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RELIABILITY MODELING OF A HYDRAULIC SYSTEM USING THE MONTE CARLO METHOD

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Abstract: This paper presents the application of the Monte Carlo simulation method in order to evaluate reliability performances of a hydraulic system. Also, it's presents the comparison between this simulation method and the analytical evaluation method based on Markov model with continuous parameter.

1. INTRODUCTION

The Monte Carlo method is a statistic method which modelates the functioning process of a system, using a probabilistic model which is tested repeatedly. This method consists in generating some pseudo random numbers, which are uniformly represented in (0,1) interval and the conversion of these numbers in non-uniform distribution values. These numbers have a certain signification in the functioning process of the system. The conversion procedures might be [1, 2]: the inversion method, the configuration method, the acceptance-rejection method.

As a rule, the reliability of the simulated system is given by the main relation:

$$\text{The system reliability} = \frac{\text{the number of situations in which the system functioned}}{\text{the total number of simulations}} \quad (1)$$

A major importance represents knowing the error involved in the established indicators.

The error (ε%) involved in the refusal probability of the system might be determined with the following relation [2]:

$$\varepsilon_{\%} = 200\sqrt{\frac{1-P_R}{N \cdot P_R}} \quad (2)$$

where, N represents the number of simulations used to estimate P_R. It is 95% probable that the estimated value of P_R to be found in the ($P_R \pm P_R \cdot \varepsilon$) interval.

The relation (2) might be also used to establish the number of simulations that are necessary to obtain a wanted accuracy.

If, for example, an (0,1±0,01) accuracy is wanted for P_R (ε%), the relation (2) indicates a number of N = 3600 simulations.

Through the Monte Carlo simulation method, it can estimate the medium and the momentary values for the reliability, performance and availability indicators, but it can also establish the distribution functions [3, 4, 6, 7, 8]:

- fixed (P_i) and momentary ((P_i)) values of the system's states probabilities;
- the successfully probability (P_S) and refusal probability (P_R) of the system;
- the total medium duration of success $\alpha(T_A)$ and refuse $\beta(T_A)$;
- the medium [$\nu(T_A)$] and momentary [$\nu(t)$] value of transition numbers in refusal states;

♦ the mean time between failures MTBF, respectively the mean time of maintenance MTM;

♦ the fault mean rate (λ_e), respectively the recovery mean rate (μ_e)

♦ the steady state values and momentary values of availability (D, D(t)) and unavailability (I, I(t)).

Advantages of the Monte Carlo method in comparison to analytical methods:

- the modeling of the system functioning is very close to reality, offering the possibility to take into consideration any well known characteristic;
- precise results for the systems reliability and availability simulation;
- simple simulation algorithms;
- the possibility of using different types of distributions which can simulate the functioning time (T_f), the failure time (T_d) or any other characteristic parameters of the system;
- the possibility to determine a large number of reliability and availability indicators of the system, presented as medium values and/or momentary values.

Main disadvantages of the method:

- simulation time is large, even in the case of a small number of elements;
- the results depend on simulations number and on random numbers generating;
- in order to obtain precise results, there might appear some problems with establishing the number of simulations and variance reducing.

2. STUDY CASE. THE FUNCTIONING SIMULATION OF TWO PUMPS IN STAND-BY REDUNDANCY

As an example, we consider a pumping station represented by two pumps, A and B, with the dimension 2x100%, simultaneously functioning, an inexhaustible water source, WS and a reservoir R, the connecting pipes are considered to be perfectly reliable (fig. 1).

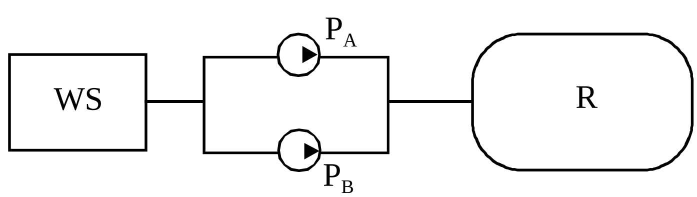

Fig. 1 Pumping stations simplified represented

The functioning of the system composed of two electro pumps might be described through a random process, which is carried sequentially in time after the cycle functioning-failure.

The electro pumps are reparable elements, so in this way, the simulation procedure must take into account this aspect.

In comparison with the Monte Carlo simulation, the analytical study of this system, with reparable elements is difficult, implying the writing and resolving complex differential equations. The simulation of time sequences will be done taking into account the fact that variates, functioning time (T_f),and failure time (T_d) have a repartition given by the exponential rule (λ=ct.). The exponential rule has the following probability distribution (F(x)):

$$F(x) = 1 - e^{-\lambda x} \quad (3)$$

Applying the method of inverse function we obtain the values $[(t_f)i, (t_d)i]$ of the variates functioning time (T_f) and failure time (T_d), distributed non-uniformly (exponentially) for an element:

$$(t_f)_i = -\frac{1}{\lambda}\ln(u_i) \quad (4)$$

$$(t_d)_i = -\frac{1}{\mu}\ln(v_i) \quad (5)$$

where:

u_i, respectively, v_i represent the values of the variates u, and v uniformly distributed on the (0,1) interval;

λ, μ - fault mean rate, respectively recovery mean rate for the element whose functioning is simulated.

The following aspects will be emphasized:

- the functioning simulation method for two electro pumps in stand-by redundancy;
- the determination of the reliability indicators which can be estimated through simulation and their method of evaluation;
- comparing the analytical results with the simulation results;
- possibilities to extend the method in order to analyze some complex schemes.

The electro pumps, being reparable and in stand-by redundancy, the following cycle will be simulated: functioning-failure. In order to study the reliability of the system, the simulation should be done at the system's first fault and, in order to study availability, simulation time (T_s) is equivalent with the element lifetime. The analysis time is T_A and for this system, the simulation might be done using the following algorithm [2]:

Step 1: the random number (u), uniformly distributed on the (0,1) interval is generated;

Step 2: u is converted in a value of functioning time, using relation (4), where parameter λ is considered constant and known;

Step 3: a random number (v) is generated and uniformly distributed on the interval (0,1);

Step 4: v is converted in a value of the failure time, using relation (5), where parameter μ is considered constant and known;

Step 5: steps 1-4 are repeated for a target period, larger than T_A;

$$\sum_{i=1}^{\text{nr. de cicluri}} [(t_f)_i + (t_d)_i] \geq T_A \quad (6)$$

Step 6: steps 1-5 are repeated for each element;

Step 7: the sequences of time simulated for each element, are compared, resulting the following situations:

- if during analysis time there is no common repair of two elements (meaning that at least one of the elements functions during the analysis time), then the system is in a successful state;
- if there is an event, a common repair of two elements during the analysis time, then the system is in a failure state.

Step 8: steps 1-7 are repeated for the desired number of simulation (N_S).

For a certain simulation, the simulated functioning time (t_f) and failure time (t_d), for the two electro pumps, might be distributed as in figure 2.

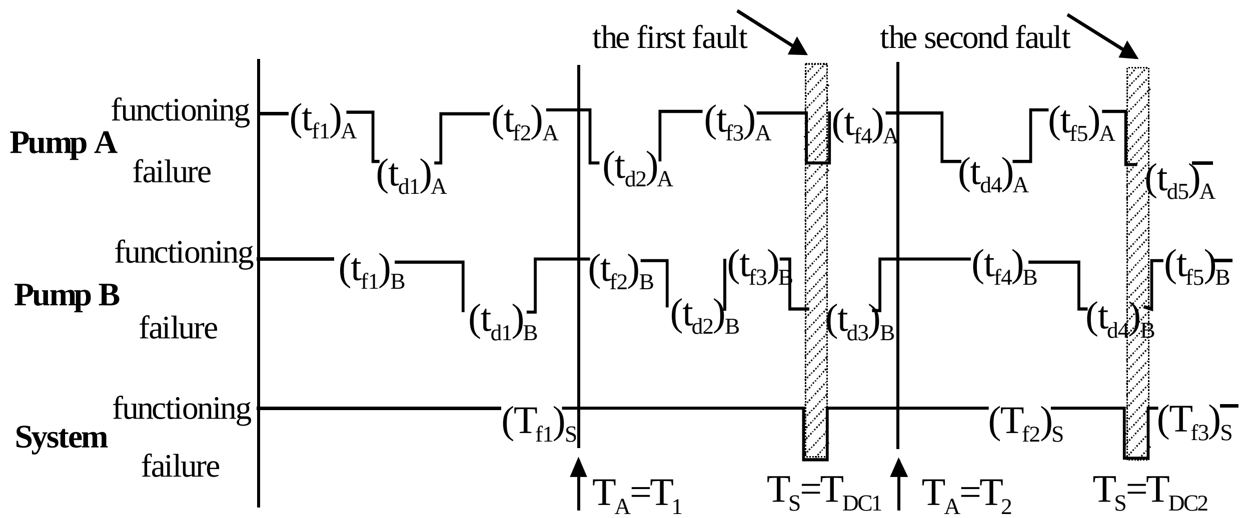

Fig. 2 Simulation of functioning and failure time for two elements in active reserve

The following notations have been done:

T_{DCi} – common failure time for the two electro pumps;
T_k – preset analysis time;
T_S – simulated failure time
$(t_{fi})_A$, $(t_{di})_A$, $(t_{fi})_B$, $(t_{di})_B$ - functioning and failure simulated time for the two electro pumps.

Analyzing figure 2, for a certain simulation we have:

- If $T_A=T_1$ then $T_1< T_S \Rightarrow$ the system functions uninterruptedly in $(0, T_A) \Rightarrow$ +1 is added to success number;
- If $T_A=T_2$ then $T_2 > T_S \Rightarrow$ the system has a fault in $(0, T_A) \Rightarrow$ +1 is added to refusal number.

Using the algorithm previously mentioned we can establish the variation in time of the reliability functions R(t) and non-reliability F(t), as well as the variation of the reliability function with the simulations number $R(N_{simulations})$.

A large simulations number $R(N_{simulations})$ represents practically the stationary value of reliability. The variation of the reliability R(t), and non-reliability F(t) functions is obtained through the simulation of functioning and failure time, until the system's first step (T_{DCi}). The value of the reliability function $[R(T_i)]$ at a given time T_i is determined with relation (1).

In order to determine the variation of the reliability function in time R(t), the reliability function will be evaluated at different periods of time $R(t_i)$ in the $[0, T_A]$ interval.

$$[\, T_1=0,\ T_2= T_1+T_{Pas} \quad \ldots \quad ,T_i= T_{i-1}+T_{Pas} \ \ldots \quad T_N= T_A \,]$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$[\, R(T_1)=1 \quad R(T_2)\ \ldots \quad , \quad R(T_i) \quad R(T_N) \,]$$

In order to establish other reliability indicators (the number of faults during the analysis time, total time of functioning or failure, the availability etc.) the algorithm might be used, mentioning the fact that the simulation of the sequences of time won't be realised until the system's first failure but for the whole period of analysis time (T_A).

The reliability and availability indicators are established on the basis of simulated functioning and failure time [9]:

➲ $\nu(T_A)$ - the faults number of the system recorded during T_A period, for any given simulation i;

$$\nu(T_A) = \frac{\sum_{i=1}^{N_S} \nu_i(T_A)}{N_S} \quad [\text{faults}/T_A] \tag{7}$$

$\nu_i(T_A)$ – the faults number of the system recorded during T_A period, for any given simulation i

➲ total time of system failure $\beta(T_A)$, during the analysis period T_A:

$$\beta(T_A) = \frac{\sum_{j=1}^{N_S} \sum_{i=1}^{ND_i} \beta_i(T_A)}{N_S} \quad [\text{hours}] \tag{8}$$

$\beta_i(T_A)$ – common defect time for the two electro pumps recorded during T_A, for any given simulation i;

➲ continuous failure time of the system (MTM):

$$MTM = \frac{\beta(T_A)}{\nu(T_A)} \quad \text{with} \quad \nu(T_A)\neq 0 \quad [\text{hours}] \tag{9}$$

➲ equivalent recovery mean rate ((μ_e):

$$\mu_e = \frac{1}{MTM} \qquad [1/h] \tag{10}$$

➲ total functioning time of the system during T_A, ($\alpha(T_A)$):

$$\alpha\,(T_A)=T_A-\beta(T_A) \quad [\text{hours}] \tag{11}$$

➲ mean time between failures (MTBF):

$$MTBF = \frac{\alpha(T_A)}{\nu(T_A)} \quad \text{with} \quad \nu(T_A) \neq 0) \quad [\text{hours}] \tag{12}$$

➲ equivalent fault mean rate (λ_e):

$$\lambda_e = \frac{1}{MTBF} \qquad [1/h] \tag{13}$$

➲ time availability ($D(T_A)$), during T_A:

$$D(T_A) = \frac{T_A - \beta(T_A)}{T_A} \quad (14)$$

➲ time unavailability ($I(T_A)$):

$$I(T_A)=1-D(T_A) \quad (15)$$

For a system made up of two electro pumps in stand-by redundancy, the comparison between the results obtained analytically (through Markov model) and these, obtained through simulation will be done [5]. The results are synthesized in table 1. The fault mean rates (λ_A, λ_B) and the recovery mean rates (μ_A, μ_B) of the two pumps, analysis time (T_A) and simulations number(N_S) have the following values: $\lambda_A=\lambda_B=9,4\cdot10^{-4}$ faults/h, $\mu_A=\mu_B=95\cdot10^{-4}$ repairs/h; T_A=8760 h; $N_S=10^4$.

Analysing the results, we can notice that the proposed indicators have almost the same values. The determination of the indicators through Monte Carlo method has been done using the programme SIMFIAB [9].

This programme allows the determination of reliability and availability indicators previously mentioned, for a system made up of two elements in stand-by redundancy, using a procedure in order to simulate functioning and failure time.

***Table 1** – Comparing the results obtained through analytical methods and by simulation for a system made up of two electro pumps in stand-by redundancy*

Analytical obtained results (Markov model)	Simulation obtained results
Number of faults: $\nu(T_A)$=1,351372 faults/year	Number of systems faults: $\nu(T_A)$=1,3307999 faults/year
Mean time between failure: MTBF=6429,71 hours	Mean time between failure: MTBF=6529,63211853802 hours
Continuos failure time: MTM=52,631579 hours	Continuos failure time: MTM=52,8746443195851 hours
Total failure time: $\beta(T_A)$=71,124844 hours	Total failure time: $\beta(T_A)$=70,36557666 ore
Total no functioning time: $\alpha(T_A)$=8688,8752 hours	Total no functioning time: $\alpha(T_A)$=8689,63442334533
System's availability: $D(T_A)$=0,9918807	System's availability: $D(T_A)$=0,9919673999
System's unavailability: $I(T_A)$=0,0081192228	System's unavailability: $I(T_A)$=0,0080326000752
Reliability function, for T_A=8760 h; R(8760)=0,256039	R(8760)=0,2836999999
$\lambda_e=1,55528\cdot10^{-4}$ h^{-1}	$\lambda_e=1,5314799\cdot10^{-4}$ h^{-1}
μ_e=0,019 h^{-1}	μ_e=0,0189126 h^{-1}

3. CONCLUSISONS

1. We can successfully study the behaviour in time of subsystems in stand-by redundancy using Monte Carlo simulation method. For this study, through sequential simulation of functioning and failure time, a certain algorithm has been chosen. The Monte Carlo simulation method allows the determination of a larger number of reliability and availability indicators, both absolute value and assessed value. Also, these indicators can be reresented with distribution functions.

2. The results obtained through Monte Carlo simulation have almost the same values then the results obtained through analytical method (Markov model). So there are possibilities to extend the method in order to analyse some complex schemes.

3. The main disavantages of Monte Carlo method are connected to the larger time of simulation; also, the results depend on simulation number necessary to obtained precise values.

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