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A STOCHASTIC APPROACH IN DESCRIBING THE VIBRATION INDUCED POSITIONING ALTERATION OF SERIAL AND PARALLEL ROBOTS' END EFFECTORS

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Abstract: The main performance counter of a positioning system is the positioning precision. The ideal positioning system performs a zero error geometrical transport of the end effector to the desired coordinates in the workspace. There are many internal and external factors that influence the precision of this operation. One of them is the vibrations phenomenon. This paper attempts to present a method for describing the alteration of the positioning precision as a result of intrinsic and extrinsic undesired vibrations.

1. INTRODUCTION

Contemporary production systems are designed to achieve an ever-increasing level of performance and quality. Performance criteria vary from technological to economical and include, but are not limited to precision, speed, safety and cost.

Automated production lines, robotized processing and integrated manufacturing are some of the solutions to the performance requirements. Speeds are getting higher and higher while precision must at least be conserved and all this with a reduced long term cost.

Speed and precision have always been the criteria for the system optimizations, but they are not independent therefore the compromise solution arise. The main precision parameter of an industrial robot is the positioning precision, i.e. how accurate the end effectors are positioned at the desired geometrical coordinates. This parameter is very sensitive to intrinsic and extrinsic perturbations, especially at vibrations.

The vibrations effects can be classified into useful effects and undesired effects. This paper aims to present a method for analyzing undesired vibrations effect in robotic systems.

2. ASPECTS OF VIBRATIONS IN ROBOTIC SYSTEMS

Vibrations' effects are omnipresent and can influence any mechanical system. Robots, whether they are serial or parallel, are mechanical systems brought to life by automated or intelligent control of their electrical, pneumatic, hydraulic or other form of energy based actuators. Therefore, they are a victim of the undesired effects of the vibrations, as any other mechanical system.

As mentioned before, one of the robots' main goals is to achieve a very accurate positioning at high speeds and accelerations. This means any factor that can alter these parameters is an undesired perturbation and must be kept under effective control, as much as possible.

In the robot design phase one should consider the vibrations as a subject of rejection. The robot-relative character of the perturbation source can lead to a classification of the vibrations in two major categories: *intrinsic vibrations* and *extrinsic vibrations*.

The intrinsic vibrations are caused by internal sources, meaning one or more robot's components are responsible. Almost all parts of the robot system can cause intrinsic vibrations. These vibrations can be and usually are generated by the mechanical parts (elastic components, flexible joints, mobile elements, etc.), but can have their source in the actuators (power drives, motors, etc.) and electronic parts, too (amplifiers, interfaces, etc). Even the control algorithms can induce undesired vibrations in a robot by the incorrect handling of the input signals and internal states.

Most of the intrinsic vibrations can be predicted, studied and prevented in the robot's design phase. Usually, the performance parameters are limited to acceptable levels from the vibrations point of view.

The extrinsic vibrations are caused by the environment the robot works in and are less predictable. Their harmful effect can be permanent or temporary. For example, if a robot is placed on a platform that is subject to vibrations, these vibrations will directly affect the entire system. Improper and non-robust design makes the robot sensible to this aspect and can even lead to amplifying the negative effects.

Usually, extrinsic vibrations cannot provide a clear deterministic model and therefore they can be regarded as stochastic perturbations that must be filtered.

Another criterion for dividing the perturbations is the chronological evolution. They can be periodical or non periodical. Also, they can be amortized or non-amortized.

Still, the approach we propose in this paper is not sensible to these classifications, but tries to offer a general model for vibrations analysis in the robotic systems.

3. POSITIONING ALTERATION MODEL FOR A TRIVIAL ROBOTIC ARM

Our statement begins with describing a model for vibrations induced positioning alteration for a trivial robot's arm. Figure 1 displays a very simple single joint robotic arm. The robot performs planar 180° rotations around the fixed point P.

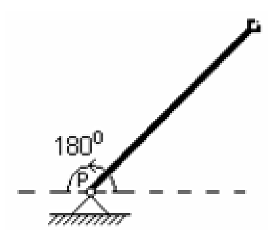

Fig. 1 – Trivial robot

Shall we consider the vibrations that propagate to the end effector and modifies its position from the desired one. These vibrations can be regarded as *transversal* or *longitudinal waves*, or as a combination of the two.

Transversal waves are usually the result of the actual robot functioning or of the environment. They appear in fast movements as the robotic arm is solid, but elastic. The transversal waves propagate toward the end effector changing its position. They can be decreasing in amplitude for amortized perturbations, permanent in the case of sustained perturbations, or even amplified by the mechanical structure. In all cases, the end effector is not positioned in the desired spot.

Longitudinal waves are produced in the elastic component, the robotic arm, and propagate toward the end effector. These vibrations are usually decreasing or permanent, as the mechanical structure does not amplify them.

Most applications present both transversal and longitudinal waves and therefore the counteracting procedures should consider them both.

As stated before, the end effector's real position is modified by these perturbations, introducing a positioning error.

A simple model for the behavior of the end effector assumes that its real position is on the surface of a sphere, as shown in figure 2.

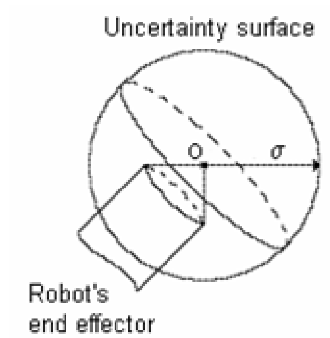

Fig. 2 – Simple spherical uncertainty surface model

Point O represents the desired position of the end effector. The real position of the end effector is somewhere on the surface of the σ radius sphere surrounding the center O. This surface is an uncertainty element for our model.

Still, this is a very simple model even for our trivial robot. A more realistic description should consider the relative orientation of the waves. The amplitude of the transversal waves is usually greater that the one of longitudinal waves in a homogenous bar. Therefore, the surface of the possible end effector's position is closer to a disc shape, as depicted in figure 3.

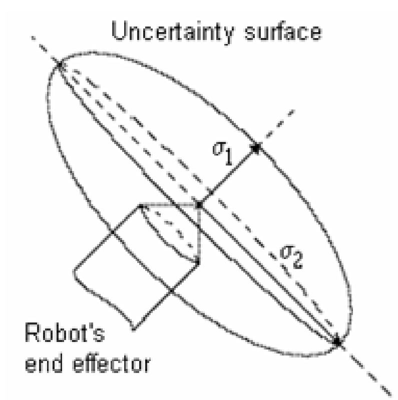

Fig. 3 – Disc shaped uncertainty surface model

The harmful effect is more prominent on the disc's greater diameter σ_2 (corresponding to the transversal waves) than on the disc's smaller diameter σ_1 (corresponding to the longitudinal waves). The disc is created perpendicular to the robot's arm axis. This model is better than the spherical surface one, but still has very strong limitations. A more precise approach should consider the volumetric disposal of the possible end effector's positions.

The simpler version considers a sphere's volume as the uncertainty space where the end effector can be found. Still, the probability distribution for this volume is not linear, meaning is more likely to find the end effector closer to the desired position. As shown in figure 4, there is a gradient of volumetric probability density that describes the positioning alteration.

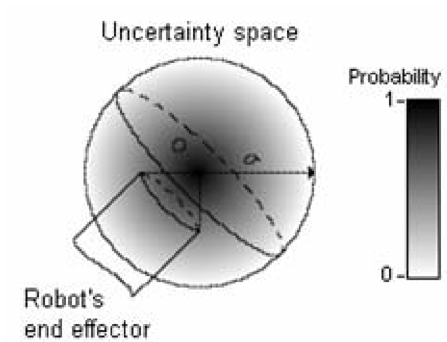

Fig. 4 – Volumetric probability density for spherical uncertainty space

A simpler approach uses a Gaussian distribution for the probability density, as in figure 5. The highest probability is around the desired position O, and 0 probability is beyond the surface of the sphere.

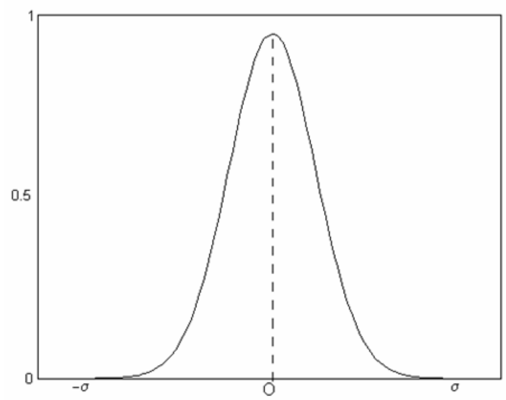

Fig. 5 – Gaussian probability density for the spherical uncertainty space

This model can be improved by considering a disc shaped volume around the desired position, as shown in figure 6. The highest probability is to find the end effector close or within the longitudinal section of the disc that is perpendicular on the robot's arm axis. This section contains the highest probability as a result of disproportionate influence of transversal and longitudinal waves.

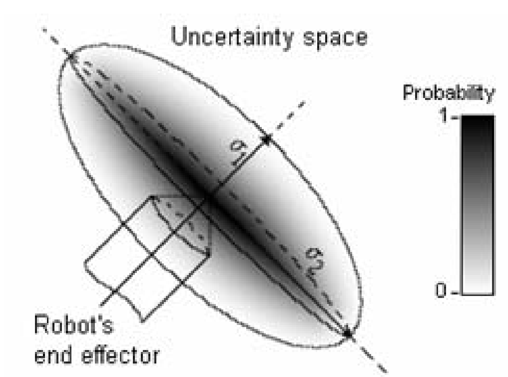

Fig. 6 – Volumetric probability density for disc shaped uncertainty space

These models apply to the trivial robot model, as the one we considered, but can be extended to more complex structures.

4. CUMULATIVE POSITIONING ALTERATION OF A SIMPLE SERIAL ROBOT

The robots are often more complicated than the trivial one we have analyzed before. Considering a generic structure of a serial robot as shown in figure 7, we apply the circular uncertainty zone model, similar to the spherical surface model, but in 2D. Also, the robot is simplified to planar movements.

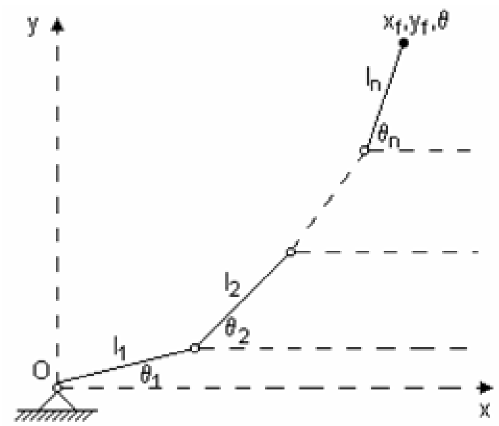

Fig. 7 – Generic serial planar robot

The coordinates of the end effector are given by (x_f,y_f,θ), l_i is the length of the i[th] bar and θ_i is the angle it makes with the *Ox* axis.

The forward kinematics model of this robot is described by the following equations:

$$\begin{cases} x_f = \sum_{i=1}^{n} \left(l_i \cdot \cos\theta_i \right) \\ y_f = \sum_{i=1}^{n} \left(l_i \cdot \sin\theta_i \right) \\ \theta = \theta_n \end{cases} \quad (1)$$

With partial angles in relation (2), the forward kinematics model changes to the equations in (3).

$$\theta_i^{'} = \theta_i - \theta_{i-1}, i = \overline{1,n}, \theta_0 = 0 \quad (2)$$

$$\begin{cases} x_f = \sum_{i=1}^{n} \left(l_i \cdot \cos\sum_{j=1}^{i} \theta_j^{'} \right) \\ y_f = \sum_{i=1}^{n} \left(l_i \cdot \sin\sum_{j=1}^{i} \theta_j^{'} \right) \\ \theta = \sum_{i=1}^{n} \theta_i^{'} \end{cases} \quad (3)$$

Altering the positioning of this system's leads to a circular cumulative uncertainty zone around (x_f,y_f).

We consider ε the deviation radius. Therefore, the alteration model is bounded by this value, meaning the value of the end effector's position projections on each axis are deviated by the [$-\varepsilon,+\varepsilon$] interval. The resulting model is described by the equations (4) and is limited to the maximum deviations and their propagation through the forward kinematics model.

$$\begin{cases} x_f = \sum_{i=1}^{n}\left(l_i \cdot \cos\sum_{j=1}^{i}\theta_j^{'}\right) \pm n\cdot\varepsilon \\ y_f = \sum_{i=1}^{n}\left(l_i \cdot \sin\sum_{j=1}^{i}\theta_j^{'}\right) \pm n\cdot\varepsilon \\ \theta = \sum_{i=1}^{n}\left(\theta_i^{'} \pm arctg\frac{\varepsilon}{l_i}\right) \end{cases} \tag{4}$$

This model also assumes that the individual undesired effects are identical and described by ε. If considering different individual effects characterized by ε_i, the model becomes:

$$\begin{cases} x_f = \sum_{i=1}^{n}\left(l_i \cdot \cos\sum_{j=1}^{i}\theta_j^{'} \pm \varepsilon_i\right) \\ y_f = \sum_{i=1}^{n}\left(l_i \cdot \sin\sum_{j=1}^{i}\theta_j^{'} \pm \varepsilon_i\right) \\ \theta = \sum_{i=1}^{n}\left(\theta_i^{'} \pm arctg\frac{\varepsilon_i}{l_i}\right) \end{cases} \tag{5}$$

The negative effect of the vibrations propagates from the fixed point O to the end effector altering its position.

5. ASPECTS OF POSITIONING ALTERATION IN PARALLEL ROBOTS

Parallel robots are usually more robust than serial robots. The higher mechanical strength and the higher work forces indicate this category of robots for applications where these are the weak points of serial robots.

Due to the direct mechanical connection of the parallel arm, the independent vibrations effect is more harmful in this case for the robots structure.

The end effector position is modified by rather complicated mathematical equations. The uncertainty space in this case can be more complex and may be defined by time functions, as the chronological evolution of the space changes. These changes can be periodically if the individual deviations don't vary, or can be stochastic if the individual deviations are changing in time.

As the end effector of the parallel robot can be a surface-like shape rather than a point-like shape, the positioning alteration materializes into a strange bouncing of the platform.

If the vibration source is well localized geometrically, the robot's designer can estimate the effect of this vibration in all the arms and can estimate the deviation of the end effector, as well as the means of reducing it.

6. CONCLUSIONS

Ever increasing quality requirements force engineers to find better solutions to the problems that arise. Systems' performance is influenced by intrinsic and extrinsic elements and can cause the system not to meet the requirements. Therefore, the designers must prevent the harmful influence as much as possible.

One of the most common perturbations of the mechanical systems is the vibrations. As they can be both intrinsic and extrinsic and their behavior is rarely deterministic, one could use a stochastic approach in analyzing them and designing the appropriate solutions for it.

A simple approach is considering the positioning alteration of the end effector as an uncertainty space around the desired position. The real position of the end effector is somewhere within this space.

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