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CONTRIBUTIONS REGARDING THE UTILIZATION OF THE CATASTROPHE SURFACE CONCEPT FOR STUDY OF THE WORM-THREAD MILLING CUTTERS WEAR IN CONDITIONS OF CYLINDRICAL GEAR WHEELS FABRICATION

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Abstract: The present paper reports the authors research regarding the catastrophe surface concept in the field of toothing manufacture of cylindrical gear wheels using worm-thread milling cutters. The investigations were engaged with the aim of control process optimization of the machine-tool with adaptive control system. Formerly, a survey of the analyzed concept on the specific research grounds is performed, being so clarified the conceptual strategy and the specific stages of such research. Afterwards, a study case is exposed, and new optimal relationships are provided for adaptive control system utilization in modern manufacturing.

1. INTRODUCTION

Experimental researches made in the case of identification of real conditions of cutting process from the viewpoint of the influence of its parameters on evolution of the worm-thread milling cutters wear, were developed basing on analysis and interpretation of kinematics of the catastrophe forms, by identification of the domains in which can be defined the process of wear studied basing on its morphology, from the viewpoint of catastrophe forms. After all, the researches goal consist in establishing of the functional dependency among the influential factors X1, X2, …, Xp and the objective function Y. In other words, is about the mathematical description of the applied factors X1, X2, …, Xp on the objective function Y [1]. Usually, among the envisaged purposes of a modeled process there are remarked, [2, 3]:

- the analysis of the system/process by means of the created model;
- the way in which the applied factors influence on the studies system/process;
- the optimization of the process respecting the counted influential factors.

A special attention in the field of research must be accorded to the mathematical model in order to identify and develope research programs that test optimize the system data acquisitions through a minimum tests. For such enterprise a close concern to the existing similar information as well as to the general knowledge and the personal experiences have to be drawn. After synthesis of those data it can be pursuit to build the mathematical model. Many authors express that an optimum mathematical model requires a series of complete cycles of investigations, in an iterative manner, such that a convergent spiral to the research goals to be achieved.

A routine experimental [5] strategy, especially used on the process optimizations, regards the involvement of the response surface concept. However, the methodology based on the use of the response catastrophe concept, here proposed, contains a set of mathematical and statistical techniques, since the statistical analysis among the mathematical description of the objective function under the many-independent variable influence is the output together with the optimization of the process itself.

In the majority of cases related with the response surface concept there is not known the analytical relationship between the objective function and the independent variables, a lack to be fill by the catastrophe present approach. Therefore, the first step regards the formulation of a mathematical regression model that approximates at the best the connection between the objective function and the independent variables. Almost all the cases that involve the response surface concept make use of one or both of the I and II rank polynomials as the mathematical models. However, is less probable that such polynomial chose to furnish a reasonable representation of the functional relation between independent variables and the objective function on the all spread range of the variables, but within a restricted limits such mathematical models behaves quite satisfactory [4]. Instead, the catastrophe surface is based on the polynomials of at least III rank, therefore including more optimal solutions . The response catastrophe method is a sequential, but continuously, one. Often in the current experimental situations it happens that the analysis starts from a surface point far from the optimum [6-10]. Currently, the handling of the response catastrophe demands its division through the planes parallel with the influential factor. The purpose of this method has the merit to sequentially indicate to experimentalist the path and especially the limit until which the response region is restricted and especiallythat area that contain the optimum point. The concerned experiment to be analyzed by means of the catastrophe surface and concepts was realized for the case of the cylindrical gear whels at I.C.M Cugir, namely the toothing realisin-duse on tooth cutting machine equiped with adaptive control of the feed used in the current fabrication for toothing of gear wheels using the worm thread milling cutters. To achieve this objective was utilized the original equipment realized by autors in special conditions at ICM Cugir.

2. THE BASIS OF CATASTROPHE THEORY

The Thom's catastrophe theory [4] basically describes how for a given system a continuous action on the *control space* (*Ck*), parameterized by *ck*'s, provides a suddenly change on its *behavior space* (*Im*), described by variables *xm*'s, through the stable singularities of the smooth map:

$$\eta(c_k, x_m): C^k \times I^m \to \Re \qquad (1)$$

being $\eta(c_k, x_m)$ called the *generic potential* of the system.

Therefore, catastrophes are given by the set of *critical points* (c_k, x_m) for which the field gradient of the generic potential vanishes:

$$M^{k\times m} = \left\{(c_k, x_m) \in C^k \times I^m \middle| \nabla_{x_m} \eta(c_k, x_m) = 0\right\} \qquad (2)$$

or, more rigorously: a catastrophe is a singularity of the map $M^{k\times m} \to C^k$.
Next, depending on the number of parameters of *Ck* (named also as *co-dimension, k*) and of the number of variables of *Im* (named also *co-rank, m*), René Thom had classified the generic potentials (or maps) given in eq. (1) as seven unfold elementary (in the sense of universally) catastrophes, i.e. many-variable (the co-rank up to two), and many-parametrical (the co-dimension up to four) polynomials, listed in Table 1. Going to a higher derivative fields of the generic potential, it will be said that the control parameters c_k^* for which the Laplacian of the generic potential vanishes,

$$\Delta_x \eta(c_k^*, x_m) = 0 \qquad (3)$$

gives the bifurcation point.

The set of control parameters c# for which the Laplacian of a critical point is non zero defines the *domain of stability* of the critical point.

Is clear now that the small perturbations of η(*c**, *n*) brings the system from a domain of stability to another, on the contrary, it is said that the system is located in a *domain of structural stability*.

Moreover, the above described cases corresponds to the equilibrium limit of a dynamical evolution of the system:

$$F\left(c_k;t;\eta(c_k;x_m);\frac{\partial\eta(c_k;x_m)}{\partial t},.....\right)=0 \tag{4}$$

where now, the behavior space is further parameterized by the temporal paths $x_m(c_k,t)$.

The connection with equilibrium is recovered through the stationary time regime imposed to the critical points.

In this way, the set of points giving a critical point in the stationary t → +∞ regime (called ω-limit) corresponds to an attractor, and form its basin, whereas the stationary regime t → −∞ (called α-limit) describes a repellor.

On the ground the catastrophe concepts modulate the equilibrium (optimal) states within an evolutionary process, the flexible processes of cutting-holding tools can be approached.

Table 1Thom's Classification for Elementary Catastrophes

Denumirea lui η	**Co-dimensiune**	**Co-putere**	**Extindere universala**
Fold(C)	1	1	x^3+ux
Head (V)	2	1	x^4+ux^2 $+vx$
Dovetail (CR)	3	1	x^5+ux^3 $+vx^2$ $+wx$
Hyperbolic umbilic center (CH)	3	2	x^3+y^3 $+uxy$ $+vx+wx$
Elliptic umbilic center (CE)	3	2	x^3-xy^2 $+u(x^2+y^2)$ $+vx+wy$
Butterfly (F)	4	1	x^6+ux^4 $+vx^3+wx^2$ $+tx$
Parabolic umbilic center (CP)	4	2	x^2y+y^4 $+ux^2+vy^2$ $+wx+ty$

3. General presentation of tooth cutting tool-machine

The tooth cutting tool-machine used in the current production of the society is equipped with adaptive control of the feed for which was established by contract the continuation of research in order to elaborate an algorithm necessary for the adaptive control of the machine and of the cutting velocity [6]. The existing system of the tooth cutting tool-machine FD 400-13CA1 instal the automatic adjustment of the feed, basing on control of the cutting process, in all tooth cutting cycles of RDC, excepting the cutting of gear worms.

The automatic adjustment of the feed basing on the charge control of the machine made by the cutting forces is made for a given diameter and revolution of FMC.

Control is realized by measuring the cutting power, and in this way, for a concrete technological case (the case of MDFMCcu SCDsi and Ca, and the technological cycle without tangential feed, these two being the most utilized in RDC fabrication) is maintained a charging as constant as possible in the cutting process. The size of charging is automatically established by a system, basing on technological data (the wear variation is in dependence with the depth of wear crater $KT_{A\gamma}$, on the release face A_{γ}, when, the wear is in the radial direction on the location face $A_{A\alpha}$) of the concrete tooth cutting case, data that will be introduced from the outside of the machine, from the control panel, before the tooth cutting be started, basing on the execution sketch of the wheal that must be processed, and these data usually contain:

- the APM revolution of FMC;
- modulus *m* of tooth cutting;
- characteristic of the processed material;
- outsets number of FMC;
- the reference feed as a maximal value admitted by STE, selected from the tooth cutting machine manual, or it will be calculated.

Realization of the approximately constant charging will be made in a continuous way by the adjustment of feed using the mechanical variation device with electromechanical servoacting.

The adaptive control of the feed installed on MDFMC used in the analyzed experiment is characterized by the following specific functions:

• accomplish commands only in the conditions of functioning of the main electro-engine, when the feed adjustment using the variation device is possible;

• performs the automatic adjustment of the feed in any tooth cutting cycle of RDC, exclusively for the gear worms;

• perform the supervision and protection of MUD FMC in the case of exceeding of nominal charging of AP, as well as in the case of exceeding of a given report of the reference power which usually is much lower then the nominal power, situation when MUD FMC will be put out of function and the situation will be signalized.

4. The method of catastrophe surface

The catastrophe surface concept is next performed for analyzing the data from two typical systems, see Table 2, engaged into flexible fabrication cutting-tool processes. Nevertheless, the parameters from Table 2 (the cutter parameters for two tips systems from the gear wheel manufacturing process) are related with those of the unfolding polynomials from Table 1 such that the variables (x, y) to remain associated with the tool's advance (f) and the cutting speed (V), respectively.

Under these premises, and being likely to preserve as much information as possible (co-dimension + co-rank) within unfolding catastrophes, from Table 1, in Fig. 1 the obtained hyperbolic umbilic (HU), elliptic umbilic (EU), butterfly (B) and parabolic umbilic (PU) surfaces are displayed, respectively.

The analyze show that only the catastrophically surfaces of CE and CP present an optimal convergent area in which the equilibrium condition is obtained, eq. 2.

However, the condition given in eq. (2) has to be applied properly on unfolding EU and PU polynomial catastrophes from Table 1 so that all considered parameters and variables in Table 2 pursuit their influences in the system's evolution.

Therefore, the optimum relationship has to follow the general form:

$$\begin{cases} \nabla_y \eta_{CE} = 0 \\ \nabla_y \eta_{CP} = 0 \end{cases} \Leftrightarrow \begin{cases} -2yx + 2uy + w = 0 \\ x^2 + 4y^3 + 2vy + t = 0 \end{cases} \tag{5}$$

Table 2. Cutting parameters for two typical systems within a flexible fabrication process in relations with the elementary unfolding catastrophe parameters from Table 1

Parametri	**Sistemul I**	**Sistemul II**
Worm cutter speed $[n] \to \{x\}$	63 [mm]	63 [mm]
Reference feed $[v_{fa}] \to [v]$	2,5 [mm/rot]	2,5 [mm/rot]
Gear wheels modulus $\{m\} \to [u]$	2 [mm]	2,5 [mm]
Cutting speed $[v] \to [y]$	Min…Max [m/min]	Min…Max [m/min]
Tooth numbers $\{z\} \to [w]$	21	62
Wear cutting on A_γ surface $[KT_{A\gamma}] \to [t]$	0,01 [μmm]	0,05 [μmm]

Relations given in eq. (5) are specific to the production programs that have to optimize the cutting process among six independent variables.

Turning to the systems I and II from Table 2, by applying the eq. (5), one has to find the Min…Max values for worm cutter rotation speed (*n* → *x*) and for the cutting speed (*v* → *y*), respectively.

In this case, the parameters u and v are first taken as small parameters, 2.5 and 6.3, to have the same rank as the parameters w and t respectively.

However, for each system, from the all-possible solutions will be taken the half of the real absolute maximum/ minimum ones.

For the y' solutions the results will be further multiply with factor 10 to achieve the desired dimension of velocity [m/min].

Thus, the searching intervals are, for the system I:

$$\begin{cases} \int^{I}_{Min} = 0.145435.....\int^{I}_{Max} = 2.29597..[mm/rot] \\ V^{I}_{Min} = 1.946335.....V^{I}_{Max} = 6.2312....[m/\min] \end{cases} \quad (6)$$

and for the system II:

$$\begin{cases} \int^{II}_{Min} = 0.177918.....\int^{II}_{Max} = 2.732625..[mm/rot] \\ V^{II}_{Min} = 1.26467......V^{II}_{Max} = 7.0677....[m/\min] \end{cases} \quad (7)$$

The minimum results are the lowest values for which the cutting process begins whereas the maximum ones indicates the catastrophic limit beyond the systems crashes its functionality in tool production.

The toothing process can take place between these above extremes.

Worth noting, however, that the results are furnished employing a complex strategy of selecting the optimum set of unfolding catastrophes that appropriately fits the three fixed parameters that determines, even implicitly, the allowed equilibrium intervals for the system's evolution (processing).

For instance, judging only the advance-speed relation it seems that higher advance demands increasing of the cutting speed, but without any other parametric involvement.

Instead, the present results, eq. (6) and (7), prescribe the inverse relation; such behavior is natural when the other parametric difference, like number of processed pieces and the wear costs, are taken into account.

Consequently, inside the production program (toothing manufacturing on the machine-tool with adaptive control system) is prescribed an elegant analytical metod for controling the parameter-variable relations, in eq (5).

Future work is required to further search for the best parameter/variable to be optimized as the processing system achieves its equilibrium and to avoid the expenses of catastrophes.

4. Conclusions

In the context of the gear wheels manufacturing process using machine-tool with adaptive control system and cutting speed there is employed the catastrophe surface concept within Thom's catastrophe theory. The complex cutting-tool systems involve many-independent variables having a parametrical contribution on the production process and its efficiency. The catastrophe theory and their concepts fully integrate such data giving also the prediction of the condition in which the concerned system works around its optimum (equilibrium) regime. The present new approach of catastrophe surface concept provides quantitative relationships among the flexible parameters of the production process and modernizing for the cutting-tool systems were presented. The actual catastrophe surface concept and their implications let open room for future study and manufacturing systems' applications.

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