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MATHEMATICAL MODEL TO CALCULATE FRESH CHARGE FLOW THROUGH THE ORIFICE CONTROLED BY THE INTAKE VALVE

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Abstract: The admission of fresh charge in the cylinder takes place due to the depression created by the piston movement from TDC to BDC after intake valve is opened. Because the variable flow section, the flow coefficient is also variable. Usually, flow coefficient through a variable section is measured experimentally. A mathematical relation between orifice flow coefficient and valve movement was introduced in order to compute fresh charge flow through the orifice controlled by the intake valve. Calculated values for cylinder pressure were compared with measured data.

1. Pressure losses in the intake manifold

In the intake manifold fresh charge pressure drops because the friction between the fluid and the manifold walls and because the variation of flow direction and flow sections. Thus, in a normally aspirated engine the pressure of fresh charge before the intake valve is smaller than atmospherically pressure.

$$p_{sa}=p_0-\Delta p_{sa} \quad (1)$$

where: - p_{sa} [MPa] – the pressure in front of the intake valve
- p_0 [MPa] – the atmospherically pressure
- Δp_{sa} [MPa] – the pressure drop in the intake manifold

The mathematical model is made for an one cylinder Diesel engine and in order to simplify calculation the air filter was taken out. The scheme for the system is presented in Figure 1.

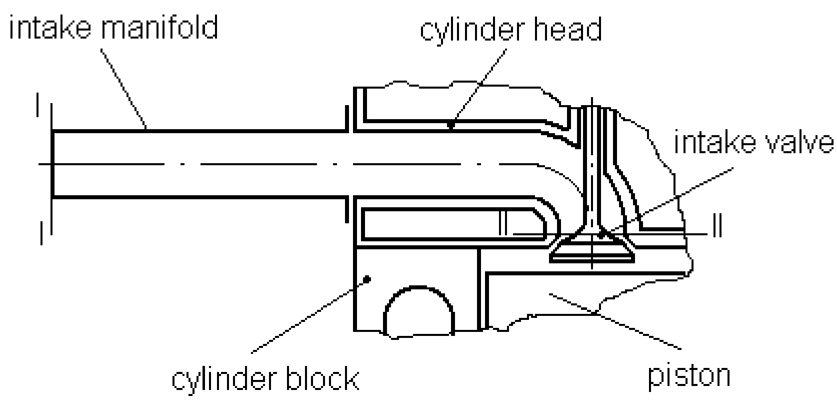

Figure 1. Computation constructive scheme

Pressure drop in the intake manifold, between sections I-I and II-II (see figure 1), has two components:
- Δp_l [MPa] – pressure drop due to friction between fresh charge and intake manifold walls (linear losses);
- Δp_s [MPa] – pressure drop due to flow section and flow direction variations (local losses);

To calculate linear losses some theoretical hypothesis were made:
- the flow in the intake manifold is considered to take place at constant temperature;

- the friction between the gas and the manifold walls depends on the gas flow speed and the quality of the walls surface (roughness);

The general formula for pressure losses in a fluid flow through a manifold is:

$$\Delta p = \gamma \cdot h_d \tag{2}$$

where: - Δp [Pa] – pressure losses
- γ [N/m^3] – specific weight
- h [m] – charge loss

Total charge losses have two components:

$$h = h_l + h_s \tag{3}$$

where: - h_l [m] – linear charge losses
- h_s [m] – local losses due to section and flow direction variations

Linear charge losses are calculated with Darcy's formula:

$$h = \lambda \cdot \frac{l}{D} \cdot \frac{W^2}{2 \cdot g} \tag{4}$$

where: - λ - the coefficient of linear charge losses
- l [m] – the length of the manifold part (Darcy's formula can be applied only for constant inner diameter manifold)
- D [m] – the inner diameter of the manifold
- W [m/s] – the flow speed
- g=9,81 m/s^2 - gravitational acceleration

Flow regime is characterized by Reynolds number, which can be calculated with the formula:

$$\mathrm{Re} = \frac{W \cdot D}{\nu} \tag{5}$$

where: - ν [m^2/s] – the kinematic viscosity
- for W and D see formula (4)

For air at 293 K:

ν=15, 1 m^2/s

The coefficient of linear charge losses is calculated by applying Moody's formula for $4 \cdot 10^3 < \mathrm{Re} < 10^7$:

$$\lambda = 0{,}0055 \cdot \left[1 + \left(20000 \cdot \varepsilon + \frac{10^6}{\mathrm{Re}}\right)\right]^{1/3} \tag{6}$$

where : - $\varepsilon = \delta / D$
- δ [m] – the mean roughness of the manifold's inner walls

As it is shown in figure 1 the manifold has the same inner diameter as the channel in the cylinder head so the entire length of the intake system is at the same inner diameter.

Local charge losses are calculated with Weissenbach formula:

$$h_s = \xi \cdot \frac{W^2}{2 \cdot g} \tag{7}$$

The local loss is given by the change with 90^0 of the flow direction. In this case [panaitescu]:

ξ=1,2

The total pressure losses will be:

$$\Delta p = \gamma \cdot (h_l + h_s) = \gamma \cdot \left(\lambda \cdot \frac{l}{D} + \xi \right) \cdot \frac{W^2}{2 \cdot g} \qquad (8)$$

For air at 293 K:
γ=11,77 N/m^3
The fresh charge pressure in front of the intake valve p_{ga} will be:

$$p_{ga} = p_0 - \Delta p \qquad (9)$$

2. The flow through the orifice controlled by the intake valve

For calculating pressure losses in the intake manifold it is necessary to know the flow speed W from the relations (4), (5), (7) and (8). This speed is equal with that in section II-II (see fig.1 and fig. 2).

Figure 2 presents a scheme for the flow through the orifice.

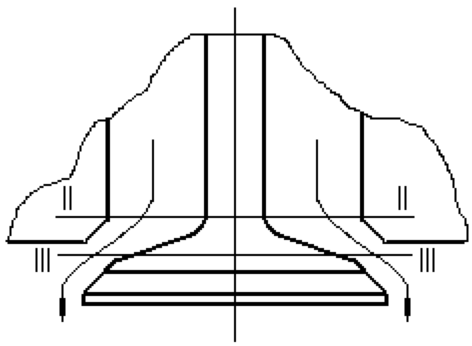

Figure 2. The flow through the orifice controlled by the intake valve

The flow rate through this orifice will be:

$$\dot{Q}_V = \mu \cdot A_{sa} \cdot \sqrt{\frac{2 \cdot |p_{ga} - p_{cil}|}{\rho}} \qquad (10)$$

where: - $\dot{Q}_V$ [m^3/s] – the flow rate through the orifice
- μ - the instantaneously flow coefficient of the orifice controlled by the intake valve
- A_{sa} [m^2] – instantaneously flow section (in section III-III from fig. 2)
- p_{ga} [Pa] – fresh charge pressure in section II-II (see rel. (9)
- p_{cil} [Pa] – fluid pressure inside the cylinder (it's considered to be equal with the pressure in section III-III from figure 2)
- ρ=1,204 kg/m^3 – the fluid's density at 293 K

The flow speed of fresh charge in section II-II will be:

$$W = \frac{4 \cdot \dot{Q}_V}{\pi \cdot D^2} \qquad (11)$$

Introducing this value in rel. (8) the pressure losses in the intake manifold can be calculated.

The mass of fresh charge that passes through the orifice in one second will be:

$$\dot{Q}_m = \rho \cdot \dot{Q}_V \qquad (12)$$

where: - $\dot{Q}_m$ [kg/s] – mass flow rate

For ρ and $\dot{Q}_V$ see rel. (10).

To simplify calculus exhaust valve is closed in TDC and intake valve is opened at the same time. In this moment inside the cylinder there is a mixture of burned gases containing carbon dioxide (CO_2), water (H_2O), oxygen (O_2) and nitrogen (N_2).

The status general equation for burned gases at the end of the exhaust process is:

$$p_e \cdot V_c = m_{ga} \cdot R_{ga} \cdot T_e \tag{13}$$

where: - p_e [Pa] – burned cases pressure in the cylinder at the end of the exhaust process

- V_c [m^3] – dead volume of the burning chamber (the volume of the burning chamber when the piston is at TDC)
- m_{ga} [kg] – the mass of burned gases situated in the cylinder at the end of the exhaust process
- R_{ga} [J/kmolK] – burned gases constant
- T_e [K] – burned gases temperature at the end of the exhaust process

The burned gases constant is determined with the general formula:

$$R_{ga} = \frac{1}{\nu_{ga}} \cdot \left(\nu_{CO_2} \cdot R_{CO_2} + \nu_{H_2O} \cdot R_{H_2O} + \nu_{O_2} \cdot R_{O_2} + \nu_{N_2} \cdot R_{N_2}\right) \tag{14}$$

where: - ν_{ga} [kmol] – total number of kmols of burned gases that occupies the dead volume at the end of the exhaust process

- ν_{CO_2}, ν_{H_2O}, ν_{O_2}, ν_{N_2} [kmol – total number of kmols of CO_2, H_2O, O_2, N_2 that occupies the dead volume at the end of the exhaust process
- R_{CO_2}, R_{H_2O}, R_{O_2}, R_{H_2} [J/kmolK] – CO_2, H_2O, O_2, N_2 constant

Now, from rel. (14) it is possible to calculate burned gases mass that's inside the cylinder at the end of the exhaust process

3. Instantaneously flow coefficient and flow area for the orifice controlled by the intake manifold

Instantaneously flow area depends on the intake valve displacement. The notations used to write the formulae for the flow area are presented in figure 3.

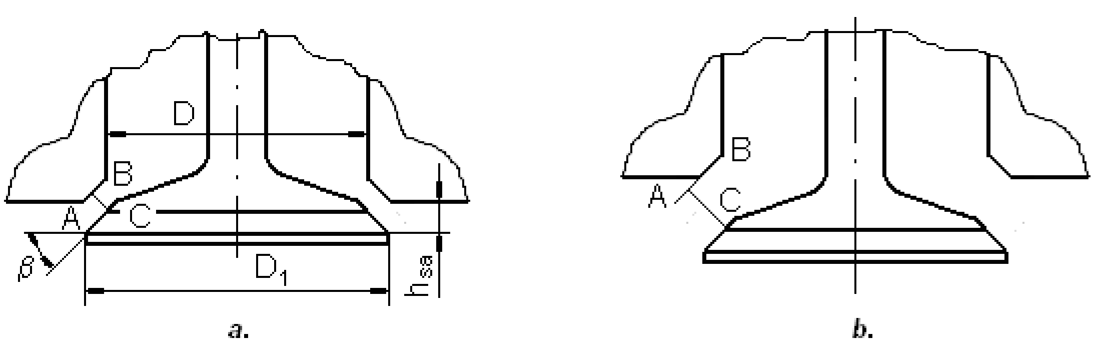

Figure 3. Intake valve dimensions

There are two formulae to calculate instantaneously flow area. If the perpendicular line raised from point C crosses the segment AB (see fig. 3), the flow area will be:

$$A_{sa} = \pi \cdot h_{sa} \cdot (D + 0{,}5 \cdot h_{sa} \cdot \sin(2\gamma)) \tag{15}$$

If the perpendicular line raised from point C is outside the segment AB:

$$A_{sa} = \pi \cdot \frac{D_1 + D}{2} \sqrt{\left(h_{sa} - \frac{D_1 - D}{2} tg\gamma\right)^2 + \left(\frac{D_1 - D}{2}\right)^2} \qquad (16)$$

where: - D [m] – the small diameter of the valve head (equal with intake manifold inner diameter as it is shown in fig 3a)

- D_1 [m] – the big diameter of the valve head (see fig. 3a)
- h_{sa} [m] – the intake valve displacement
- β [^{0}] – the inclination of the valve head

The critical displacement, when the perpendicular line raised from point C crosses the segment AB in point A, is:

$$h_{cr} = \frac{D_1 - D}{\sin(2\gamma)} \qquad (17)$$

For $h_{sa} \leq h_{cr}$ the formula to calculate flow area will be rel. (15) and if $h_{sa} > h_{cr}$ rel. (16) will be applied.

For the instantaneously flow coefficient through the orifice it's variation with the valve displacement is considered to be the one from figure 4.

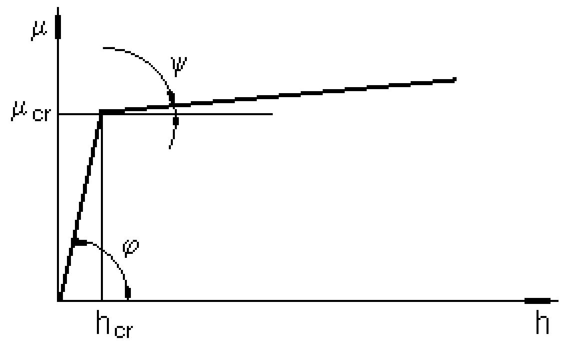

Figure 4. Instantaneously flow coefficient variation

It's obviously that:

- for $h_{sa} \leq h_{cr}$

$$\mu = h_{sa} \cdot tg\varphi \qquad (18)$$

- and for $h_{sa} > h_{cr}$

$$\mu = \mu_{cr} + \left(h_{sa} - h_{cr}\right) \cdot tg\psi \qquad (19)$$

4. Heat exchange between fresh charge and combustion chamber walls

Combustion chamber walls are heated by the hot gases burned in the previous cycle. At a stabilized regime wall temperature at the end of the exhaust process is almost constant. During the intake process fresh charge is heated due to the contact with the hot walls. Because of the turbulent flow inside the cylinder convective heat transfer occurs in the entire mass of the fluid.

At the end of the exhaust process residual burned gases that remains in the cylinder are at a temperature T_e which is higher than the wall temperature T_p. So, after intake valve opens and the first amounts of fresh charge flow inside the cylinder, residual burned gases heat the fresh charge and the walls. When the quantity of residual gases becomes smaller in comparison with the quantity of fresh charge in the cylinder, gas temperature in combustion chamber drops and heat exchange takes place from the walls to the working fluid.

Applying Newton's law, the heat flux exchanged between combustion chamber walls and fluid in a convective heat transfer is:

$$\dot{Q} = \chi \cdot S \cdot \left(t_p - t_f\right) \qquad (20)$$

where: - $\dot{Q}$ [W] – heat flux
- χ [W/m^{2}K] – heat exchange coefficient
- S [m^2] – heat exchange surface
- t_p [K] – combustion chamber walls temperature
- t_f [K] – fluid temperature

$$\chi = Nu \cdot \frac{\lambda_f}{l} \qquad (21)$$

where: - Nu – Nusselt non-dimensional criteria
- λ_f [W/mK] - thermal conductivity of the fluid (air)
- l [m] – the characteristic dimension (in this case is d the cylinder bore)

To calculate Nu criteria the following empirical equation determined by Dittus-Boelter was used:

$$Nu = 0{,}023 \cdot \mathrm{Re}^{0{,}8} \cdot \mathrm{Pr}^{n} \qquad (22)$$

where: - Re – Reynolds non-dimensional criteria

$$\mathrm{Re} = \frac{W_{III-III} \cdot d}{\nu} \qquad (23)$$

- $W_{III-III}$ [m/s] – fluid flow speed in section III-III (see fig. 2)
- d [m] – cylinder inner diameter
- ν [m^2/s] – the kinematic viscosity of air
- Pr – Prandtl non-dimensional criteria

$$\mathrm{Pr} = \frac{\eta \cdot c_p}{\lambda_f} \qquad (24)$$

- η [Ns/m^2] – the dynamic viscosity
- c_p [J/kgK] – the specific heat at constant pressure

For λ_f see relation (21)

n=0,3 if $t_f>t_p$ and n=0,4 if $t_f<t_p$
- t_f [K] – the fluid temperature
- t_p [K] – the walls temperature

Now it is possible, by applying rel. 20, to calculate the heat exchanged between the working fluid and the cylinder walls in a time interval $\Delta\tau$:

$$Q_p = \dot{Q} \cdot \Delta\tau \qquad (25)$$

- Q_p [J] – the exchanged between the working fluid and the cylinder walls

The fresh charge enters in the cylinder at the environmental temperature T_0 and it's heated until reaches the temperature of the fluid inside the cylinder.

$$Q_f = m_\tau \cdot c_p \cdot (t_f - T_0) \qquad (26)$$

- Q_f [J] – the heat received by the fresh charge that enters in the cylinder in the time interval $\Delta\tau$
- m_τ [kg] – the amount of fresh charge that enters in the cylinder in the time interval $\Delta\tau$

The total heat exchanged by the working fluid in the time interval $\Delta\tau$ will be:

$$Q = Q_p - Q_f \qquad (27)$$

The fluid temperature inside the cylinder will be:

$$t_{f_2} = t_{f_1} + \frac{Q}{m_f \cdot c_p} \tag{28}$$

- t_{f_1}, t_{f_1} [K] – the initial, respective the final temperature of the working fluid

The pressure of the working fluid inside the cylinder at one moment will be:

$$p_{cil_\alpha} = \frac{m_\alpha \cdot R_f \cdot t_{f_2}}{V_\alpha} \tag{29}$$

where: - p_{cil_α} [Pa] – the instantaneously pressure of the working fluid inside the cylinder
- m_α [kg] – the mass of fluid that's inside the cylinder at that moment
- R_f [J/kgK] – the working fluid constant
- t_{f_2} [K] – the working gas temperature
- V_α [m^3] – the instantaneously volume of the cylinder

$$V_\alpha = V_c + \frac{\pi \cdot d^2}{4} \cdot s_p \tag{30}$$

- V_c [m^3] – the dead volume of the cylinder
- d [m] – the cylinder inner diameter
- s_p [m] – the piston displacement

$$s_p = r \cdot \left[1 - \cos\alpha + \frac{\Lambda}{4} \cdot (1 - \cos 2\alpha)\right] \tag{31}$$

- r=S/2 [m] – where S [m] the piston stroke
- Λ=r/b – the connecting rod ratio
- b [m] – the connecting rod length
- α [^{0}] – rotational angle of the crankshaft

5. The mathematical model

The initial conditions in the cylinder are the pressure and the temperature of the burned gases at the end of the exhaust process. For calculation the mean pressure at the end of the exhaust process for 1000 cycles was taken in consideration. The temperature was measured in the exhaust manifold. In the moment when the intake valve is opening fluid speed in section II-II (see fig. 2) is W=0.

Now it is possible to calculate the mass of burned gases that occupies the dead volume at the end of the exhaust process.

$$m_{ga} = \frac{p_e \cdot V_c}{R_{ga} \cdot T_e} \tag{32}$$

where – p_e [Pa] – burned gases pressure at the end of the exhaust process
- T_e [K] – burned gases temperature at the end of the exhaust process
- R_{ga} [[J/kgK] – burned gases constant

With rel. (12) the mass rate of fresh charge through the orifice controlled by the intake valve can be calculated. Further, applying rel. (10), results the flow rate and the flow speed of fresh charge in section II-II is calculated with rel. (11). In the moment in which intake valve is opening the pressure in front of it is $p_{ga}=p_0$ (the atmospheric pressure). The flow speed in section III-III is calculated with the relation:

$$W_{cil} = \frac{Q_v}{A_{sa}} \tag{33}$$

Further pressure losses and pressure in front of the intake valve are calculated.

The time interval for computation was fixed for 0,6 degrees of crankshaft rotation which corresponds to 10^{-4} s. The mass of fresh charge that enters in the cylinder in this time interval will be:

$$m_{fp_\alpha} = Q_m \cdot \Delta t \tag{34}$$

Now, with relations (25), (26) and (27) heat exchange is calculated, and after that the temperature t_{f_2} is determined with rel. (28). The pressure in the cylinder is calculated with (29) and it's introduced as initial data for the next pace.

6. Conclusions

To validate the model was made a comparison between calculated and measured pressure inside the cylinder. This is shown in figure 5.

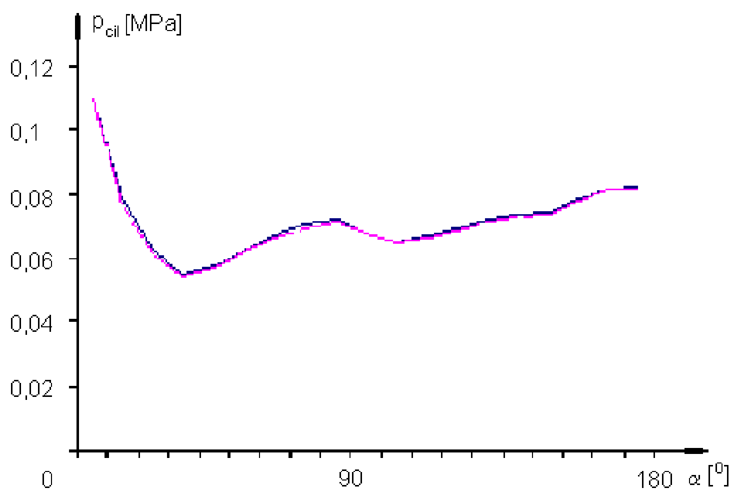

Figure 5. Comparison between calculated and measured data

As figure 5 shows, the model offers a sufficient precision to calculate fresh charge flow through the orifice controlled by the intake valve. The hypothesis that the flow coefficient has a linear variation is not good only when the flow section is very small, at the beginning of intake process.

This model can also be used to calculate exhaust process

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