ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

THE RELIEVED SURFACE OF THE TOOTHING KNIVES

Ioan PANTEA, Mircea VEREŞ, Anamaria MADURA, Nicolae CRECAN
Universitatea din Oradea

Key words: normal line, tangent, relative speed, chipping edge, toothing knife;

Abstract. The problem of calculating the geometry of the knives used for toothing conical gears with curved teeth is the following: knowing the generating curve of the plane wheel, it is required to determine the profile of the chipping edge of the knife and the profile of the abrasive tool used for relieving that will ensure a minimal amount of errors on resharpening. The optimal shape of the positioning surfaces of the knife, as a mathematical expression of the conditions imposed upon the tool, is Archimedes's helix.

1. Introduction

In this paper, the tangent vector to the lateral positioning surface of knives used for toothing conical gears with curved teeth (profiled surface) is determined with the purpose of using it on simulating the kinematics of the processing technology of the profile of toothing knives. The problem of calculating the geometry of the toothing knives used for milling conical gears with curved teeth is the following: knowing the generating curve of the plane wheel it is required to determine the profile of the chipping edge of the knife and the profile of the abrasive tool used for relieving that would ensure a minimal amount of errors on resharpening. The optimal form of the positioning surfaces of the knife, as a mathematical expression of the conditions imposed upon the tool, is Archimedes' helix. The profile of the chipping edge in the calculus section is determined by intersecting the relieving surface plane and the generating surface.

2. The positioning surface and the chipping edge

The generating curves of the flanks of the plane generating wheels are defined through the parametric equations [4]

$$X_p = \begin{bmatrix} x_p(p) \\ y_p(p) \\ z_p(p) \end{bmatrix} \qquad (1)$$

where p is an independent parameter.

The toothing knife is positioned with the definition section of the generating curve in an axial plane. The M_{dp} matrix [3] is considered and it represents the transfer from the system linked to the generating curve to the system linked to the relieving device. The position vector of the S_d surface (figure 1) is:

$$X_d = M_{dp} X_p = \begin{bmatrix} x_d(p, v, \varepsilon) \\ y_d(p, v, \varepsilon) \\ z_d(p, v, \varepsilon) \end{bmatrix} \tag{2}$$

where p and v are the independent parameters of the S_d surface and ε is an angular parameter that positions P_γ in S_d.

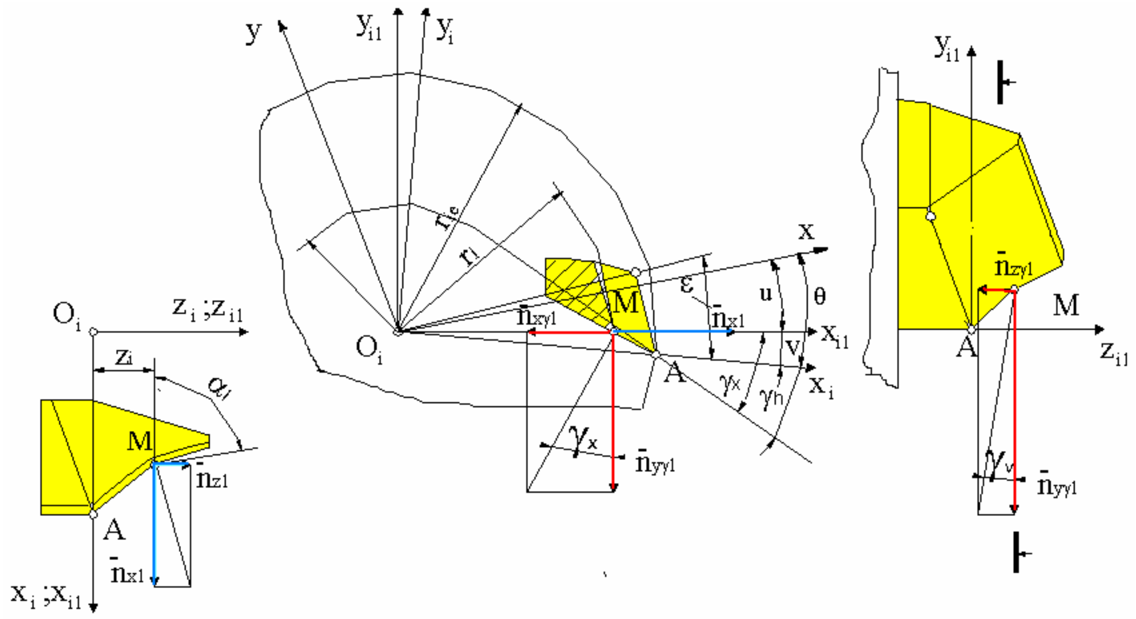

Fig. 1

By definition, the exact chipping edge M_a is the intersection curve between the generating surface of the S_d device and the liberation plane of the P_γ knife.

Taking into consideration that the P_γ liberation plane passes through a point in which the director parameters A, B, C of the N_γ normal line to it are defined, its expression is:

$$A \cdot x + B \cdot y + C \cdot z + D = 0 \tag{3}$$

The intersection between (2) and (3) is written:

$$A \cdot x_d + B \cdot y_d + C \cdot z_d + D = 0 \tag{4}$$

or:

$$f_1(p, v, \varepsilon) = 0 \tag{5}$$

For an imposed ε and discrete values of the p parameter, the v rotation parameter is determined and the M_a exact chipping edge's equation is:

$$X_a = \begin{bmatrix} x_a(p,\varepsilon) \\ y_a(p,\varepsilon) \\ z_a(p,\varepsilon) \end{bmatrix} = X_a(p,\varepsilon) \tag{6}$$

3. The generated surface

The normal line to the chipping edge is determined from the vector perpendicularity condition of the tangent to the chipping edge and the relative speed in the relieving process:

$$\overline{N} = \bar{t} \times \bar{v} \tag{7}$$

or under component form:

$$N = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix} = \begin{bmatrix} t_y \cdot v_z - t_z \cdot v_y \\ t_z \cdot v_x - t_x \cdot v_z \\ t_x \cdot v_y - t_y \cdot v_x \\ 0 \end{bmatrix} \tag{8}$$

In order to obtain the equations of the theoretical profile of the disk and of the normal line to the relieving surface in the contact points of the chipping edge with the disk's surface, that define the contact conditions [2], the projections of the relative displacement speed vector in the $x_iy_iz_i$ system must be determined and, for the profile of the chipping edge with various degrees of resharpening, the projections of the vector in the $x_ky_kz_k$ system (fig.1).

The projections of the v_i vector will be proportional to the partial derivatives, found from the equations of the coordinate systems' transformation after the θ parameter.

$$\begin{bmatrix} v_{xi} \\ v_{yi} \\ v_{zi} \\ 0 \end{bmatrix} = \begin{bmatrix} -y_i + \dfrac{\partial A_x}{\partial \theta}\cos(\theta) \\ x_i + \dfrac{\partial A_x}{\partial \theta}\sin(\theta) \\ -\dfrac{\partial A_z}{\partial \theta} \\ 0 \end{bmatrix} \tag{9}$$

The projections of the v_i vector in the xyz system are obtained using the coordinate transformation matrixes [4].

The t_i tangent to the chipping edge is obtained as an intersection line of the P_γ relieving plane determined by the n_γ vector and the P_e tangent plane to the positioning surface determined by the n_e vector in the considered point of the profile. In the $x_{i1}y_{i1}z_{i1}$ coordinate system, the N_γ normal line to the relieving plane and the N_i normal line to the generated surface are situated in a plane normal to the chipping edge.

The tangent to the edge is given by the expression [2]:

$$\overline{t_i} = \overline{n_\gamma} \times \overline{n_{ie}} \tag{10}$$

cu : $u = \theta - v$ (11)

By introducing in (8) the tangent values from (10) and the relative speed ones form (9), the following normal line is obtained:

$$N = \begin{bmatrix} \cos(u+\gamma_x)\dfrac{1}{\tan(\alpha)} + \sin(u)\cos(\gamma_h)\tan(\gamma_v) \\ \sin(u+\gamma_x)\dfrac{1}{\tan(\alpha)} + \cos(u)\cos(\gamma_h)\tan(\gamma_v) \\ -\cos(\gamma_x) \\ \end{bmatrix} \times \begin{bmatrix} -y \\ x \\ A_z \end{bmatrix} \tag{12}$$

The obtained generated surface will be:

$$X_g = \begin{bmatrix} [x(p)\cos(\varphi) + z(p)\sin(\varphi) + r]\cos(v) \\ [x(p)\cos(\varphi) + z(p)\sin(\varphi) + r]\sin(v) \\ z(p)\cos(\varphi) - x(p)\sin(\varphi) + 1 \\ 1 \end{bmatrix} \tag{13}$$

4. Conclusions

For the modeling of the assembly made up of a I order tool and a II order tool, after establishing the axes system for determining the generating surface, the chipping edge, it is necessary to calculate the normal line, the tangent and the relative speed on relieving, so that the motion equations obtained under a matrix form lead through solving to determining the abrasive tool's surface, the real positioning surface and the real chipping edge.

References

1. Litvin F. L., *Gear Geometry and Applied Theory.* University of Illinois, Chicago. PTR Prentice Hall, Englewood Cliffs, New Jersey 0763. ISBN-13-211095-4, 1994.
2. Pantea I., *Contribuții privind tehnologia sculelor de danturat roți dințate conice cu dinți curbi.*Teză de doctorat, Universitatea din Oradea, 2004.
3. Pantea, I. *Studiul teoretic al sculelor de danturat roți conice cu dinți curbi. Sisteme de coordonate, matrici de transformare.* Analele Universității Oradea, pg. 215-218.
4. Pantea I., *Studiul teoretic al sculelor de danturat roți conice cu dinți curbi. Suprafața generatoare*. Analele Universității Oradea, 2002, pg. 219-223.