

## THE INFLUENCE OF THE BRIDGE SKEWNESS ON THE DYNAMIC RESPONSE TO THE ACTION OF MOBILE LOADINGS

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**Abstract:** The article presents the results of a study related to the influence of the structure skewness on the dynamic response to the action of mobile loadings. The studied structure is a railway bridge with the following structure: (i) superstructure, made up of 9 prefabricated girders, prestressed with adherent chords; (ii) infrastructures of massive abutment-type, founded directly on building-foundations. The deck skewness is  $52^{\circ}$ .

### 1. INTRODUCTION

The study related to the structures dynamic response to the convoys' action is made up of two stages, namely:

**Study stage I** consisting in determining the structures' own vibration characteristics, by retaining the bending-type vibrations on a vertical direction with waves throughout the length of the bridge. This study was performed by modeling the structures with finite elements in a spatial assembly by using general software.

**Study stage II** consisting of determining the dynamic coefficient. To this purpose, the first two authors designed a series of specific calculation programs performing both the necessary calculations and the graphic representations of the results.

### 2. THE STRUCTURE'S OWN VIBRATION CHARACTERISTICS

#### 2.1. Representation of the structure

The transversal section is made up of 9 prefabricated girders, prestressed with adherent chords  $L=16.00\text{m}$ . The girders measure  $0.80\text{m}$  in height, (Fig. 1).

The calculation span measures  $15.50\text{m}$ . The floor skewness is  $52^{\circ}$ . The overslab varies between  $12$  and  $20$  cm in thickness. The bridge asphalt layers have a total thickness of  $11$  cm. The infra-structures are of massive abutment-type, being grounded directly on concrete-foundations. The girders offer support on infra-structures via the neoprene bearing devices. The design class is E.

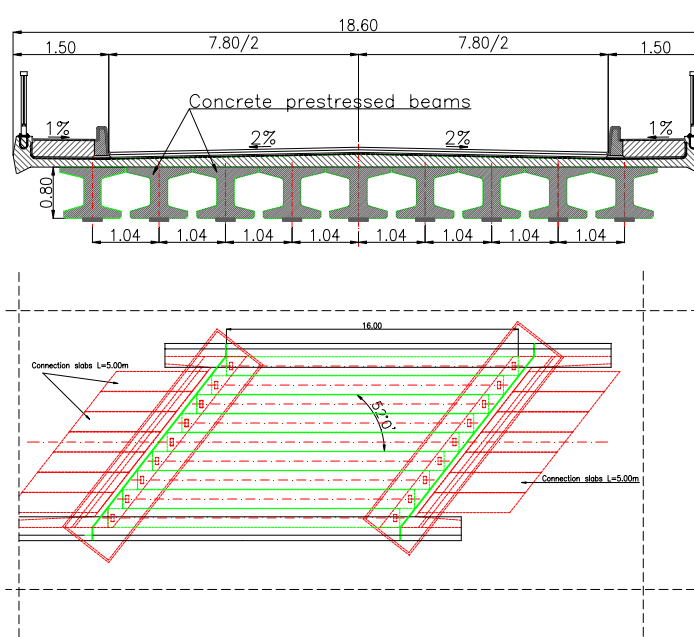


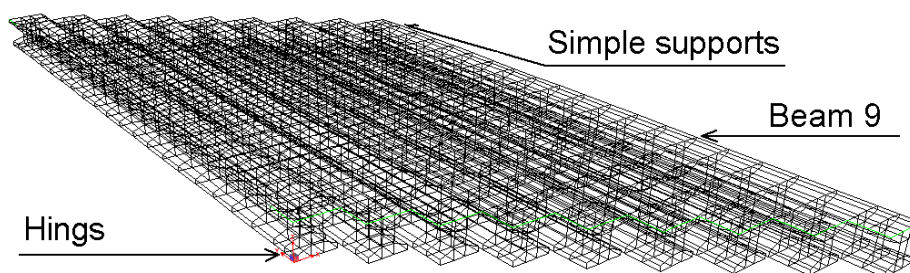
Fig. 1: Transversal section. Plan

## 2.2. Calculation model

The calculation model comprises only the super-structure, as the infra-structures are rigid and the foundation ground is rigid. Taking into account the transversal section structure, the supra-structure was modeled with tri- and bi-dimensional finite elements assembled in a spatial model [4, 6].

Hence, the pre-stressed concrete girders were modeled with 8-nods tri-dimensional finite elements. The girder's joints simulate a spatial articulated bearing at one end and a simple bearing – with a transversal joint crossing the bridge – at the other end. The overslab was modeled with bi-dimensional finite elements with constant equivalent thickness. The connections between the 9 girders and between them and the slab are spatial articulations-type. The girders endings are united by a girder modeled with one-dimensional finite elements.

To point out the skewness influence, two models were realized: (i) the one following the girders' skewness « SKEW », (Fig. 2); (ii) and one where the endings of the girders make up a perpendicular section with girders – « STRAIGHT ».



**Fig. 2: Calculation model – « SKEW »**

The calculation model numbers 3312 nods, 1386 finite tri-dimensional elements, 594 bi-dimensional finite elements, and 92 one-dimensional finite elements. The number of equations is 10452. The loading for the dynamic study comes from the mass of resistance elements and of non-structural elements, as well as 20% of the calculation convoy mass. The structure mass is 216.533 kNs<sup>2</sup>/m.

The calculation models with tri-dimensional elements were checked by comparing the static displacements obtained on these models with the one obtained on a model with one-dimensional equivalent elements.

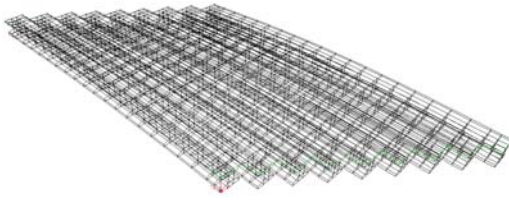
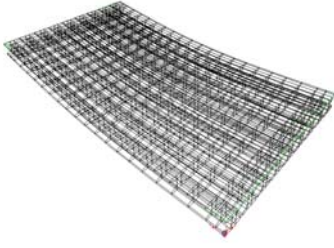
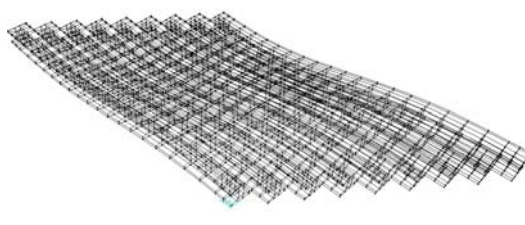
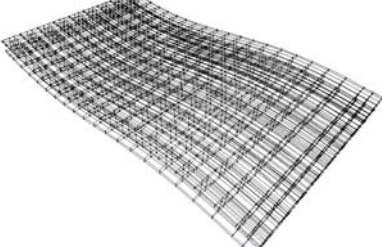
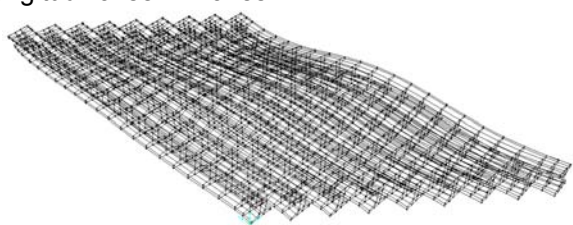
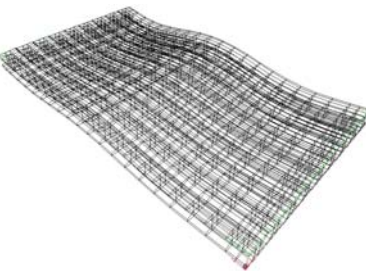
## 2.3. The structure's own dynamic response

To engage almost 90% of the modal mass of the structure, 15 own vibration modes were necessary. Thus, transversally to the bridge, the engaged modal mass is 88.24%, longitudinally – 89.88%, while on the vertical direction the engaged modal mass is 82.09%.

For the study of the dynamic response to the action of mobile loadings one needs the own vibration modes of bending-type on vertical plan with waves throughout the length of the bridge. From the study of the two models the following observations can be made, resumed in table 1. The fundamental mode is of bending-type on vertical plan, in the case of both calculation models. Up to mode 7, the own vibration vectors are of the same type. Starting with mode 8, the own vibration vectors, the own vibration vectors differ from one model to the next. The vibration of bending-type on vertical plan with semi-waves

throughout the length of the bridge is manifested in modes 1, 5, and 14 in case of the « SKEW » model and in modes 1, 5, and 15 in case of the « STRAIGHT » model.

Table 1

SKEW	STRAIGHT
<p><b>Module 1</b> (<math>T_1=0.2072s</math>; <math>\omega_1^O=30.32rad/s</math>): is the basic vibration mode. The vibration form is of bending-type on vertical plan with longitudinal semi-wave.</p> 	<p><b>Module 1</b> (<math>T_1=0.2405s</math>; <math>\omega_1^D=26.13rad/s</math>) the same vibration type as in the case of the SKEW model.</p> 
<p><b>Module 2</b> (<math>T_2=0.1504s</math>): the vibration form is of torsion-type compared to the longitudinal symmetry axis.</p>	<p><b>Module 2</b> (<math>T_2=0.1589s</math>): the same vibration type as in the case of the SKEW model.</p>
<p><b>Module 3</b> (<math>T_3=0.1066s</math>): the vibration is of vertical bending-type, transversal to the bridge with a single semi-wave on the whole supra-structure.</p>	<p><b>Module 3</b> (<math>T_3=0.1087s</math>): the same vibration type as in the case of the SKEW model.</p>
<p><b>Module 5</b> (<math>T_5=0.0643s</math>; <math>\omega_5^O=97.72rad/s</math>): the own form is of a bending on vertical plan with two semi-waves in the longitudinal direction of the bridge.</p> 	<p><b>Module 5</b> (<math>T_5=0.0651s</math>; <math>\omega_5^D=96.52rad/s</math>) the same vibration type as in the case of the SKEW model.</p> 
<p><b>Module 14</b> (<math>T_{14}=0.0283s</math>; <math>\omega_{14}^O=222.02rad/s</math>) is a vibration of vertical bending-type with three longitudinal semi-waves.</p> 	<p><b>Module 15</b> (<math>T_{15}=0.0267 s</math>; <math>\omega_{15}^D=235.33rad/s</math>): is a vibration of vertical bending-type with three longitudinal semi-waves.</p> 

The examination of the **dynamic response** of the « SKEW » model, leads to the following comments: (i) the value of the vibration period situates the studied structure among those with rigid behaviour; (ii) the skewness triggers an increase in the dynamic rigidity of the basic vibration mode by 12%; (iii) the time period of the torsion mode is 27% smaller than the period of the basic mode; (iv) the period of mode 3 is already half the basic period; (v) the period of mode 15 is 10 times smaller than the basic one; (vi) the superior modes of bending-type on vertical plan represents 31% and, respectively, 14% from the first vibration mode of the same type.

### 3. THE CALCULATION OF THE DYNAMIC COEFFICIENT

#### 3.1. Theoretical approach

In order to determine the dynamic coefficient, the first two authors designed a series of calculation programs for different models of the mobile loading. In the study presented in this article, two of these programs will be used for two models for the mobile loading.

**The first modeling** is the one in which the vehicle is represented by a unitary concentrated force. Five traveling speeds of the vehicle are taken into consideration, namely:  $v_0 = 5$  km/h, speed at which one considers the static deformation is obtained,  $v_1 = 60$  km/h,  $v_2 = 90$  km/h,  $v_3 = 120$  km/h, and  $v_4 = 160$  km/h. The structure is modeled as a girder rested simply for which there can be taken into account the first three own vibration forms of bending-type on vertical plan with waves throughout the length of the bridge. The program named „LID 2” is made up of two modules, namely (1) LID2 – the calculation program; (2) DESLIDM – the drawing program.

The program allows the calculation of the static and dynamic deformations in three sections of the girder  $\left(\frac{1}{3}l, \frac{1}{2}l, \frac{2}{3}l\right)$  for the 200 positions of the P force, as well as the calculation of the dynamic coefficient in these 200 positions, indicating its maximum value, the position of the section where this value is developing and of the corresponding section where the force acts.

By this study one can showcase the influence of the structure's superior vibration modes on its dynamic response to the action of the mobile loadings.

**The second modeling** of the girder-vehicle system has the following characteristics (Fig. 3): a) the girder is simply supported. The traveling surface is modeled without or with unevenness. Structural damping can also be introduced. The dynamic characteristics of the structure are introduced by the pulsation of the first vibration mode on bending-type on vertical plan. The numerical calculation is performed only for the section in the center of the girder; b) the vehicle is unique, modeled as a system with a single degree of freedom, comprising a suspended mass and an unsuspended one, the elastic characteristic and the damping characteristic. The unsuspended mass of the vehicle stays in permanent contact with the traveling surface.

The mathematical method is the step-by-step integration, by using central differences. The integration step is

$\frac{1}{400}t$  (where  $t$  is the time necessary for the

vehicle to travel at  $v$  speed, the span girder  $l$ ). The vehicle traveling speeds are the same with the ones in the previous modeling.

The calculation program called „OSIE 1” is made up of three modules, namely: (1) INDOS 1 - program for data uploading; (2) CALCOS 1 – calculation program; (3) DESOS 1 – drawing program [2].

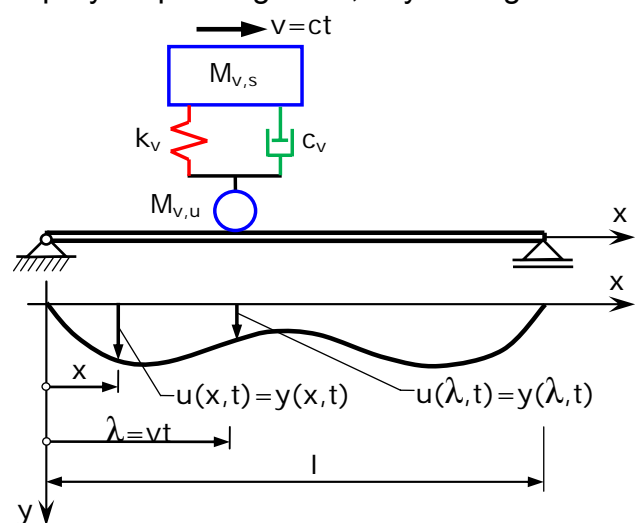


Fig. 3. The scheme for the calculation of the dynamic coefficient in the OSIE1 program

The dynamic coefficient is obtained as a ratio between the value of the maximum dynamic traveling and the value of the maximum static traveling – traveling of the central section of the girder.

### 3.2. The influence of the superior vibration modes of bending-type on vertical plan on the dynamic response under the action of mobile loadings

The data necessary for the study is:

- for the structure: the calculation span  $l_c=15.5$  m; mass  $M=21.65$  t, the own pulsation of the first three own vibration modes of bending-type on vertical plan  $\omega_1^O=30.32$  rad/s,  $\omega_2^O=97.72$  rad/s,  $\omega_3^O=222.02$  rad/s for the «SKEW» model and  $\omega_1^D=26.13$  rad/s,  $\omega_2^D=96.52$  rad/s,  $\omega_3^D=235.33$  rad/s for the «STRAIGHT» model.
- for the vehicle: the unitary concentrated force.

Fig. 4 gives an example of the manner in which the calculation program and the drawing one present the results of the study.

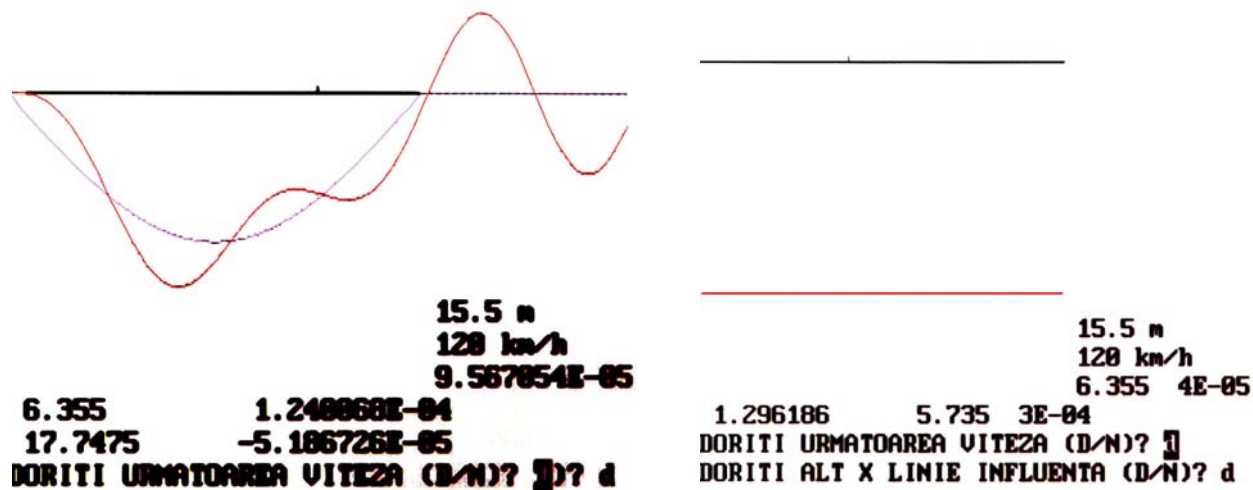


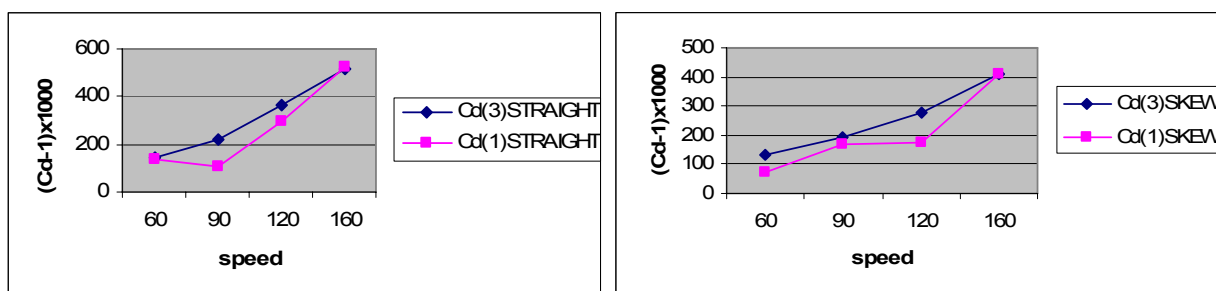
Fig. 4. The manner of presenting the results in the „LIED-2” program

An additional inscription will be made, namely: **Cd(3)** = the dynamic coefficient corresponding to the situation, when three own vibration modes of bending-type on vertical plan are taken into consideration and with **Cd(1)** = when only the first vibration characteristic of bending-type on vertical plan is taken into account.

Table 2

Speed (km/h)	<b>Cd(3)<sub>SKEW</sub></b>	<b>Cd(1)<sub>SKEW</sub></b>	<b>Cd(3)<sub>SKEW</sub> / Cd(1)<sub>SKEW</sub></b>	%
60	1.132	1.071	1.057	(+) 5.7
90	1.191	1.169	1.019	(+) 1.9
120	1.278	1.172	1.090	(+) 9.0
160	1.410	1.410	1.000	0
Speed (km/h)	<b>Cd(3)<sub>STRAIGHT</sub></b>	<b>Cd(1)<sub>STRAIGHT</sub></b>	<b>Cd(3)<sub>STRAIGHT</sub> / Cd(1)<sub>STRAIGHT</sub></b>	%
60	1.147	1.138	1.008	(+) 0.8
90	1.220	1.106	1.103	(+) 10.3
120	1.364	1.296	1.052	(+) 5.2
160	1.520	1.525	0.997	(-) 0.3

The variation of the dynamic coefficient is presented in the graphics below. To have a better view, the values of the dynamic coefficient were processed according with the formula  $(C_d - 1) \times 1000$ .



From the study of the values there result the following:

1. The influence of the superior modes varies from 0 % to 9 % for the SKEW mode and from 0 to 10.3 % for the STRAIGHT mode. One can observe that the sensitivity of the SKEW mode is at the speed of 120 km/h, while the one for the STRAIGHT mode is at 90 km/h. There results that, for the SKEW model, the influence of the own vibration modes come into work later than in the case of the STRAIGHT mode.
2. A completely special case is represented by the speed of 160 km/h for which, in fact, from the point of view of this structure (both for the STRAIGHT model and for the SKEW one) there is no influence of the vibrations from the superior modes. This shows that during the period needed for the vehicle to cross the bridge at this speed, the superior modes do not come into action.
3. The values of the dynamic coefficients for the SKEW model are smaller than the ones for the dynamic coefficients related to the STRAIGHT model with one exception, namely the value corresponding to the STRAIGHT model at the traveling speed of 90 km/h and the case when only the first pulsation is taken into consideration.

### 3.3. The skewness's influence on the dynamic coefficient

The data necessary for the study is:

- for the structure: the calculation span  $l_c=15.5$  m; mass  $M=21.65$  t, the own pulsation of the first vibration mode of bending-type on vertical plan  $\omega_1^O=30.32$  rad/s for the « SKEW » model and  $\omega_1^D=26.13$  rad/s for the « STRAIGHT » model. From a previous study made by the authors in relation with the structural damping, there resulted that for the study at loadings from convoys the fraction from the critical damping for bridges made up of prefabricated girders should be 0.04.
- for the vehicle: the suspended mass = 2.1 t; unsuspended mass = 0.21 t; resort rigidity = 290 tf/m; resort damping= 4.2 tf.s/m.

Fig. 5 gives an example of the manner in which the calculation and drawing program presents the study results. In table 3 there are presented the primary results of the study.

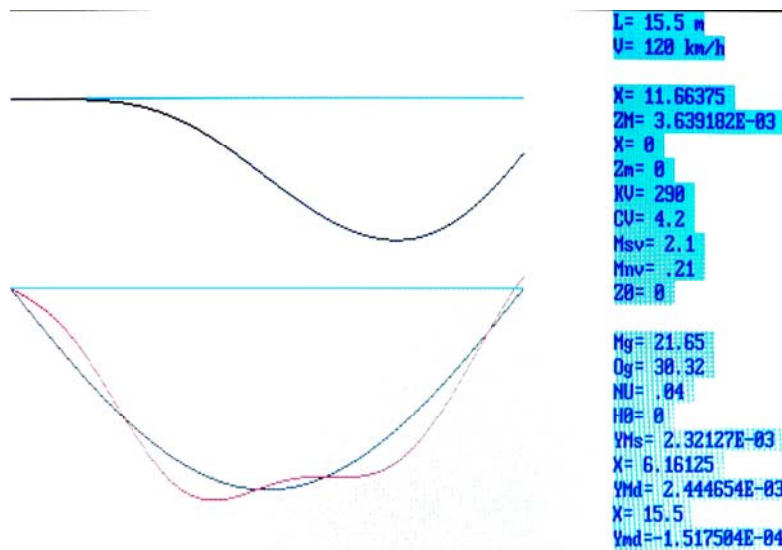
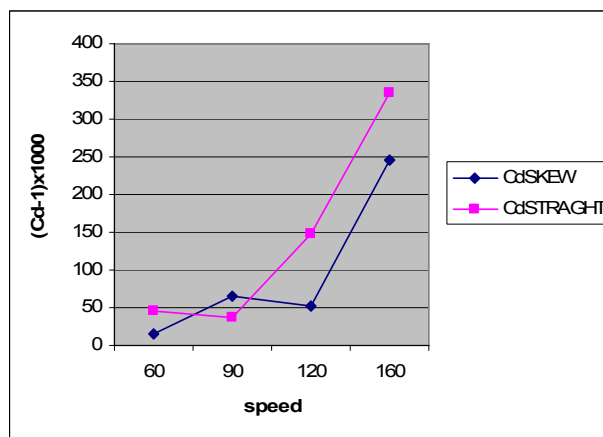


Fig. 5 The manner of presenting the results in the „OSIE 1” program

Table 3 Primary results

S K E W	Static arrow (m) = 2.32127 E-3		
	Speed (km/h)	Dynamic arrow (m)	Dynamic coefficient $Cd_{SKEW}$
	60	2.358010 E-3	1.016
	90	2.474339 E-3	1.066
	120	2.444654 E-3	1.053
160	2.892622 E-3	1.246	
S T R A I G H T	Static arrow (m) = 3.125398 E-3		
	Speed (km/h)	Dynamic arrow (m)	Dynamic coefficient $Cd_{STRAIGHT}$
	60	3.269940 E-3	1.046
	90	3.244816 E-3	1.038
	120	3.583418 E-3	1.147
160	4.169083 E-3	1.334	



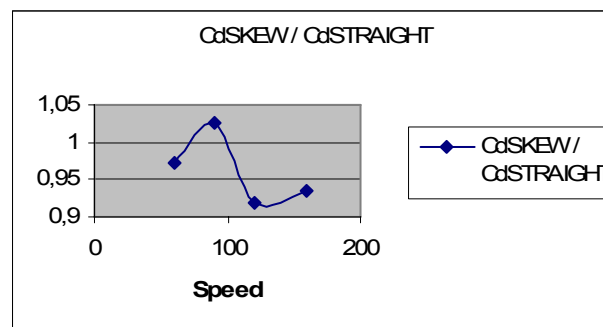
Following the study of the obtained values, the comments below can be made:

1. The ratio of the static arrows under the action of the mobile loadings shows that the SKEW model is 29% more rigid than the STRAIGHT model.

2. These two models have different behaviours at a speed of 90 km/h. The STRAIGHT model registers a decrease in the value of the dynamic coefficient, while the SKEW model records a slight increase of it.
3. The biggest difference between the values of the dynamic coefficients is at a speed of 120 km/h (– 8.2 %), table 4.

Table 4

Speed (km/h)	$\frac{C_{d_{SKEW}}}{C_{d_{STRAIGHT}}}$	%
60	0.971	(-) 2.9% more rigid
90	1.026	(+) 2.6 % more flexible
120	0.918	(-) 8.2% more rigid
160	0.934	(-) 6.6% more rigid



#### 4. CONCLUSIONS

1. From the point of view of the own dynamic response, the structure is classified in a rigid behaviour. The sensitivity of the structure is of bending-type on vertical direction.
2. The skewness triggers an increase in the rigidity of the fundamental vibration mode as around 12%.
3. For the SKEW model, the influence of the own superior vibration modes of bending-type on vertical plan is minimal at 160 km/h – 0% and maximal at a speed of 120 km/h – around 9 %.
4. From the point of view of the response to mobile loadings, the behaviour of the SKEW model is more rigid than the one of the STRAIGHT model – maximum 8%.
5. It is necessary to determine the dynamic coefficients customized to each particular structure and speed.

#### REFERENCES

- [1] Biggs M. J. (1964) – *Introduction to Structural dynamics*, Massachusetts Institute of Technology
- [2] Bucur C., Bucur M. (1994) – Program de calcul și desen “OSIE1” – Buletinul Științific al UTCB nr. 1 – 1994 pp. 75-85
- [3] Bucur C., Bucur V. (2005) - *Railway Dislevelments Influence on the Dynamic Response of Bridges - Reliability and Performance on Bridge & Transportation Infrastructure Engineering* – Ed. Societății „Matei-Teiu Botez”, Iași, Editori C. Ionescu, F. Paulet-Crainiceanu, R. Andrei, ISBN 973-7962-57-5 – pag. 28-39
- [4] Bucur C, Moise I.D. (2006) - The spectral response of a railroad bridge with high skew – ROMANIA – International Conference on Bridges / Dubrovnik, Croatia 2006, „Bridges” Proceedings Secon HDGH2006: lucrarea 36/Sesiunea 2/pag. 375-382, ISBN 953-95428-0-4
- [5] Capatu C. – *Poduri din beton precomprimat* – Ed. Tehnică, București, 1983
- [6] Hambly E.C. - *Bridge Deck Behaviour*, CHAPMAN & HALL, 1976
- [7] Jenkins D. - *Bridge Deck Behaviour Revisited* - BSc MEngSci MIEAust MICE  
[http://www.interactiveds.com.au/BRIDGE\\_DECK\\_BEHAVIOUR.doc](http://www.interactiveds.com.au/BRIDGE_DECK_BEHAVIOUR.doc)
- [8] Radu P.I., E. Negoescu, P. Ionescu – *Poduri din beton armat* – Ed. Didactica și Pedagogică, București 1981
- [9] (1995-1998) Contract, responsabil C. Bucur - *Cercetări privind răspunsul structurilor de poduri de cale ferată la acțiunea dinamică a convoaielor* – Ministerul Cercetării și Tehnologiei

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