OVER THE PULSATION OF OWN VIBRATIONS OF BARS STRUCTURES WITH THIN WALLS SUPPORTED ON ELASTIC ENVIRONMENT

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Abstract : It will be first presented the modality of obtaining differential equations of vibration of bar frames with thin walls of open sections, supported on elastic environment Winkler, with the form of a system with triple infinity of differential equations of second degree. Then, it will be established that the algebraic equation of the pulsations and it is shown how their coefficients obtain the interpretation of dynamic reactions and can be calculated with the aid of basic system of the frame with the method of displacements, as influence coefficients.

1. INTRODUCTION

It is considered, for generality, a certain frame of bars with thin walls with open sections having bars supported on elastic environment (fig. 1).

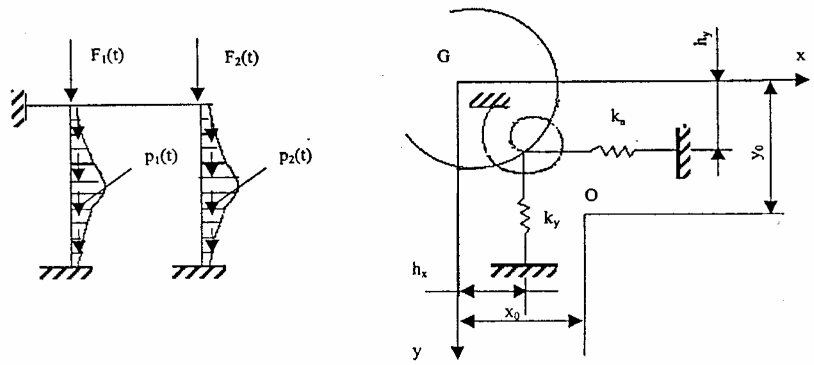

Fig 1. Frame of bars with thin walls on open sections supported on elastic environment

If *u*, *v* and *φ* will be linear displacements of the transversal section of a bar in main direction of their inertia in process of vibration of the frame and respectively their rotation around the center of circumvolution - contortion, kinetic energy and potential energy of deformation of the entire frame will be:

$$E_c = \sum\int m\left[\left(\frac{\partial u}{\partial v}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 - 2x_0\frac{\partial v}{\partial t}\frac{\partial \varphi}{\partial t} + 2y_0\frac{\partial u}{\partial t}\frac{\partial \varphi}{\partial t} + y_x^2\left(\frac{\partial^2 u}{\partial t\partial t}\right)^2 + y_y\left(\frac{\partial^2 v}{\partial t\partial t}\right)^2 + y_0^2\left(\frac{\partial \varphi}{\partial t}\right)^2 + y_\infty^4\left(\frac{\partial \varphi}{\partial t\partial t}\right)^2\right]dz \quad (1)$$

$$E_p = \frac{1}{2}\sum\int\left[EI_y\left(\frac{\partial^2 u}{\partial z^2}\right) + EI_x\left(\frac{\partial^2 v}{\partial z^2}\right) + GI_1\left(\frac{\partial \varphi}{\partial z^2}\right)^2 + EI_\infty\left(\frac{\partial^2 \varphi}{\partial z^2}\right)^2\right]dz \quad (2)$$

The integral being extended on the length of each bar of the frame and the amount of all bars of the frame.

The mechanical achievement of the reactions of the elastic environment, considered after the hypothesis of Winkler, for the entire frame will be:

$$L = -\frac{1}{2}\sum\int\left\{k_x u^2 + k_y v^2 + \left[k_x\left(y_0 - h_x\right)^2 + k_y(x_0 - h_x)^2 - k\varphi\right]\varphi^2 + 2k_x(y_0 - h_y)u\varphi - 2k_y(x_0 - h_x)v\varphi\right\}dz$$

Decomposing functions of displacement and rotation of *u (z,t)*, *v(z,t)* and *φ(z,t)* of the section in process of vibration of the frame, after functions of the circumvolution-contortion of the bars of the frame in basic system of the method of displacement under action of displacements on direction of supplementary connection under form:

$$\begin{aligned} u(z,t) &= \sum_{k=1}^{n} T_k^n(t), Z_k^u(z) \\ v(z,t) &= \sum_{k=1}^{n} T_k^v(t), Z_k^v(z) \\ \varphi(z,t) &= \sum_{k=1}^{n} T_k^\varphi(t), Z_k^\varphi(z) \end{aligned} \quad (3)$$

And then writing Lagrange equation for vibratory movement of the frame:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial q_i}\right) - \frac{\partial}{\partial q_1}\left(E_c - E_p\right) = Q_i \text{ ; i=1,2,….n}$$

Considered, in a generalized manner Q_I being function unknown in time T_u (t), T_i^V (t), T_i^φ (t), and generalized forces, $Q_i = \frac{\overline{\partial L}}{\partial q_i}$.

Results the system of differential equation of second homogenous linear order and correlated coefficients:

$$C\frac{d^2T}{dt^2} + (D + F)T = 0 \ , \quad (4)$$

T, being hyper vector:

$$T^T = \{T_1, T_2, ..., T_n\} \quad (5)$$

having as components,

$$T_k^n = \{T_k^n(t), T_k^v(t), T_k^\varphi(t)\} \quad (6)$$

and C, D and F square cellular matrix

$$C=\begin{Bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{Bmatrix}; D=\begin{Bmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ D_{n1} & D_{n2} & \cdots & D_{nn} \end{Bmatrix}; F=\begin{Bmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & F_{22} & \cdots & F_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ F_{n1} & F_{n2} & \cdots & F_{nn} \end{Bmatrix} \qquad (7)$$

having as elements

$$C_{ik}=\begin{Bmatrix} c_{ik}^{uu} & 0 & c_{ik}^{u\varphi} \\ 0 & c_{ik}^{vv} & c_{ik}^{v\varphi} \\ c_{ik}^{\varphi u} & c_{ik}^{\varphi v} & c_{ik}^{\varphi\varphi} \end{Bmatrix}; D_{ik}=\begin{Bmatrix} d_{ik}^{uu} & 0 & 0 \\ 0 & d_{ik}^{vv} & 0 \\ 0 & 0 & d_{ik}^{\varphi\varphi} \end{Bmatrix}; F_{ik}=\begin{Bmatrix} f_{ik}^{uu} & 0 & f_{ik}^{u\varphi} \\ 0 & f_{ik}^{vv} & f_{ik}^{v\varphi} \\ f_{ik}^{\varphi u} & f_{ik}^{\varphi v} & f_{ik}^{\varphi\varphi} \end{Bmatrix} \qquad (8)$$

Equations (4) expresses own vibrations of the frame on elastic environment. In this case, all components of hyper vector *T* varies harmonic in time,

$$T = T_o \sin pt \qquad (9)$$

so, resulting the algebraic equation of determination of pulsations *p*:

$$|D+F-p^2C|=0 \qquad (10)$$

One with equation:

$$|R|=0 \qquad (11)$$

Elements of hyper matrix R are square matrix of third order,

$$R_{ik}=\begin{Bmatrix} r_{ik}^{uu} & 0 & r_{ik}^{u\varphi} \\ 0 & r_{ik}^{vv} & r_{ik}^{v\varphi} \\ r_{ik}^{\varphi u} & r_{ik}^{\varphi v} & r_{ik}^{\varphi\varphi} \end{Bmatrix} \qquad (12)$$

which have as elements,

$$\begin{aligned} & r_{ik}^{uu}=d_{ik}^{uu}+f_{ik}^{uu}-c_{ik}^{uu}; \quad r_{ik}^{vv}=d_{ik}^{vv}+f_{ik}^{vv}-c_{ik}^{vv}; \quad r_{ik}^{\varphi\varphi}=d_{ik}^{\varphi\varphi}+f_{ik}^{\varphi\varphi}-c_{ik}^{\varphi\varphi} \\ & r_{ik}^{u\varphi}=f_{ik}^{u\varphi}-p^2c_{ik}^{u\varphi}; \quad r_{ik}^{v\varphi}=f_{ik}^{v\varphi}-p^2c_{ik}^{v\varphi}; \\ & r_{ik}^{\varphi u}=f_{il}^{\varphi u}-p^2c_{ik}^{\varphi u}; \quad r_{ik}^{v\varphi}=f_{ik}^{v\varphi}-p^2c_{ik}^{\varphi v} \end{aligned} \qquad (13)$$

The structure of coefficients of matrix (13) has the form:

$$d_{ik}^{uu}=\sum EI_y\int_0^i \frac{dZ_i^u(z)}{dz}\frac{dZ_k^u}{dz}dz;$$

$$d_{ik}^{\varphi\varphi}=\sum EI_{\omega}\int_{0}^{i}\frac{dZ_i^{2\varphi}(z)}{dz^2}\frac{dZ_k^{2\varphi}(z)}{dz^2}dz+\sum GI_i\int\frac{dZ_i^{\varphi}(z)}{dz}\frac{dZ_k^{\varphi}}{dz}dz;$$

$$c_{ik}^{uu}=\sum m\int_{0}^{i}Z_i^u(z)Z_k^u(z)dz+\sum mr_y^2\int_{0}^{i}\frac{dZ_i^u(z)}{dz}\frac{dZ_k^u(z)}{dz}dz;$$

$$c_{ik}^{\varphi\varphi}=\sum mr_0^2\int_{0}^{i}Z_i^{\varphi}(z)Z_k^{\varphi}(z)dz+\sum mr_{\omega}^4\int_{0}^{i}\frac{dZ_i^{\varphi}}{dz}\frac{dZ_k^{\varphi}(z)}{dz}dz; \quad (14)$$

$$c_{ik}^{u\varphi}=\sum my_0\int_{0}^{i}Z_i^u(z)Z_k^{\varphi}(z)dz; \qquad c_{ik}^{v\varphi}=-\sum mx_0\int_{0}^{1}Z_i^v(z)Z_k^{\varphi}(z)dz;$$

$$f_{ik}^{uu}=\sum k_x\int_{0}^{1}Z_i^u(z)Z_i^u(z)dz; \qquad f_{ik}^{u\varphi}=\sum\int_{0}^{i}k_x(y_0-h_y)Z_i^u(z)Z_k^{\varphi}(z)dz;$$

$$f_{ik}^{\varphi\varphi}=\sum\int_{0}^{i}\left[k_x\left(y_0-h_y\right)^2+k_y(x_0-h_x)^2+k_{\varphi}\cdot\varphi\right]Z_i^{\varphi}(z)Z_k^{\varphi}(z)dz;$$

2. CONCLUSIONS

Comparing the equation (10) of free pulsations of frame on elastic environment with equation of free vibration, in absence of elastic environment

$$|D-p^2C|=0 \quad (15)$$

it can be observed that the influence of the elastic environment of support is made of increasing pulsation of free vibrations.

Still, it can be shown that [4] the elements of the matrix R_{ik} having name of dynamic reactions can be interpreted as influence coefficients, common in calculus of frames in conformity with the method of displacements [1].

In this case, the order of matrix C, D and F will be equal with the degree of non determination of the frame in the calculus in conformity with method of displacements.

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