ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

THEORETICAL STUDIES REGARDING THE DYNAMICS OF THE RIGID BODY WITH ELASTIC BEARINGS AND STRUCTURAL SYMMETRIES, EXCITED BY HARMONICAL FORCES AND COUPLES

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Keywords: mechanical elastic systems, dynamic parameters, structural symmetry, forced vibration

Résumé: This paper is an analyze of the dynamical features of a mechanical elastic system excited by harmonical forces and/or couples. For two different types of structural symmetry of the system, there are some uncoupled subsystems with coupled moving. The author of the study determines the analytical expressions of the amplitudes of forced vibrations, pointing out the influences of structural parameters of the system (masses, mass inertia, elasticities, aso.).

1. INTRODUCTION

The main problems of the dynamical analyze of the mechanical systems with elastic or/and viscous-elastic bearings are the next:
1)the modal moving analyze (natural frequencies, eigen values)
2)the amplitudes of harmonical vibration (straight vibration, rotational vibration)
3)the forces and couples transmitted to the foundation through the bearings

The analyze of the above dynamical parameters is very important starting even from designing of any kind of mechanical systems, no matter of the vibration source: technological vibration, useless/undesired vibration (from driven engine, from mechanical transmission, from work equipment), environment vibration.

Because the most part of the exciting forces or couples are due to the cyclic moving (straight or angular) of pieces or mechanical equipment, these are harmonical or poly-harmonical. The type of excitations is inertial with the frequency much higher than the natural frequencies of the mechanical system. That's why, the model with elastic bearings is good enough for dynamical analyze of the technological equipment.

Acc. to [1], the matrix expression of the differential moving equation is

$$\underline{A}\,\underline{\underline{\ddot{q}}} + \underline{C}\,\underline{\underline{q}} = \underline{\underline{f}} \ , \tag{1}$$

where $\underline{A}$ is the matrix of inertia;

$\underline{C}$-matrix of elasticities;

$\underline{\underline{q}}$-the vector of displacements;

$\underline{\underline{\ddot{q}}}$-the vector of accelerations;

$\underline{\underline{f}}$-the vector of disturbing forces.

2. THE DISTURBING FORCES AND COUPLES

In order to determine the elements of the vector of disturbing forces $\underline{\underline{f}}$ corresponding to the six independent movings of the rigid body, it has to take into consideration the model from ***figure 1***. Considered harmonical disturbing forces and couples are the most common for mechanical equipment:

►eccentric vertical forces (***F***$_z$) and longitudinal forces (***F***$_y$);

►pitching couple (**M**$_y$) and rolling couple (**M**$_x$).

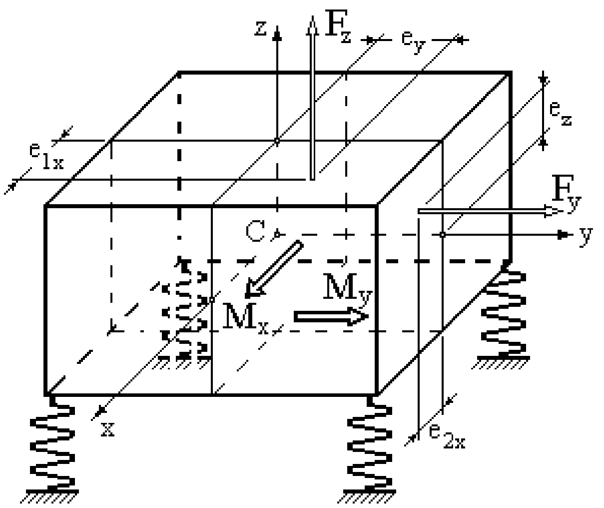

Fig. 1. Rigid body with four elastic bearings excited by some harmonical forces and couples

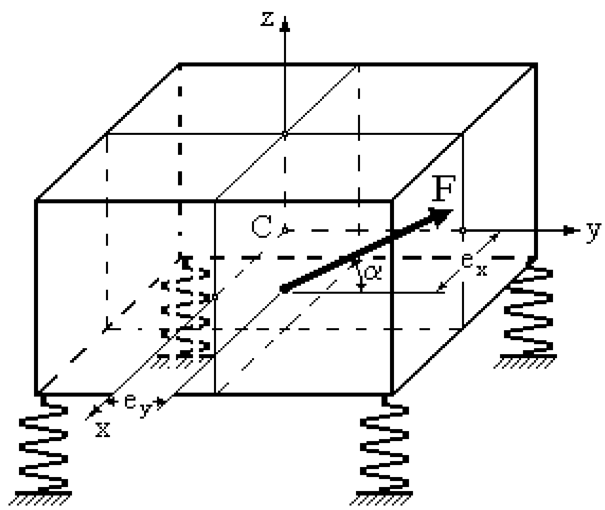

Fig. 2. Rigid body with four elastic bearings excited by an inclined harmonical force

The ***figure 2*** shows the model of the rigid body with four elastic bearings excited by an inclined harmonical force ***F*** into a vertical-longitudinal plane. This is the case of some technological equipment driven by one direction vibrators like industrial feeders and chargers/dispensers, conveyors, classifiers aso..

For the forces and couples from figure 1 and figure 2, the vectors of disturbing forces are the next:

►for rolling couple **M**$_x$

$$\underline{\underline{f}} = \left[0,0,0,M_{0x}\sin\omega t,0,0\right]^{\mathrm{T}} \tag{2}$$

►for pitching couple **M**$_y$

$$\underline{\underline{f}} = \left[0,0,0,0,M_{0y}\sin\omega t,0\right]^{\mathrm{T}} \tag{3}$$

►for eccentric vertical force ***F***$_z$

$$\underline{\underline{f}} = \left[0,0,F_{0z}\sin\omega t,e_y F_{0z}\sin\omega t,-e_{1x}F_{0z}\sin\omega t,0\right]^{\mathrm{T}} \tag{4}$$

►for eccentric longitudinal force ***F***$_y$

$$\underline{\underline{f}} = \left[0,F_{0y}\sin\omega t,0,-e_z F_{0y}\sin\omega t,0,e_{2x}F_{0y}\sin\omega t\right]^{\mathrm{T}} \tag{5}$$

►for inclined force ***F*** with α angle

$$\underline{\underline{f}} = F_0\sin\omega t\left[0,\cos\alpha,\sin\alpha,e_y\sin\alpha,-e_x\sin\alpha,e_x\cos\alpha\right]^{\mathrm{T}} \tag{6}$$

3. THE AMPLITUDES OF THE FORCED VIBRATION OF THE RIGID BODY WITH UNCOUPLED MOVING

Many technological equipment have a symmetrical structure after vertical planes and, at the same time, they have four identical bearings (position, elasticity). In this case, the dynamical analyze is facilitated by decoupling the six moving of the rigid body into subsystems with less coupled moving, that meaning less equations for the systems of differential moving equations. Thus, the analytical expressions of the amplitudes of steady-state forced vibration are more simple, setting off the influence of structural features (dimensions, masses, inertia, elasticity) on them.

3.1. Rigid body with a vertical axis of symmetry

Acc. to [2], the moving of the rigid body with six degree of freedom is decoupling into four subsystems; these displacements are measured by the coordinates (X, φ_y), (Y, φ_x), Z and φ_z.

For §2 types of harmonical excitation, the differential moving equations and the amplitudes of forced vibrations are the next:

►*Excitation by rolling couple M_x*

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_yY + 4\left(b^2k_z + h^2k_y\right)\varphi_x = M_{0x}\sin\omega t \end{cases} \tag{7}$$

●the amplitudes of forced vibration:

$$A_Y = \frac{-4hk_yM_{0x}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{8}$$

$$A_{\varphi_x} = \frac{\left(4k_y - m\omega^2\right)M_{0x}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{9}$$

►*Excitation by pitching couple M_y*

●the differential moving equations:

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4\left(h^2k_x + a^2k_z\right)\varphi_y = M_{0y}\sin\omega t \end{cases} \tag{10}$$

●the amplitudes of forced vibration:

$$A_X = \frac{4hk_xM_{0y}}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{11}$$

$$A_{\varphi_y} = \frac{\left(4k_x - m\omega^2\right)M_{0y}}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{12}$$

►*Excitation by eccentric vertical force F_z*

●the differential moving equations:

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4\left(h^2k_x + a^2k_z\right)\varphi_y = -e_{1x}F_{0z}\sin\omega t \end{cases} \tag{13}$$

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_yY + 4\left(b^2k_z + h^2k_y\right)\varphi_x = e_yF_{0z}\sin\omega t \end{cases} \tag{14}$$

$$m\ddot{Z} + 4k_zZ = F_{0z}\sin\omega t \tag{15}$$

●the amplitudes of forced vibration:

$$A_X = \frac{-4hk_xe_{1x}F_{0z}}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{16}$$

$$A_{\varphi_y} = \frac{-\left(4k_x - m\omega^2\right)e_{1x}F_{0z}}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{17}$$

$$A_Y = \frac{-4hk_ye_yF_{0z}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{18}$$

$$A_{\varphi_x} = \frac{\left(4k_y - m\omega^2\right)e_yF_{0z}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{19}$$

$$A_Z = \frac{F_{0z}}{4k_z - m\omega^2} \tag{20}$$

►*Excitation by eccentric longitudinal force F_y*

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = F_{0y}\sin\omega t \\ J_x\ddot{\varphi}_x + 4hk_yY + 4\left(b^2k_z + h^2k_y\right)\varphi_x = -e_zF_{0y}\sin\omega t \end{cases} \tag{21}$$

$$J_z\ddot{\varphi}_z + 4a^2k_y + b^2k_x\varphi_z = e_{2x}F_{0y}\sin\omega t \tag{22}$$

●the amplitudes of forced vibration:

$$A_Y = \frac{F_{0y}\left\{\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] + 4hk_ye_z\right\}}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{23}$$

$$A_{\varphi_x} = -\frac{F_{0y}\left[e_z\left(4k_y - m\omega^2\right) + 4hk_y\right]}{\left(4k_y - m\omega^2\right)\left[4\left(b^2k_z + h^2k_y\right) - J_x\omega^2\right] - 16h^2k_y^2} \tag{24}$$

$$A_{\varphi_z} = \frac{e_{2x}F_{0y}}{4\left(a^2k_y + b^2k_x\right) - J_z} \tag{25}$$

►*Excitation by inclined longitudinal force F (figure 2)*

●the differential moving equations:

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4\left(h^2k_x + a^2k_z\right)\varphi_y = -e_xF_0\sin\alpha\sin\omega t \end{cases} \tag{26}$$

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = F_0\cos\alpha\sin\omega t \\ J_x\ddot{\varphi}_x + 4hk_yY + 4\left(b^2k_z + h^2k_y\right)\varphi_x = e_yF_0\sin\alpha\sin\omega t \end{cases} \tag{27}$$

$$m\ddot{Z} + 4k_zZ = F_0\sin\alpha\sin\omega t \tag{28}$$

$$J_z\ddot{\varphi}_z + 4a^2k_y + b^2k_x\varphi_z = e_xF_0\cos\alpha\sin\omega t \tag{29}$$

●the amplitudes of forced vibration:

$$A_X = \frac{-4hk_xe_xF_0\sin\alpha}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{30}$$

$$A_{\varphi_y} = \frac{-\left(4k_x - m\omega^2\right)e_xF_0\sin\alpha}{\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_x\omega^2\right] - 16h^2k_x^2} \tag{31}$$

$$A_Y = F_0 \frac{\left[4\left(b^2 k_z + h^2 k_y\right) - J_x \omega^2\right]\cos\alpha - 4hk_y e_y \sin\alpha}{\left(4k_y - m\omega^2\right)\left[4\left(b^2 k_z + h^2 k_y\right) - J_x\omega^2\right] - 16h^2 k_y^2} \qquad (32)$$

$$A_{\varphi_x} = F_0 \frac{\left(4k_y - m\omega^2\right)e_y \sin\alpha - 4hk_y \cos\alpha}{\left(4k_y - m\omega^2\right)\left[4\left(b^2 k_z + h^2 k_y\right) - J_x\omega^2\right] - 16h^2 k_y^2} \qquad (33)$$

$$A_Z = \frac{F_0 \sin\alpha}{4k_z - m\omega^2} \qquad (34)$$

$$A_{\varphi_z} = \frac{e_x F_0 \cos\alpha}{4\left(a^2 k_y + b^2 k_x\right) - J_z} \qquad (35)$$

3.2. Rigid body with a vertical-longitudinal plane of symmetry

The moving of the rigid body with a vertical-longitudinal plane of symmetry is decoupled into two subsystems with the displacements after the coordinates $\left(X,\varphi_y,\varphi_z\right)$ and $\left(Y,Z,\varphi_x\right)$. For the same harmonical excitation like in §3.1, the differential moving equations and the amplitudes of the forced vibrations are:

►*Excitation by rolling couple M_x* is moving the subsystem $\left(Y,Z,\varphi_x\right)$

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_y Y + 4hk_y\varphi_x = 0 \\ m\ddot{Z} + 4k_z Z + 2k_z\left(b_3 - b_2\right)\varphi_x = 0 \\ J_x\ddot{\varphi}_x + 4hk_y Y + 2k_z\left(b_3 - b_2\right)Z + 2\left[k_z\left(b_2^2 + b_3^2\right) + 2h^2 k_y\right]\varphi_x = M_{0x}\sin\omega t \end{cases} \qquad (36)$$

●the amplitudes of forced vibration:

$$A_Y = -\frac{4hk_y}{\Delta_1} M_{0x}\left(4k_z - m\omega^2\right) \qquad (37)$$

$$A_Z = -\frac{2k_z}{\Delta_1} M_{0x}\left(b_3 - b_2\right)\left(4k_y - m\omega^2\right) \qquad (38)$$

$$A_{\varphi_x} = \frac{1}{\Delta_1} M_{0x}\left(4k_y - m\omega^2\right)\left(4k_z - m\omega^2\right) \quad , \qquad (39)$$

where

$$\Delta_1 = \left(4k_y - \omega^2 m\right)\left(4k_z - \omega^2 m\right)\left\{2\left[k_z\left(b_2^2 + b_3^2\right) + 2h^2 k_y\right] - \omega^2 J_x\right\} - \\ -\left[2k_z\left(b_3 - b_2\right)\right]^2\left(4k_y - \omega^2 m\right) - \left(4hk_y\right)^2\left(4k_z - \omega^2 m\right) \qquad (40)$$

►*Excitation by pitching couple M_y* is moving the subsystem $\left(X,\varphi_y,\varphi_z\right)$

●the differential moving equations:

$$\begin{cases} m\ddot{X} + 4k_x X - 4hk_x\varphi_y - 2k_x\left(b_3 - b_2\right)\varphi_z = 0 \\ J_y\ddot{\varphi}_y - 4hk_x X + 4\left(h^2 k_x + a^2 k_z\right)\varphi_y + 2hk_x\left(b_3 - b_2\right)\varphi_z = M_{0y}\sin\omega t \\ J_z\ddot{\varphi}_z - 2k_x\left(b_3 - b_2\right)X + 2hk_x\left(b_3 - b_2\right)\varphi_y + 2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right]\varphi_z = 0 \end{cases} \qquad (41)$$

●the amplitudes of forced vibration:

$$A_X = -\frac{4hk_x}{\Delta_2} M_{0y}\left\{k_x\left(b_3 - b_2\right)^2 - \left[4a^2 k_y + 2k_x\left(b_2^2 + b_3^2\right) - J_z\omega^2\right]\right\} \qquad (42)$$

$$A_{\varphi_y} = \frac{1}{\Delta_2} M_{0y} \left\{ -\left[2k_x(b_3 - b_2)\right]^2 + \left(4k_x - m\omega^2\right)\left[4a^2k_y + 2k_x\left(b_2^2 + b_3^2\right) - J_z\omega^2\right]\right\} \quad (43)$$

$$A_{\varphi_z} = \frac{2hmk_x\omega^2}{\Delta_2} M_{0y}(b_3 - b_2) \quad , \quad (44)$$

where

$$\begin{aligned}\Delta_2 = &\left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_y\omega^2\right]\left\{2\left[2a^2k_y + k_x\left(b_2^2 + b_3^2\right)\right] - J_z\omega^2\right\} + \\ &+ 2(-4hk_x)\left[2hk_x(b_3 - b_2)\right]\left[-2k_x(b_3 - b_2)\right] - \\ &- \left[2hk_x(b_3 - b_2)\right]^2\left(4k_x - m\omega^2\right) - \\ &- \left[-2k_x(b_3 - b_2)\right]^2\left[4\left(h^2k_x + a^2k_z\right) - J_y\omega^2\right] \\ &- (-4hk_x)^2\left\{2\left[2a^2k_y + k_x\left(b_2^2 + b_3^2\right)\right] - J_z\omega^2\right\}\end{aligned} \quad (45)$$

►*Excitation by eccentric vertical force* $\boldsymbol{F_z}$ is moving both subsystems $(X, \varphi_y, \varphi_z)$ and (Y, Z, φ_x)

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_yY + 4hk_y\varphi_x = 0 \\ m\ddot{Z} + 4k_zZ + 2k_z(b_3 - b_2)\varphi_x = F_{0z}\sin\omega t \\ J_x\ddot{\varphi}_x + 4hk_yY + 2k_z(b_3 - b_2)Z + 2\left[k_z\left(b_2^2 + b_3^2\right) + 2h^2k_y\right]\varphi_x = e_yF_{0z}\sin\omega t \end{cases} \quad (46)$$

$$\begin{cases} m\ddot{X} + 4k_xX - 4hk_x\varphi_y - 2k_x(b_3 - b_2)\varphi_z = 0 \\ J_y\ddot{\varphi}_y - 4hk_xX + 4\left(h^2k_x + a^2k_z\right)\varphi_y + 2hk_x(b_3 - b_2)\varphi_z = -e_{1x}F_{0z}\sin\omega t \\ J_z\ddot{\varphi}_z - 2k_x(b_3 - b_2)X + 2hk_x(b_3 - b_2)\varphi_y + 2\left[2a^2k_y + k_x\left(b_2^2 + b_3^2\right)\right]\varphi_z = 0 \end{cases} \quad (47)$$

●the amplitudes of forced vibration:

$$A_Y = \frac{4hk_y}{\Delta_1} F_{0z}\left[2k_z(b_3 - b_2) - e_y\left(4k_z - m\omega^2\right)\right] \quad (48)$$

$$\begin{aligned}A_Z = -\frac{F_{0z}}{\Delta_1}&\left\{(4hk_y)^2 + 2e_yk_z(b_3 - b_2)\left(4k_y - m\omega^2\right) - \right. \\ &\left. - \left[2k_z\left(b_2^2 + b_3^2\right) + 4h^2k_y - J_x\omega^2\right]\left(4k_y - m\omega^2\right)\right\}\end{aligned} \quad (49)$$

$$A_{\varphi_x} = \frac{F_{0z}}{\Delta_1}\left(4k_y - m\omega^2\right)\left[e_y\left(4k_z - m\omega^2\right) - 2k_z(b_3 - b_2)\right] \quad (50)$$

$$A_X = -\frac{4hk_x}{\Delta_2} e_{1x}F_{0z}\left\{k_x(b_3 - b_2)^2 -- \left[4a^2k_y + 2k_x\left(b_2^2 + b_3^2\right) - J_z\omega^2\right]\right\} \quad (51)$$

$$A_{\varphi_y} = \frac{e_{1x}F_{0z}}{\Delta_2}\left\{\left[2k_x(b_3 - b_2)\right]^2 - \left(4k_x - m\omega^2\right)\left[4a^2k_y + 2k_x\left(b_2^2 + b_3^2\right) - J_z\omega^2\right]\right\} \quad (52)$$

$$A_{\varphi_z} = -\frac{2hk_x}{\Delta_2} e_{1x}F_{0z}(b_3 - b_2)m\omega^2 \ , \quad (53)$$

where Δ_1 is like (40) and Δ_2 has the expression (45).

►*Excitation by eccentric longitudinal force* $\boldsymbol{F_y}$ is moving both subsystems $(X, \varphi_y, \varphi_z)$ and (Y, Z, φ_x)

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_y Y + 4hk_y \varphi_x = F_{0y} \sin \omega t \\ m\ddot{Z} + 4k_z Z + 2k_z (b_3 - b_2)\varphi_x = 0 \\ J_x \ddot{\varphi}_x + 4hk_y Y + 2k_z (b_3 - b_2) Z + 2\left[k_z\left(b_2^2 + b_3^2\right) + 2h^2 k_y\right]\varphi_x = -e_z F_{0y} \sin \omega t \end{cases} \tag{54}$$

$$\begin{cases} m\ddot{X} + 4k_x X - 4hk_x \varphi_y - 2k_x (b_3 - b_2)\varphi_z = 0 \\ J_y \ddot{\varphi}_y - 4hk_x X + 4\left(h^2 k_x + a^2 k_z\right)\varphi_y + 2hk_x (b_3 - b_2)\varphi_z = 0 \\ J_z \ddot{\varphi}_z - 2k_x (b_3 - b_2) X + 2hk_x (b_3 - b_2)\varphi_y + 2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right]\varphi_z = e_{2x} F_{0y} \sin \omega t \end{cases} \tag{55}$$

●the amplitudes of forced vibration:

$$A_Y = \frac{F_{0y}}{\Delta_1}\left\{\left(4k_z - m\omega^2\right)\left[2k_z\left(b_2^2 + b_3^2\right) + 4h^2 k_y - J_x \omega^2\right] - \left[2k_z(b_3 - b_2)\right]^2 + e_z\left[4hk_y\left(4k_z - m\omega^2\right)\right]\right\} \tag{56}$$

$$A_Z = \frac{2k_z}{\Delta_1} F_{0y} (b_3 - b_2)\left[4hk_y + e_z\left(4k_y - m\omega^2\right)\right] \tag{57}$$

$$A_{\varphi_x} = -\frac{F_{0y}}{\Delta_1}\left(4k_z - m\omega^2\right)\left[4hk_y + e_z\left(4k_y - m\omega^2\right)\right] \tag{58}$$

$$A_X = \frac{e_{2x} F_{0y}}{\Delta_2} 2k_x (b_3 - b_2)\left(4a^2 k_z - J_y \omega^2\right) \tag{59}$$

$$A_{\varphi_y} = \frac{e_{2x} F_{0y}}{\Delta_2} 2hk_x (b_3 - b_2) m\omega^2 \tag{60}$$

$$A_{\varphi_z} = -\frac{e_{2x} F_{0y}}{\Delta_2}\left\{(4hk_x)^2 - \left(4k_x - m\omega^2\right)\left[4\left(h^2 k_x + a^2 k_z\right) - J_y \omega^2\right]\right\} \tag{61}$$

►*Excitation by inclined force **F** (figure 2)* is moving both subsystems

●the differential moving equations:

$$\begin{cases} m\ddot{Y} + 4k_y Y + 4hk_y \varphi_x = F_0 \cos\alpha \sin \omega t \\ m\ddot{Z} + 4k_z Z + 2k_z (b_3 - b_2)\varphi_x = F_0 \sin\alpha \sin \omega t \\ J_x \ddot{\varphi}_x + 4hk_y Y + 2k_z (b_3 - b_2) Z + 2\left[k_z\left(b_2^2 + b_3^2\right) + 2h^2 k_y\right]\varphi_x = e_y F_0 \sin\alpha \sin \omega t \end{cases} \tag{62}$$

$$\begin{cases} m\ddot{X} + 4k_x X - 4hk_x \varphi_y - 2k_x (b_3 - b_2)\varphi_z = 0 \\ J_y \ddot{\varphi}_y - 4hk_x X + 4\left(h^2 k_x + a^2 k_z\right)\varphi_y + 2hk_x (b_3 - b_2)\varphi_z = -e_x F_0 \sin\alpha \sin \omega t \\ J_z \ddot{\varphi}_z - 2k_x (b_3 - b_2) X + 2hk_x (b_3 - b_2)\varphi_y + 2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right]\varphi_z = e_x F_0 \cos\alpha \sin \omega t \end{cases} \tag{63}$$

●the amplitudes of forced vibration:

$$A_Y = -\frac{F_0 \cos\alpha}{\Delta_1}\left\{\left[2k_z(b_3 - b_2)\right]^2 - \left(4k_z - m\omega^2\right)\left[2k_z\left(b_2^2 + b_3^2\right) + 4h^2 k_y - J_x \omega^2\right]\right\} + \frac{F_0 \sin\alpha}{\Delta_1} 4hk_y\left[2k_z(b_3 - b_2) - e_y\left(4k_z - m\omega^2\right)\right] \tag{64}$$

$$A_Z = -\frac{F_0 \sin\alpha}{\Delta_1}\left\{2k_z e_y (b_3 - b_2)\left(4k_y - m\omega^2\right) + \left(4hk_y\right)^2 - \left(4k_y - m\omega^2\right)\left[2k_z\left(b_2^2 + b_3^2\right) + 4h^2 k_y - J_x \omega^2\right]\right\} + \frac{F_0 \cos\alpha}{\Delta_1} 8hk_y k_z (b_3 - b_2) \tag{65}$$

$$A_{\varphi_x} = \frac{F_0}{\Delta_1}\Big[-4hk_y \cos\alpha\left(4k_z - m\omega^2\right) - \\ -2k_z \sin\alpha\left(b_3 - b_2\right)\left(4k_y - m\omega^2\right) + e_y \sin\alpha\left(4k_y - m\omega^2\right)\left(4k_z - m\omega^2\right)\Big] \quad (66)$$

$$A_X = \frac{2k_x}{\Delta_2} e_x F_0 \left\{\cos\alpha\left(b_3 - b_2\right)\left(4a^2k_z - J_y\omega^2\right) - 2h\sin\alpha\left[4a^2k_y + k_x\left(b_3 + b_2\right)^2 - J_z\omega^2\right]\right\} \quad (67)$$

$$A_{\varphi_y} = \frac{e_x F_0 \sin\alpha}{\Delta_2}\left\{\left[2k_x\left(b_3 - b_2\right)\right]^2 - \left(4k_x - m\omega^2\right)\left[4a^2k_y + 2k_x\left(b_2^2 + b_3^2\right) - J_z\omega^2\right]\right\} + \\ + \frac{e_x F_0 \cos\alpha}{\Delta_2} 2hk_x\left(b_3 - b_2\right)m\omega^2 \quad (68)$$

$$A_{\varphi_z} = \frac{-e_x F_0}{\Delta_2}\left\{2hk_x \sin\alpha\left(b_3 - b_2\right)m\omega^2 + \\ + \cos\alpha\left[\left(4hk_x\right)^2 - \left(4k_x - m\omega^2\right)\left[4\left(h^2k_x + a^2k_z\right) - J_y\omega^2\right]\right]\right\} \quad (69)$$

4. CONCLUSIONS

a)thanks of structural symmetries of the rigid body and of his elastic bearings, the moving are decoupled, in this way the differential equations can be analytically solved;

b)analytical expressions of the amplitudes can be used to point out the influence of the structural features on the dynamic characteristics of the rigid body;

c)the dynamic study of different technological equipment can have two kind of scopes:

1-finding the optimal parameters for increasing the technological efficiency;

2-reducing the level of vibration transmitted to human operator, to environment or to equipment himself;

d)for a "good" choice of structural parameters, the steady-state forced vibration can be reduced or even repealed, especially rotational vibration which are very dangerous.

5. BIBLIOGRAPHIE

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