

## Unitary Method of Synthesis on Profile of Rotary Disc-Cam

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**Keywords:** Rotary Disc-cam, Translating / Oscillating Follower, De-phasing Angle, Theoretical Profile

### Abstract

A unitary method of the geometrical and kinematic synthesis used for the exact design of the profile of the rotary disc-cam and translating or oscillating follower is presented. This method uses the direct movement of the rotary cam, without a reversing movement of the follower around the fixed cam. This unitary method relies on the boundary conditions imposed by the follower using the movement law and the minimum transmission angle or maximum pressure angle. In the case of a roll follower, first the theoretical curve of the cam profile is calculated and then is described in polar coordinates. All cases of mechanisms with translating, oscillating follower, with tip, roll or flat follower are considered. A phase displacement angle or de-phasing angle is highlighted. In the mobile system that is stiffened by the cam, it is defined as the difference between the cam rotation angle and the position angle of the vector radius.

### 1. Introduction

The movement law of the follower can be accurately described if the cam profile is calculated using a numerical analytical method [1, 4, 6, 8, 11, 22]. The present paper refers to the exact synthesis of the curve defined as the profile of the rotary disc-cam and roller or flat follower [4, 8, 9, 10, 17, 19, 21] without a reversing movement [1, 5, 16, 18, 20]. In 1970, [2] K. Hain proposed a method for the optimization of the mechanism with cam in order to obtain good maximal transmissibility, output angle of the swing and minimal acceleration.

In 1988, [7] J. Angeles and C. Lopez-Cajun presented the optimal synthesis of the cam mechanism with oscillating flat-face follower. In 1979, [3] F. Giordano et al. investigated the influence of measurement errors in the kinematic analysis of the cam. In 1985 [6] P. Antonescu has presented an analytical method for the synthesis of the mechanism with cam and flat follower.

The author has continued [11, 15, 20] with the geometrical and kinematic synthesis of the mechanism with rotary cam and balance follower. The analytical method implemented in this paper takes into account the de-phasing angle. This angle is defined [20] as the difference between the rotation angle of the cam and the position angle of the vector radius of the cam-follower contact point, which is measured in the disc-cam plane. The variation of the de-phasing angle has been analyzed [11, 12, 14, 18] in the case of the synthesis of the disc-cam profile with a roll balancing follower.

This paper describes a *unitary method* for the geometrical and kinematic synthesis of the rotating cam profile in 4 variants: with the translating and oscillating follower, which can have a roll or a flat form (with a sole face).

## 2. Mechanism with disc-cam and tip / roller follower

A law of movement is considered as  $s = s(\varphi)$  for the translating follower (fig. 2.1) or  $\psi = \psi(\varphi)$  for the oscillating follower (fig. 2.2). In the case of the translating follower with tip (the radius roll is null), the linear oscillation is constrained between  $s_{\min} = s_0$  and  $s_{\max} = s_0 + h$ , where  $h$  is linear stroke.

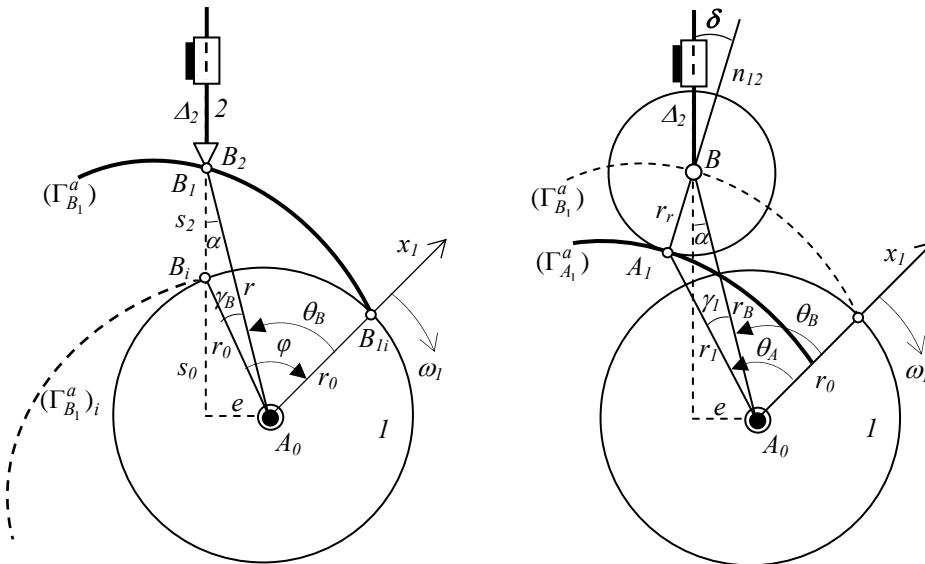


Fig. 2.1. Kinematic schema of mechanism with disc-cam and translating tip / roller follower

In the case of the oscillating rotating follower with tip (when the radius roll is null), the angular oscillation is constrained (fig. 2.2) between  $\psi_{\min} = \psi_0$  and  $\psi_{\max} = \psi_0 + \beta$ , where  $\beta$  is angular stroke.

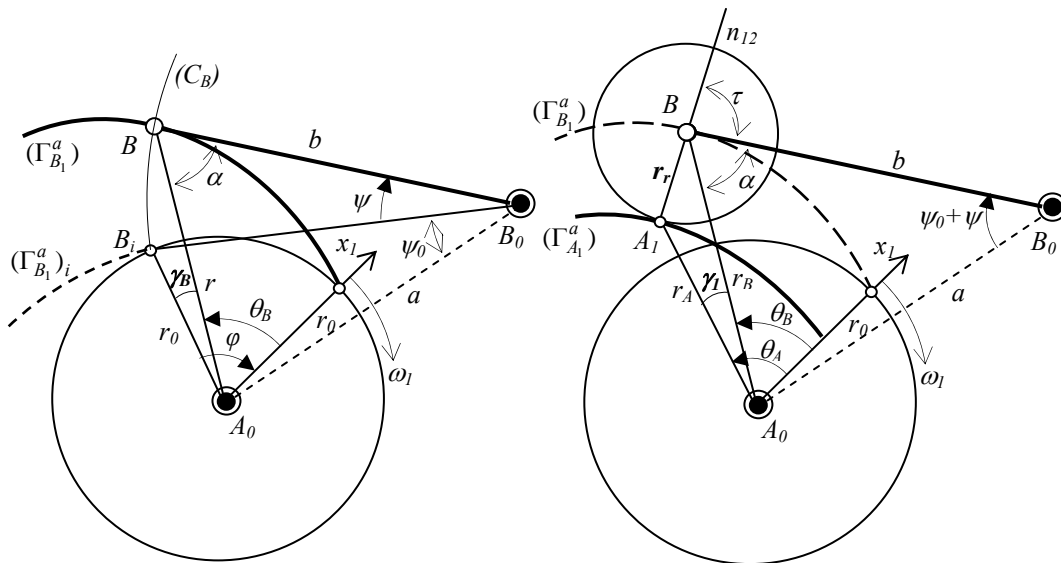


Fig. 2.2. Kinematic schema of mechanism with disc-cam and tip / roller oscillating follower

The functions of the displacement of the follower are obtained from the imposed initial conditions  $s = s(\varphi)$  (fig. 2.1) or  $\psi = \psi(\varphi)$  (fig. 2.2). In the case of reduced velocity:

$$s' = ds/d\varphi; \quad \psi' = d\psi/d\varphi \quad (2.1)$$

and in the case of reduced acceleration:

$$s'' = d^2s/d\varphi^2; \quad \psi'' = d^2\psi/d\varphi^2 \quad (2.2)$$

In order to obtain the base circle radius  $r_0$  of the disc-cam, the maximum values of the reduced linear ( $s'_M = s'_{\max}$ ) or angular velocity ( $\psi'_m = \psi'_{\max}$ ) are calculated at the ascending and descending stroke of the follower.

If the length of the oscillating (rocker) follower 2 with tip  $B_0B = b$  is considered (fig. 2.2), the linear reduced velocity of point  $B$  (the center of the roll) is determined by:

$$v_B^r = b \cdot \psi' = b \cdot \omega_2 / \omega_1 \quad (2.3)$$

where  $\omega_1 = d\varphi/dt$  and  $\omega_2 = d\psi/dt$  are the angular velocity of the disc-cam 1 and the oscillating follower 2, respectively.

The laws of symmetrical ascending and descending movement are specified from initial conditions [11, 14]. The segments  $e$  and  $s_0$  (defining the point  $A_0$ ) are graphically determined (fig. 2.3).

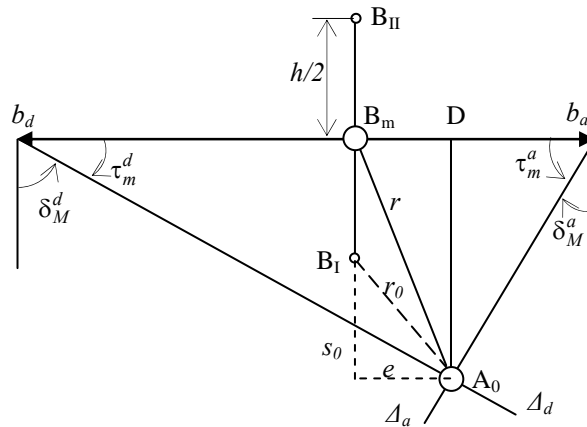


Fig. 2.3. Graphical calculation of the rotation center of the disc-cam with translating follower

From the right angle triangles  $A_0Db_a$ ,  $A_0Db_d$  (fig. 2.3, 2.4) the follower formulas are obtained:

a) Translating follower (fig. 3):

$$e = \frac{s_M'^a \cdot tg\tau_m^a + s_M'^d \cdot tg\tau_m^d}{tg\tau_m^a + tg\tau_m^d}; \quad s_0 = \frac{(s_M'^a - s_M'^d) \cdot tg\tau_m^a \cdot tg\tau_m^d}{tg\tau_m^a + tg\tau_m^d} - \frac{h}{2} \quad (2.4a, 2.5a)$$

b) Oscillating rotating follower (fig. 2.4):

$$e^* = b \cdot \frac{\psi_M'^a \cdot tg\tau_m^a + \psi_M'^d \cdot tg\tau_m^d}{tg\tau_m^a + tg\tau_m^d}; \quad s_0^* = b \cdot \frac{(\psi_M'^a - \psi_M'^d) \cdot tg\tau_m^a \cdot tg\tau_m^d}{tg\tau_m^a + tg\tau_m^d} - b \cdot \sin\left(\frac{\beta}{2}\right) \quad (2.4b, 2.5b)$$

In formulas (2.4) and (2.5), the notations are as follows:

$s_M'^a, \psi_M'^a$  - Maximum reduced velocity ( $B_m b_a$ ) for the ascending movement of the follower ( $>0$ ); for the oscillating follower (fig. 2.4) is considered  $BB_m=0$ .

$s_M'^d, \psi_M'^d$  - Maximum reduced velocity ( $B_m b_d$ ) for the descending movement of the follower ( $<0$ );





### 5. Synthesis of mechanisms with plate cam and flat follower

The mechanism with the translating (fig. 5.1) flat follower and with the oscillating (fig. 5.2) flat follower with the eccentric, respectively are considered.

In the case of the translating flat follower (fig. 5.1), if the line segment AB (tangential to the profile  $\Gamma_1$  curve) is perpendicular to the linear guidance  $\Delta_2$  of the follower 2, the transmission angle  $\tau$  has the optimal value ( $\tau = 90^\circ$ ).

The orthogonal flat follower imposes the geometrical condition that the profile curve  $\Gamma_1$  (both ascending and descending) should have a convex shape, so that the curvature is positive (fig. 5.1).

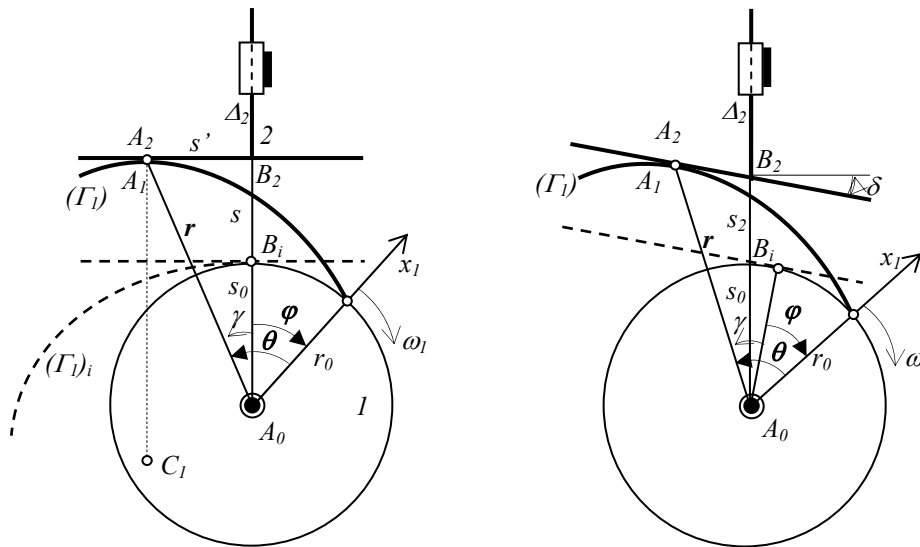


Fig. 5. 1. Kinematic schema of mechanism with disc-cam and the orthogonal / non-orthogonal translating flat follower

The length of the curvature radius is [20]:

$$\rho_{A_1} = A_1C_1 = r_0 + s + s'' > 0 \tag{5.1}$$

The condition for the basis circle radius results from formula (5.1) [4, 11, 19]:

$$r_0 > -(s + s'') \tag{5.1'}$$

The length of the line segment  $A_1B_1$  (fig. 5.1) is equal to the reduced velocity of the flat follower 2:

$$A_1B_1 = s' = ds / d\phi = \dot{s} / \omega_1 \tag{5.2}$$

The polar coordinates  $r$  and  $\theta$  of the ascending profile curve  $\Gamma_{A_1}^a$  are calculated with the formulas:

$$r = \sqrt{(r_0 + s)^2 + (s')^2} ; \theta = \phi + \gamma \tag{5.3, 5.4}$$

In formula (5.4)  $\gamma$  is the de-phasing angle and is calculated with the following formula (fig. 5.1):

$$\gamma = \arctg\left(\frac{s'}{r_0 + s}\right) \tag{5.5}$$

For the descending movement of the flat follower, the polar coordinates are obtained from the formulas:

$$r = \sqrt{(r_0 + h - s)^2 + (s')^2}; \theta = \varphi - \gamma \quad (5.6, 5.7)$$

In the case of the oscillating flat follower (fig. 5.2), if  $A_0D \perp AB$ , the length of the line segment  $AD$  is equal to the projection of the reduced velocity of the point  $A_2$  [13, 20]:

$$AD = v_{A_2}^r \cdot \cos \alpha = \rho \cdot \psi' \cdot \cos \alpha = \rho_0 \cdot \psi' \quad (5.8)$$

where (fig. 5.2):  $\rho = AB_0$ ;  $\rho_0 = AB$ ;  $\text{tg} \alpha = e / \rho_0$ .

For the ascending movement of the oscillating flat follower (fig. 5.2), the polar coordinates are:

$$r = \sqrt{a^2 + \rho^2 - 2a \cdot \rho \cdot \cos(\psi_0 + \psi - \alpha)}; \theta = \varphi + \gamma; \quad (5.9, 5.10)$$

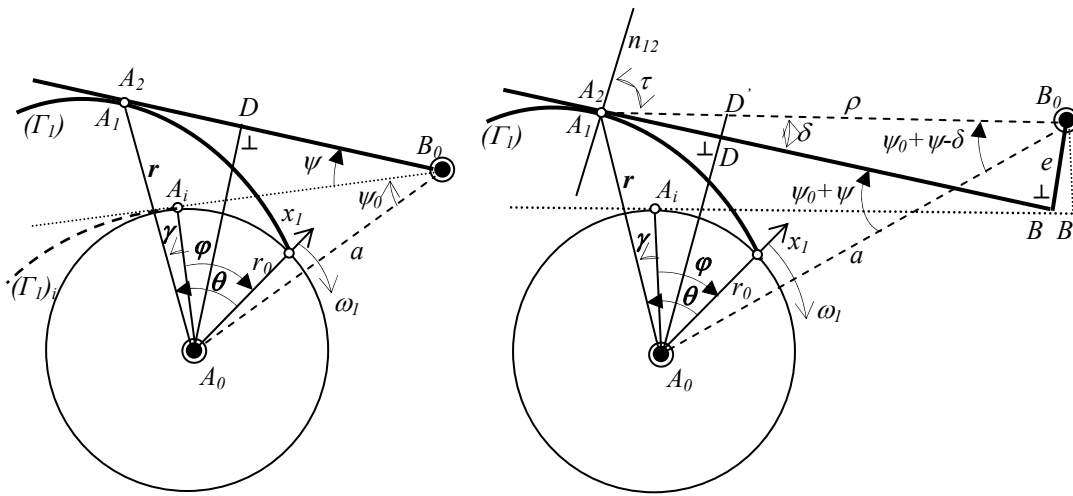


Fig. 5.2. Kinematic schema of mechanism with disc-cam and oscillating flat follower

where (fig. 5.2):

$$\psi_0 = \arcsin\left(\frac{r_0 + e}{a}\right); \rho = \frac{a}{\cos \alpha} \cdot \frac{\cos(\psi_0 + \psi)}{1 - \psi'}; \quad (5.11, 5.12)$$

$$\delta = \arctg\left(\frac{e \cdot (1 - \psi')}{a \cdot \cos(\psi_0 + \psi)}\right); \gamma = \gamma^* - \psi; \quad (5.13, 5.14)$$

$$\gamma^* = \arctg\left(\frac{a}{a - e} \cdot \frac{\psi'}{(1 - \psi') \cdot \text{tg}(\psi_0 + \psi)}\right) \quad (5.15)$$

For the descending movement of the oscillating flat follower, the polar coordinates are:

$$r = \sqrt{a^2 + \rho^2 - 2a \cdot \rho \cdot \cos(\psi_0 + \beta - \psi - \delta)}; \theta = \varphi + \gamma \quad (5.16, 5.17)$$

If it is considered that the eccentricity length  $e$  is opposite to the fixed joint  $B_0$  (fig. 8) then the angle  $\alpha$  changes the sign as can be seen in formula (5.18):

$$r = \sqrt{a^2 + \rho^2 - 2a \cdot \rho \cdot \cos(\psi_0 + \psi + \delta)} \quad (5.18)$$

The optimal case for the mechanism with rotating flat follower is obtained if  $e=0$ ,  $\delta=0$ ,  $\rho=\rho_0$  when the transmission angle is  $\tau=90^\circ$ . In this case the formulas (5.18), (5.10) and (5.15) become:

$$r = \sqrt{a^2 + \rho_0^2 - 2a \cdot \rho_0 \cdot \cos(\psi_0 + \psi)}; \theta = \varphi + \gamma^* - \psi; \quad (5.19, 5.20)$$

$$\gamma^* = \arctg\left(\frac{\psi'}{(1-\psi') \cdot \text{tg}(\psi_0 + \psi)}\right) \quad (5.21)$$

## 6. Numerical example

The synthesis of a mechanism with disc-cam and translating flat follower (fig. 5.1), which is used as distribution system of the internal combustion engine, is considered [11,14,20].

The movement law of the follower is sinusoidal (fig. 6.1), where the functions  $s(\varphi)$ ,  $s'(\varphi)$  and  $s''(\varphi)$  are represented for the ascending displacement of follower 2.

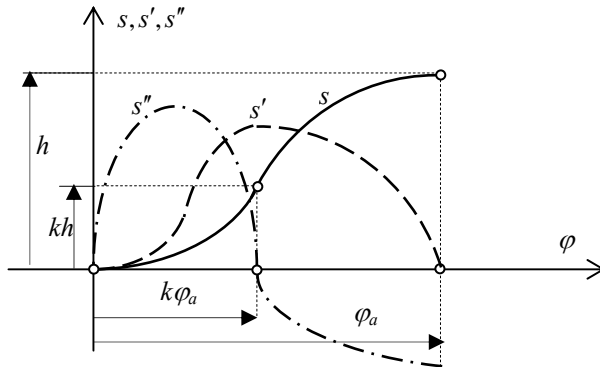


Fig. 6.1. Diagrams of the translating follower movement

For the interval  $\varphi \in [0, k\varphi_a]$  where  $\varphi_a$  is the ascending angle, the movement equations [11, 14, 20] are (fig. 6.1):

$$s = \frac{\pi h}{[4(1-k) + k\pi]} \left( \frac{\varphi}{\varphi_a} - \frac{k}{\pi} \cdot \sin\left(\frac{\pi \cdot \varphi}{k \cdot \varphi_a}\right) \right) \quad (6.1)$$

$$s' = \frac{\pi h}{[4(1-k) + k\pi] \cdot \varphi_a} \left( 1 - \cos\left(\frac{\pi \cdot \varphi}{k \cdot \varphi_a}\right) \right) \quad (6.2)$$

$$s'' = \frac{\pi^2 h}{[4(1-k) + k\pi] \cdot k\varphi_a^2} \cdot \sin\left(\frac{\pi \cdot \varphi}{k \cdot \varphi_a}\right) \quad (6.3)$$

For the interval  $\varphi \in [k\varphi_a, \varphi_a]$ , the movement equations are [11], [14], [20]:

$$s = \frac{\pi h}{[4(1-k) + k\pi]} \left( k + \frac{4(1-k)}{\pi} \cdot \sin\left(\frac{\pi \cdot (\varphi - k\varphi_a)}{2(1-k) \cdot \varphi_a}\right) \right) \quad (6.4)$$

$$s' = \frac{2\pi h}{[4(1-k) + k\pi] \cdot \varphi_a} \cdot \cos\left(\frac{\pi \cdot (\varphi - k\varphi_a)}{2(1-k) \cdot \varphi_a}\right) \quad (6.5)$$

$$s'' = \frac{-\pi^2 h}{[4(1-k) + k\pi] \cdot (1-k) \cdot \varphi_a^2} \cdot \sin\left(\frac{\pi \cdot (\varphi - \varphi_a)}{2(1-k) \cdot \varphi_a}\right) \quad (6.6)$$



For  $s_{\max} - s_{\min} = h = 15.5\text{mm}$ ,  $k = 0.3$  and  $\varphi_a = \varphi_d (=70^\circ)$ ,  $s_{ma}'' = -39.122\text{mm}$  result from equation (6.6). From equation (5.1')  $r_0 > 23.622\text{mm}$  is calculated and  $r_0 = 40\text{mm}$  is chosen [11]. Polar coordinates  $r$  and  $\theta$  of the curve  $\Gamma_1^a$  (fig. 5.1) are calculated with the formulas (5.3, 5.4, 5.5). For the numerical calculus of profile  $\Gamma_1$  of the disc-cam (fig. 6.2) a program has been elaborated.

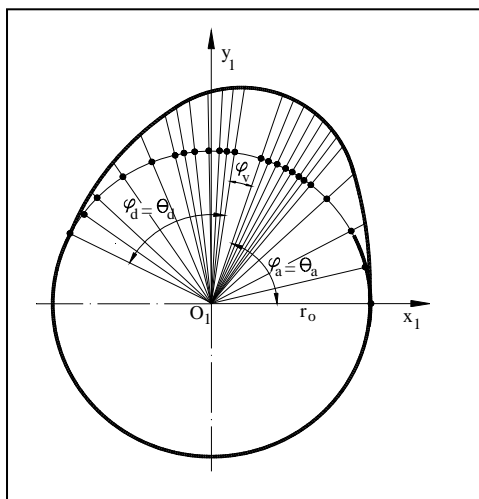


Fig. 6.2. Symmetrical profile of disc-cam

## 7. Conclusions

This paper presents a unitary method for the analytical synthesis of the disc-cam profile, when the movement law is imposed on the follower with tip, roll or flat face.

The method uses the direct movement of the rotary cam, without a reversing movement of the follower around the fixed considered disc-cam.

This unitary method relies on the boundary conditions imposed by the follower using the movement law and the minimum transmission angle or maximum pressure angle. In the case of a roll follower, the theoretical curve of the cam profile is first calculated and then described in polar coordinates.

A phase displacement angle or de-phasing angle was highlighted; it is defined as the difference between the cam rotation angle and the position angle of the vector radius.

All the cases of the synthesis of mechanisms with translating or oscillating follower, with tip, roll or flat follower are considered.

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A unitary method of the geometrical and kinematic synthesis used for the exact design of the profile of the rotary disc-cam and translating or oscillating follower is presented. This method uses the direct movement of the rotary cam, without a reversing movement of the follower around the fixed cam. This unitary method relies on the boundary conditions imposed by the follower using the movement law and the minimum transmission angle or maximum pressure angle. In the case of a roll follower, first the theoretical curve of the cam profile is calculated and then is described in polar coordinates. All cases of mechanisms with translating, oscillating follower, with tip, roll or flat follower are considered. A phase displacement angle or de-phasing angle is highlighted. In the mobile system that is stiffened by the cam, it is defined as the difference between the cam rotation angle and the position angle of the vector radius.

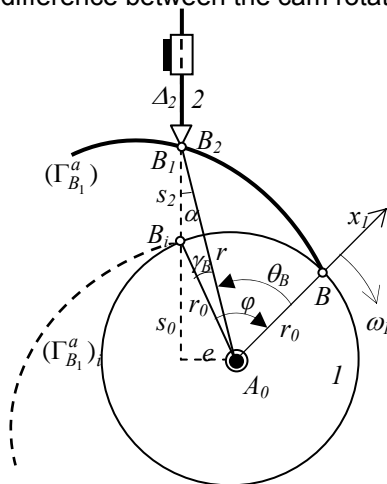


Fig. 2.1. Kinematic schema of mechanism with disc-cam and translating tip / roller follower

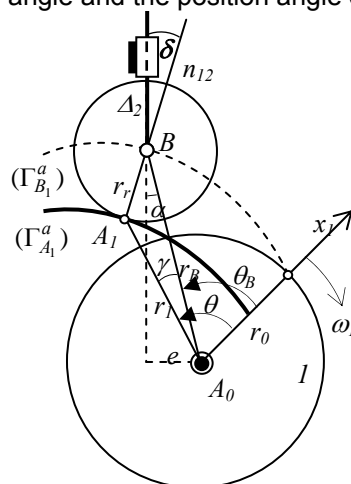


Fig. 4. 2. Geometrical schema for calculating the real profile  $\Gamma_{A_1}^a$  with oscillating follower

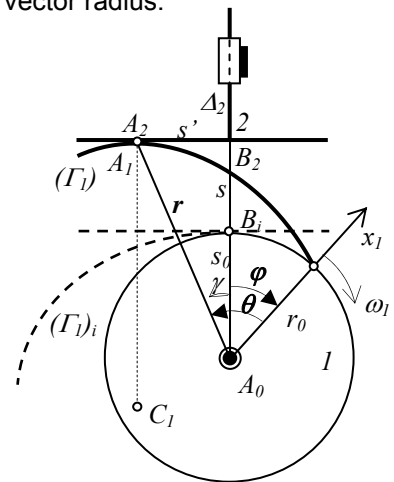


Fig. 5.1. Orthogonal translating flat follower

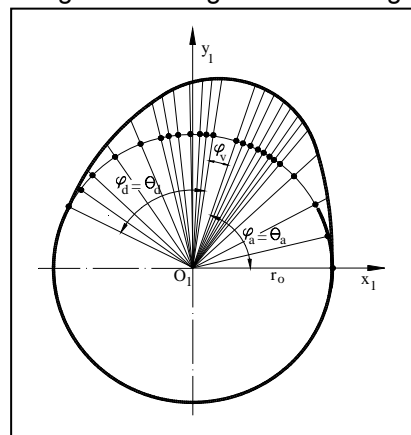


Fig. 6.2. Symmetrical profile of disc-cam

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