MATHCAD MEDIUM`S APLICATIONS IN AXIAL GAS TURBINE DESIGN

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Abstract: A complete set of simple formulas for designing a gas turbine are provided. After assumptions for numerical values a MathCad program was developed for solving formulas. A modelling of the designed rotor was made using ANSYS, a general purpose finite element modelling package.

1. A THEORETICAL APPROACH OF THE DESIGN AND EVALUATION OF A GAS TURBINE ROTOR

The aim of the paper is to design an axial turbine, with reaction, able to realize a shaft power P=10.887W, having an efficiency $\eta'_u \geq 0.88$ using Mathcad.

Assumptions for numerical values:

- The gas flow capacity: 61,3kg/s.
- The status parameters at the turbine entry:

  $T'3 := 1100$ K, $p'3 := 3.595 \cdot 10^5$ Pa, $c'3 := 196$ m/s.
- The coefficient of air excess: $\lambda := 3.866$.
- Turbine shaft speed : $n := 8886$ RPM

Chosen constructive solutions for the turbine's stage: diameters between 622mm and 886 mm, the diameter of the average fibre being 754mm.

$$D := \begin{pmatrix} 0.622 \\ 0.688 \\ 0.754 \\ 0.820 \\ 0.886 \end{pmatrix} \text{m}$$

It results the specific shaft work: $lu := 176.7 \cdot 10^3$ J/kg, value that is realised in one stage.

According to the air excess $\lambda := 3.866$, it results:

$i'3 := 1184 \cdot 10^3$ J/kg $i'4 := 1008 \cdot 10^3$ J/kg $T'4 := 935$ K

The average exponent of the isentropic evolution is: $\chi := 1.364$.

The coefficient of gas output velocity, λ_4, depends on the destination (application) of the turbine, influence of turbine losses, mechanical stress, etc.

We have chosen: $\lambda 4 := 0.6$

It results the gasses evacuation speed:

$$c4 := \lambda 4 \cdot \sqrt{2 \cdot i'4 \cdot \frac{\chi - 1}{\chi + 1}} \text{ m/s} \qquad c4 = 334.29 \text{ m/s}$$

Tangential speed:

$$u := \frac{\pi \cdot D \cdot n}{60} \text{ m/s} \qquad u = \begin{pmatrix} 289.398 \\ 320.106 \\ 350.813 \\ 381.521 \\ 412.229 \end{pmatrix} \text{ m/s}$$

From the mechanical's work expression $lu := u \cdot (c3'u + c4'u)$

It results:

$$\frac{lu}{u} = \begin{pmatrix} 610.578 \\ 552.005 \\ 503.686 \\ 463.146 \\ 428.645 \end{pmatrix} \text{ m/s}$$

Maximum efficiency η'_u is reached if $c_{4u}=0$.

In this case

$$c3'u = \begin{pmatrix} 610.578 \\ 552.005 \\ 503.686 \\ 463.146 \\ 428.645 \end{pmatrix} \text{ m/s.}$$

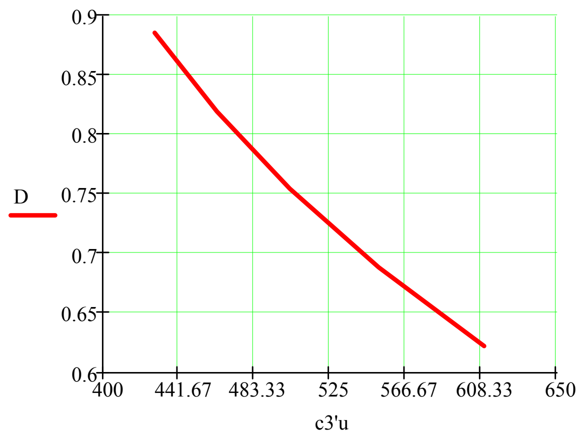

Fig.1. "D" values in dependences of c3'u

As $\alpha_{3'} \in (16,35)^\circ$, small rates meets the case for action turbines, the big ones to large fluid flow capacities. We have chosen $\alpha 3' := 28deg$ $(\cos(28deg) = 0.883)$.

It results:

$$c3' := \frac{c3'u}{\cos(\alpha 3')} \text{ m/s}$$

$$c3' = \begin{pmatrix} 691.522 \\ 625.185 \\ 570.46 \\ 524.545 \\ 485.471 \end{pmatrix} \text{ m/s.}$$

We estimate the loss coefficient in the stator φ, φ=0,97. The corresponding rate of the speed coefficient $\lambda_{c3'}$ at critical regime became:

$$\lambda c3'cr := \phi \cdot \left[1 - \frac{(\chi - 1) \cdot (1 - \phi^2)}{2} \right]$$

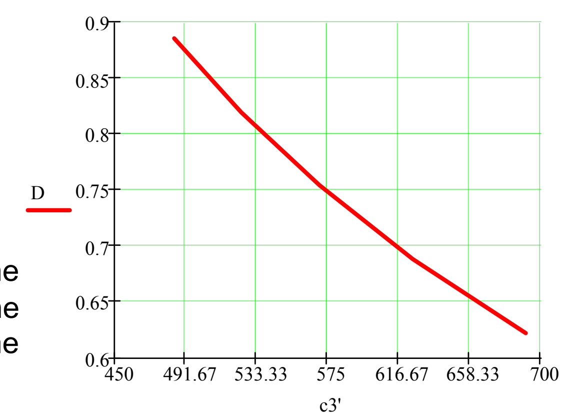

Fig.2. "D" values in dependences of c3'

$\lambda c3'cr = 0.96$

The pressure losses in the stator are estimated with the coefficient:

$$\sigma pf := \frac{\pi \cdot \left(\frac{\lambda c3'}{\phi}\right)}{\pi \cdot (\lambda c3')}$$

$\sigma pf = 1.031$

$\lambda c3' := 0.94$.

We can assign the global geometry of the represented turbine by the areas of the turbine's sections $A_{3'}$ and A_4, respective to the blade heights h_3, $h_{3'}$ and h_4. We use the general expression for flow capacity:

$\dot{M} = 0.0396 \frac{p'}{\sqrt{T'}} Aq(\lambda)\sin\alpha$ with applied to fundamental sections leads to the rates:

$$A3' := \frac{M'}{0.0396 \cdot \sin(\alpha 3') \cdot \frac{p'3}{\sqrt{T'3}}} \, m^2$$

$A3' = 0.304\, m^2$; $A3 = 0.287\, m^2$;

$A4 = 0.323\, m^2$.

We calculate from the expression:

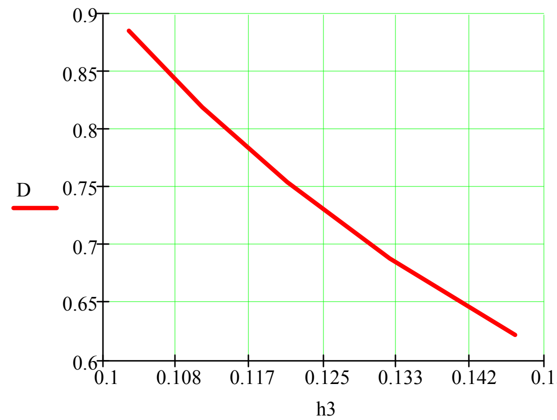

Fig.3. "D" values in dependences of h3

$$h := \frac{A}{\pi \cdot D} \qquad h3' := \frac{A3'}{\pi \cdot D} \qquad h3 := \frac{A3}{\pi \cdot D} \qquad h4 := \frac{A4}{\pi \cdot D}$$

$$h3' = \begin{pmatrix} 0.156 \\ 0.141 \\ 0.128 \\ 0.118 \\ 0.109 \end{pmatrix} m \qquad h3 = \begin{pmatrix} 0.147 \\ 0.133 \\ 0.121 \\ 0.111 \\ 0.103 \end{pmatrix} m \qquad h4 = \begin{pmatrix} 0.165 \\ 0.149 \\ 0.136 \\ 0.125 \\ 0.116 \end{pmatrix} m$$

It results a small divergence of the flow channel, in admissible limits, avoiding the appearance of gas separation. In this case, the divergence angle of the flow channel is smaller than 5º.

The thermodynamic calculus of the turbine assumes the determination of the principal thermodynamic parameters of hot gases in sections 3-3 şi 4-4, considering that:

$$p = p'\pi(\lambda); \; T = T'\Theta(\lambda); \; \rho = \frac{p}{R'T}$$

It results:

$p3 = 3.39 \times 10^5$ Pa; $T3 = 1.083 \times 10^3$ K; $\rho_3 = 562.739$ kg/m³;

$p4 = 2.251 \times 10^5$ Pa; $T4 = 920.975$ K; $\rho 4 = 439.524$ kg/m³.

For the above state parameter values, we assign the following expanding degree of gas:

$\delta T := \frac{p3}{p4}$ $\delta T = 1.506$; $\delta' T := \frac{p'3}{p'4}$ $\delta' T = 3.011$

In section 3'- 3', where $c3' := 570.44$ m/s, $\alpha 3' := 28deg$, we obtain:

$c3'u := c3' \cdot \cos(\alpha 3')$ $c3'u = 503.669$ m/s;

$c3'a := c3' \cdot \sin(\alpha 3')$ $c3'a = 267.805$ m/s;

$w3'u := c3'u - u$

$$w3'u = \begin{pmatrix} 214.271 \\ 183.563 \\ 152.855 \\ 122.147 \\ 91.44 \end{pmatrix} \text{ m/s;}$$

$w3'a := c3'a$

$w3'a = 267.805$ m/s;

$$w3' = \begin{pmatrix} 281.042 \\ 250.334 \\ 219.627 \\ 188.919 \\ 158.211 \end{pmatrix} \text{ m/s;}$$

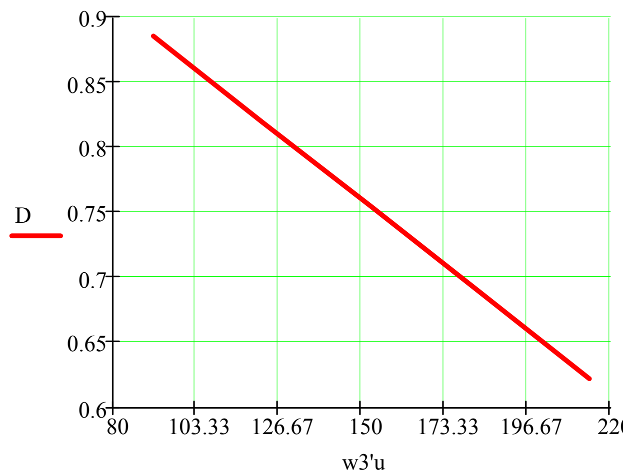

Fig.4. "D" values in dependences of w3'u

$$\beta 3' := \operatorname{atan}\left(\frac{w3'a}{w3'u}\right); \qquad \beta 3' = \begin{pmatrix} 0.896 \\ 0.97 \\ 1.052 \\ 1.143 \\ 1.242 \end{pmatrix}$$

In section 4-4, where c_4=334,3m/s and α_4=90, it results:

$c4u := c4 \cdot \cos(\alpha 4)$ m/s $c4u = 0$ m/s;

$c4a := c4 \cdot \sin(\alpha 4)$ m/s;

$c4a = 334.29$ m/s;

$w4u := c4u + u$

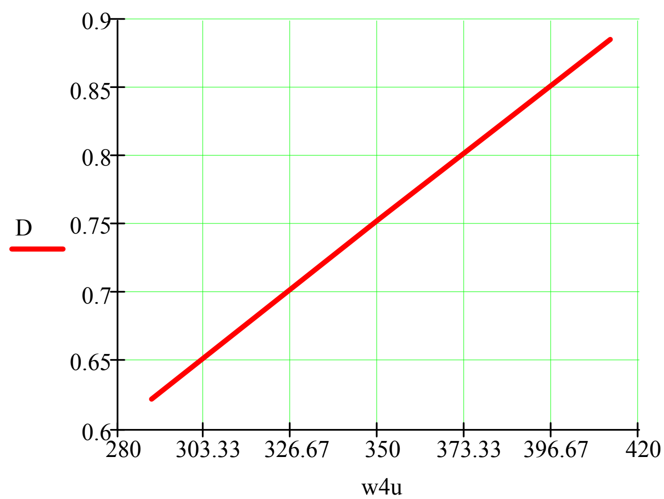

Fig.5. "D" values in dependences of w4u

$$w4u = \begin{pmatrix} 289.398 \\ 320.106 \\ 350.813 \\ 381.521 \\ 412.229 \end{pmatrix}$$ m/s;

$$w4a := c4a$$;

$$w4a = 334.29$$ m/s;

$$w4 = \begin{pmatrix} 623.688 \\ 654.396 \\ 685.104 \\ 715.811 \\ 746.519 \end{pmatrix}$$ m/s;

$$\beta 4 := \operatorname{atan}\left(\left(\frac{w4a}{w4u}\right)\right) \qquad \beta 4 = \begin{pmatrix} 0.857 \\ 0.807 \\ 0.761 \\ 0.72 \\ 0.681 \end{pmatrix}$$;

The thermodynamic parameters in relative move, became, considering that

$$i'_w = i + \frac{w^2}{2}; \qquad p'_w = p_w\left(\frac{T'_w}{T_w}\right)^{\frac{\chi}{\chi-1}}$$

In section 3'- 3':

$$i'w3' := i'3 + \frac{w3'^2}{2} \qquad i'w3' = \begin{pmatrix} 1.223\times 10^6 \\ 1.215\times 10^6 \\ 1.208\times 10^6 \\ 1.202\times 10^6 \\ 1.197\times 10^6 \end{pmatrix}$$ kJ/kg;

$$p'w3' := p'3\cdot\left(\frac{T'3}{T3}\right)^{\frac{\chi}{\chi-1}} \qquad p'w3' = 3.804\times 10^5$$ Pa;

And in section 4-4:

$$i'w4 := i'4 + \frac{w4^2}{2} \qquad i'w4 = \begin{pmatrix} 1.202 \times 10^6 \\ 1.222 \times 10^6 \\ 1.243 \times 10^6 \\ 1.264 \times 10^6 \\ 1.287 \times 10^6 \end{pmatrix} \text{ kJ/kg;}$$

$(T'w4 := 993.84$ K);

$$p'w4 := p'4 \cdot \left(\frac{T'w4}{T4}\right)^{\frac{\chi}{\chi-1}} \qquad p'w4 = 1.588 \times 10^5 \text{ Pa.}$$

It results a total pressure loss coefficient in the mobile blades network:

$$\sigma'pm := \frac{p'w4}{p'w3'} \qquad \sigma'pm = 0.417;$$

Based on the above results, we can assign the fundamental parameters of the turbine, the reaction degree ρ_T, the charge coefficient ι_u, the relative efficiency of the turbine without using the output gas velocity, η_u.

The reaction degree:

$$\rho T := \frac{\Delta ipmed}{lu}$$

where:

$$\Delta ipmed := \frac{1}{2} \cdot \left[\left(\frac{w4}{\psi}\right)^2 - w3'^2\right]$$

$$\Delta ipmed = \begin{pmatrix} 1.76 \times 10^5 \\ 2.059 \times 10^5 \\ 2.359 \times 10^5 \\ 2.66 \times 10^5 \\ 2.962 \times 10^5 \end{pmatrix} \text{ kJ/kg}$$

Fig.6. "D" values in dependences of i'w3'

Where we estimated:

$$\psi := 0.95$$

$$lu := i'3 \cdot \left[1 - \left(\frac{p4}{p3}\right)^{\frac{\chi-1}{\chi}}\right]$$

$$lu = 1.226 \times 10^5 \text{ J/kg}$$

It results:

$$\rho T = \begin{pmatrix} 1.436 \\ 1.679 \\ 1.924 \\ 2.17 \\ 2.416 \end{pmatrix}$$

The charge coefficient became: $\iota u := \dfrac{lu}{u^2}$ $\qquad \iota u = \begin{pmatrix} 1.464 \\ 1.197 \\ 0.996 \\ 0.842 \\ 0.722 \end{pmatrix}$

small rate because of the axial outlet of the gases.

The relative efficiency of the turbine

$$\eta u := 2 \cdot \frac{lu}{c0^2}$$

where: $c0 := \sqrt{2 \cdot i'3 \cdot \left[1 - \left(\dfrac{p4}{p'3} \right)^{\frac{\chi-1}{\chi}} \right]}$ m/s $\qquad c0 = 527.451$ m/s

The relative efficiency of the turbine: $\eta u = 0.881$.

2. ANSYS MODELLING OF THE AXIAL GAS TURBINE ROTOR

We present some aspect concerning ANSYS modelling. First step is the geometric drawing for the analysed rotor (see Fig.7.). Next step is the meshing of the surfaces (see Fig.8.). We proceed to the mechanical and thermal loading as close as possible to the real case. Using “Solve” command, ANSYS software calculates and displays Figure 9 the results of numerical calculus.

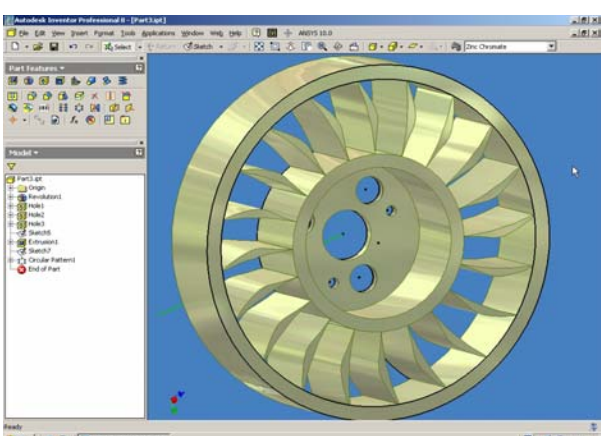

Fig.7. Geometrical model

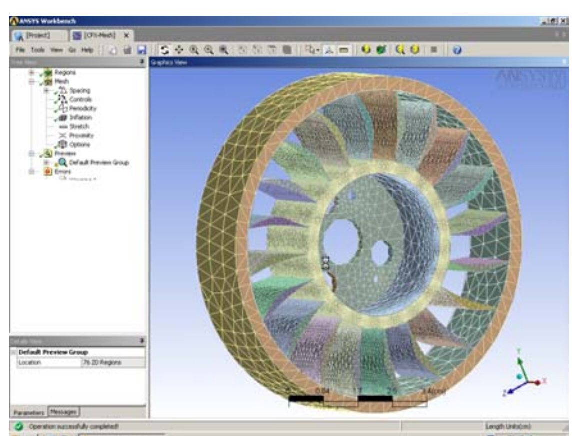

Fig.8. Meshed model

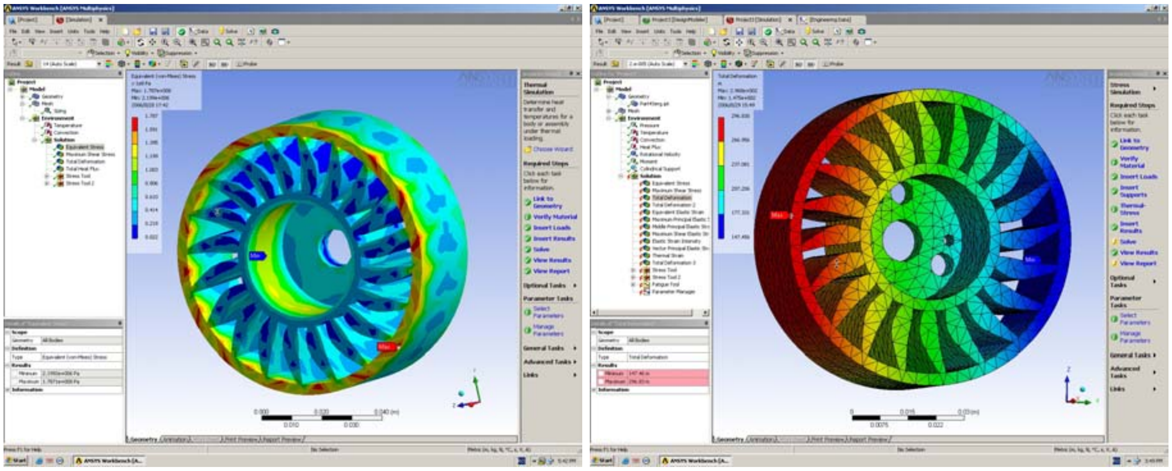

Fig.9. Results of calculus

As seen in the Figure 10 and above, one can model and get results for equivalent von Misses stress, total deformation, safety factor in dependence of constant amplitude load (Fig 11).

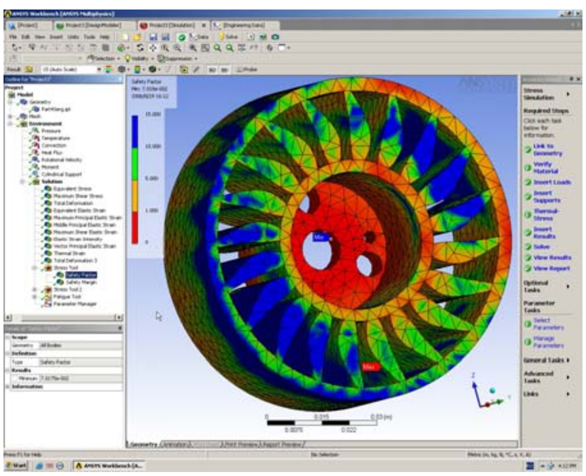

Fig.10. Safety factor

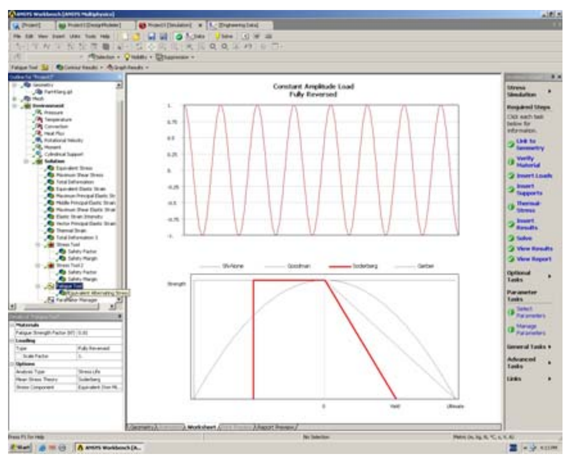

Fig.11. Constant amplitude load

3. CONCLUSIONS

By using two powerful software packages like MathCad and ANSYS, the design of the rotor of a gas turbine can be easy done and modelled on. These encouraging results will determine us to approach the complete design of a gas turbine.

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