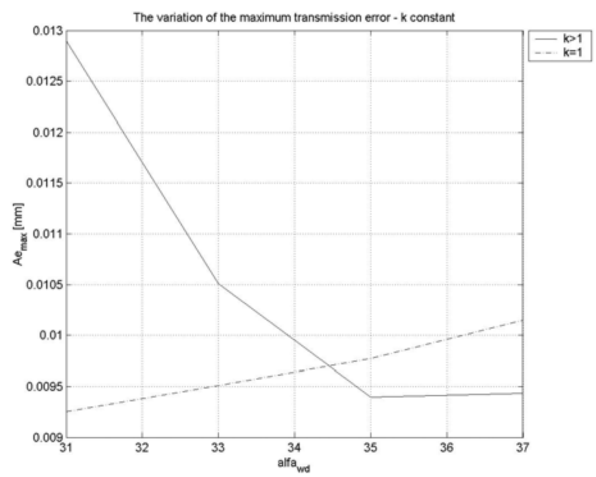

Fig.4 The variation of the maximum transmission error for the asymmetric gears 1-4 / symmetric gears 9-12

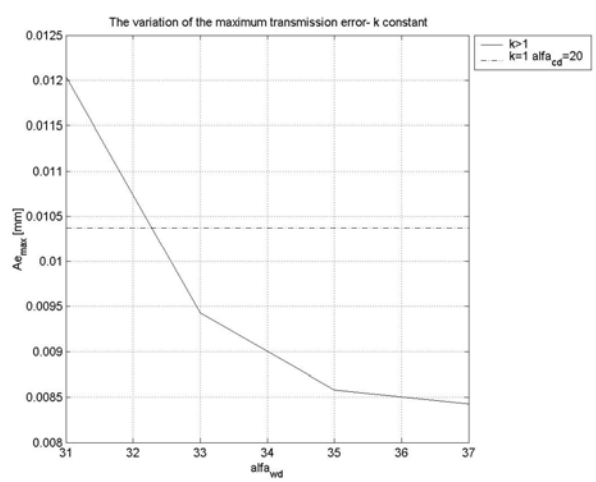

Fig.5 The variation of the maximum transmission error for the asymmetric gears 1-4 in relation with the ordinary symmetric gear 13

From figure 3 one can see that the maximum transmission error is biger for the coefficient of asymmetry higher than one, because in this case the maximum normal force to the tooth profile is biger in relation with the gear with coefficient of asymmetry smaller than one. From figure 4 and 5 results that for the biger values of the pressure angles the maximum transmission error of the direct asymmetryc gears are smaller in relation with the symmetric gears. The inverted asymmetric gears, with smaller values of the transmission error, are also better in relation with the ordinary symmetric gear.

6. Concluzii

The asymmetric gear with the geometrycal parameters established, by choosening one of the asymmetrical profile as active profile, represent two asymmetric gears with different coefficient of asymmetry and different functional parameters. For one can study and compare these gears, in the same time on the same diagrams, it is necesary to introduce the degree of asymmetry. The degree of asymmetry indicate the ratio between the diameters of the base circles of the two different involute profiles and the coefficient of asymmetry indicate also which is the active profile.

Because, for the same initial data: center distance, numbers of teeth, transmited power and rotation speed, by choosening the pressure angles and the coefficient of asymmetry, the designing engineer can obtain gears with very different performances, it is useful to know how the designing variables can be choose to obtain optimum parameter.

For a small value of the maximum transmission error it is better to design asymmetric gears with biger values of the degree of asymmetry, at the same degree of asymmetry with biger values of the pressure angles and to use the inverted profile as active profile.

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