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THE STUDY OF DYNAMIC INSTABILITY OF THE STRUCTURES FROM BARS WITH THIN WALLS SUPPORTED ON ELASTIC ENVIRONMENT

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Abstract : Considering differential equations of the parametric vibrations of the bar frames with thin walls, supported on elastic frame, will be established equations of the critical pulsation of the excitation at limits of dynamic instability fields, having the form of algebraically equations of superior degree whose coefficients contain geometrical and elastically parameters of the frame. It is shown then that the dynamic instability fields will be situated as in the case of absence of elastic environment, being translated towards the area of higher frequencies.

1. INTRODUCTION

Considering a frame with thin walls and open sections on elastic environment of Winkler type, auctioned externally by variable periodic forces in time, which produce in bars of the frame of axial forces, also variable periodically in time, differential equations of the parametric forced vibrations are [4]:

$$C\frac{d^2T}{dt^2}+[D+F-\alpha\, a-\beta\phi(t)B]T=0 \tag{1}$$

T being an infinite vector, $T^T=\{T_1,T_2,T_3,........\}$, having as components vectors of time functions, corresponding to three displacements of the transversal section of the bars (two linear displacements in main directions of inertia and a rectangular displacement of rotation around the center of circumvolution - contortion), $T_K^T=\{T_k^u(t)T_k^v(t)T_k^\varphi(t)\}$. C, D, F, A, B being square infinite hyper matrix containing geometrical and elastic characteristics of the bar and the support environment and of the bar and the support environment and α and β two parameters.

2. LIMITS OF DYNAMIC INSTABILITY FIELDS

Equations (1) are a system of differential equation of second order, linear, homogenous and with variable coefficients. Establishing solutions of such equations beoing a problem known in the theory of equations with periodic coefficients [1], we will admit for Φ (t), harmonic simple form: Φ(t)=cosθt.

Representing the practical case that is most frequent and fundamental, given the fact that periodical excitation most of time can be disintegrated in simple harmonics. In this case, equations (1) will be written:

$$C\frac{d^2T}{dt^2}+[D+F-\alpha A-\beta\cos\theta B]T=0 \tag{2}$$

As the limits of the fields of dynamic instability of the system of differential equations (2) must admit periodic solution with the period of excitation and double of this period, for this will be searched form solutions:

$$T = \sum_{k=2,4,6,...}^{\infty} \left(a_k \sin\frac{k\theta t}{2} + b_k \cos\frac{k\theta t}{2} \right) \tag{3}$$

$$T = \frac{1}{2} b_0 + \sum_{k=2,4,6,...}^{\infty} \left(a_k \sin\frac{k\theta t}{2} + b_k \cos k\theta t \right) \tag{4}$$

With the aid of solutions (3) there are critical pulsations θ at limits of instability fields 1,3,5…..

$$\begin{vmatrix} D-F-\alpha A \pm \frac{1}{2}\beta B - \frac{1}{4}\theta^2 C & -\frac{1}{2}\beta B & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A - \frac{9}{4}\theta^2 C & -\frac{1}{2}\beta B \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A - \frac{25}{4}\theta^2 C \end{vmatrix} = 0 \tag{5}$$

With the aid of solutions (4) will be obtained determination equations of critical pulsations at limits of instability fields 2,4,6….

$$\begin{vmatrix} D-F-\alpha A & -\frac{1}{2}\beta B & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A - 4\theta^2 C & -\frac{1}{2}\beta B \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A - 16\theta^2 C \end{vmatrix} = 0 \tag{6}$$

$$\begin{vmatrix} D-F-\alpha A & -\frac{1}{2}\beta B & 0 & 0 \\ -\frac{1}{2}\beta B & D-F-\alpha A - \theta^2 C & -\frac{1}{2}\beta B & 0 \\ 0 & -\frac{1}{2}\beta B & D-F-\alpha A - 4\theta^2 C & -\frac{1}{2}\beta B \\ 0 & 0 & -\frac{1}{2}\beta B & D-F-\alpha A - 16\theta^2 C \end{vmatrix} = 0 \tag{7}$$

In practical cases of calculus, the determinants of the equations (5), (6) and (7) will be limited, obviously at determinants of *n* order and this way, those equations will be algebraically equations of *3n* degree, determining with approximation the critical pulsations of the excitation at limits of the first *3n* field of dynamic instability. When amplitude of the variable component of the assignment is very low (β→0) equations of critical pulsations at limits of fields of dynamic instability become:

$$\left| D + F - \alpha A - \frac{1}{4} k^2 \theta^2 \right| = 0; \quad k=1,2,3,.... \tag{8}$$

Equations of the pulsations of free pulsations of frame supported on elastically environment and loaded by static components of the excitation of parameter α is:

$$\left|D + F - \alpha A - \omega^2 C\right| = 0 \tag{9}$$

Confronting (8) and (9)will be deduced:

$$t = \frac{2\omega}{k}; \quad k=1, 2,3,\ldots\ldots \tag{10}$$

that is critical pulsation of the burden is an entire submultiples of double of the pulsations of free vibrations of the frame and absence of the static attempts and elastically support.

Borders of the first field of dynamic instability, main domain, the largest and most important from practical point of view, will be obtained with sufficient precision, considering only the first element of the determinant in the equations (5):

$$\left|D + F - \alpha A \pm \frac{1}{2}\beta B - \frac{1}{4}\theta^2 C\right| = 0 \tag{11}$$

Equations (11) can be written:

$$\left|R\right| = 0 \tag{12}$$

R being the cellular matrix having as elements square, third degree matrix,

$$\begin{Bmatrix} r_{ik}^{uu} & 0 & r_{ik}^{u\varphi} \\ 0 & r_{ik}^{vv} & r_{ik}^{v\varphi} \\ r_{ik}^{\varphi u} & r_{ik}^{\varphi v} & r_{ik}^{\varphi\varphi} \end{Bmatrix} \tag{13}$$

Whose elements are:

$$\begin{aligned}
r_{ik}^{uu} &= d_{ik}^{uu} + h_{ik}^{uu} - \alpha a_{ik}^{uu} \pm \frac{1}{2}\beta b_{ik}^{uu} - \frac{1}{4}\theta^2 c_{ik}^{uu}; \\
r_{ik}^{vv} &= d_{ik}^{vv} + h_{ik}^{vv} - \alpha a_{ik}^{vv} \pm \frac{1}{2}\beta b_{ik}^{vv} - \frac{1}{4}\theta^2 c_{ik}^{vv}; \\
r_{ik}^{\varphi\varphi} &= d_{ik}^{\varphi\varphi} + h_{ik}^{\varphi\varphi} - \alpha a_{ik}^{\varphi\varphi} \pm \beta b_{ik}^{\varphi\varphi} - \frac{1}{4}\theta^2 c_{ik}^{\varphi\varphi}; \\
r_{ik}^{u\varphi} &= h_{ik}^{u\varphi} - \alpha a_{ik}^{u\varphi} \pm \beta b_{ik}^{u\varphi} - \frac{1}{4}\theta^2 c_{ik}^{u\varphi}; \\
r_{ik}^{v\varphi} &= h_{ik}^{v\varphi} - \alpha a_{ik}^{v\varphi} \pm \beta b_{ik}^{v\varphi} - \frac{1}{4}\theta^2 c_{ik}^{v\varphi};
\end{aligned} \tag{14}$$

Elements of matrix (13) can be easily calculated in differential equations (1), time functions T(t) are given by decompositions in series of functions of the displacements of transversal sections of the frame's bars, in conformity with displacement of the bars as part of the basic system of the method of displacement under the action of unit displacement, auctioning on basic system. These elements, named also dynamic reactions, can be written with the aid of charts of dynamic reactions of the two types of bard, interfering in the method of displacements and situated on elastic environment.

The structure of the elements of the coefficients of dynamic reactions is complex enough,

$$d_{ik}^{uu} = \sum EI_y \int_0^1 \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$d_{ik}^{\varphi\varphi} = \sum EI_\omega \frac{dZ_i^{2\varphi}}{dz^2}\frac{dZ_k^{2\varphi}}{dz^2}dz + \sum GI_i \int_0^i \frac{dZ_i^\varphi}{dz}\frac{dZ_k^\varphi}{dz}dz;$$

$$c_{ik}^{uu} = \sum m\int_0^i Z_i^u(z)Z_k^u(z)dz + \sum mr_y^2 \int_0^i \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$f_{ik}^{\varphi\varphi} = \sum \int_0^i \left[k_x(y_0 - h_y)^2 + k_y(x_0 - h_x)^2 + k_\varphi \cdot \varphi\right] Z_i^\varphi(z)Z_k^\varphi(z)dz; \qquad (15)$$

$$a_{ik}^{uu} = \sum \int_0^i N_0(z)\frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$a_{ik}^{\varphi\varphi} = \sum (e_y\beta_1 + e_x\beta_2 + r_0^2)\int_0^i N_0(z)\frac{dZ_i^\varphi(z)}{dz}\frac{dZ_k^\varphi(z)}{dz}dz;$$

$$b_{ik}^{vv} = \sum \int_0^i N_i(t)\frac{dZ_i^v(z)}{dz}\frac{dZ_k^v(z)}{dz}dz;$$

$$b_{ik}^{v\varphi} = -\sum (x_0 - e_0)\int_0^i N_i(z)\frac{dZ_i^v(z)}{dz}\frac{dZ_k^\varphi(z)}{dz}dz;$$

$$c_{ik}^{vv} = \sum m\int_0^i Z_i^v(z)Z_k^v(z) + \sum mr_x^2 \int_0^i \frac{dZ_i^v(z)}{dz}\frac{dZ_k^\varphi(z)}{dz}dz; 1$$

They containing geometrical and elastic characteristics of the bar, as well as parameters of the excitation burden and elastic characteristics of the support environment.

The equation of free pulsations of the frame that was supported elastically is:

$$\left| D + F - \alpha A \pm \frac{1}{2}\beta B - \omega^2 C \right| = 0 \qquad (16)$$

3. CONCLUSIONS

Taking into account the equations (11) and (16) it can easily be observed that the presence of elastic environment allows unmodified the width of the fields of dynamic instability, realizing only their translation toward higher frequency

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