ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

ON THE CONTROLLER DESIGN METHODS FOR HIGH-ORDER DYNAMICAL SYSTEMS

Dumitru NICOARA
Prof. Dr., Transilvania University of Brasov
tnicoara@unitbv.ro

Keywords: controller design, large-scale dynamic systems, projection methods

Abstract: In this paper, reduced order controller for high-order dynamical systems will be obtained by matching a subset of the frequency and power moments. High and low frequency moments are being simultaneously matched. A straightforward approach to constructing reduced-order models was obtained by explicitly computing 2*r* moments of the original system, using numerical algorithms implemented in MATLAB.

1. INTRODUCTION

The design of low-order controllers for high-order plants is a challenging problem, both theoretically as well as from a computational point of view.
Designs for control of large systems and structures are often based on mathematical models constructed by finite element techniques together with experimental data, and so frequently, they yield large state-space dimensions (on the order of tens of thousands to millions) upon discretization. Advanced controller design methods such as LQG/LTR loop-shaping, H_2/H_∞ control design, μ-synthesis and linear matrix inequalities (LMIs) typically produce controllers with orders comparable to the order of the plant. Highly accurate models desired for feedback control often lead to high-order controllers. These high-order controllers are not practical for real-time applications. It is an important problem to consider how to achieve reduced-order models and controllers while maintaining the desired performance during real-time implementation.

In general, however, the order of these modern controllers tends to be too high for practical use. For many reasons, simple controllers are preferred over complex ones. Thus, model reduction methods capable of addressing controller reduction problems are of primary importance to allow the practical applicability of modern controller design methods for high-order systems.

2. CONTROLLER ORDER REDUCTION METHODS

2.1. Projection methods

Consider a linear, continuous time- invariant (LTI) large-scale dynamical system or plant, single-input single-output (SISO)

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}u(t)\\ y(t) &= \mathbf{C}\mathbf{x}(t)+\mathbf{D}u(t),\ \mathbf{x}(0)=\mathbf{x}_0\end{aligned} \quad \Leftrightarrow \quad \sum = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix} \in \mathrm{R}^{(n+p)\times(n+m)} \tag{1}$$

where the state vector $\mathbf{x}(t)\in \mathrm{R}^n$, and $u(t), y(t)$ are the scalar input and output, respectively. The transfer function is

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}, \tag{2}$$

and we can rewrite it as

$$\mathbf{G}(s) = \mathbf{C}\,G_B(s) = G_C^T(s)\mathbf{B}, \tag{3}$$

where $G_B(s)$ and $G_C^T(s)$ are the solution to the linear systems

$$(s\mathbf{I} - \mathbf{A})G_B(s) = \mathbf{B} \tag{4}$$
$$G_C^T(s)\,(s\mathbf{I} - \mathbf{A}) = \mathbf{C} \tag{5}$$

and the model reduction problem becomes the one of finding approximate solutions $G_{B,m}(s)$ and $G_{C,m}^T(s)$ to $G_B(s)$ and $G_C^T(s)$, respectively, such that the Petrov-Garlekin conditions [8] are satisfied. In this case one can obtain the reduced-order models as

$$\dot{\mathbf{x}}_r(t) = \underbrace{\mathbf{W}^T\mathbf{A}\mathbf{V}}_{:=\mathbf{A}_r}\mathbf{x}_r(t) + \underbrace{\mathbf{W}^T\mathbf{B}}_{:=\mathbf{B}_r}\mathbf{u}(t) \Leftrightarrow \dot{\mathbf{x}}_r(t) = \mathbf{A}_r\mathbf{x}_r(t) + \mathbf{B}_r\mathbf{u}(t) \tag{6}$$

$$\mathbf{y}_r(t) = \underbrace{\mathbf{C}\mathbf{V}}_{:=\mathbf{C}_r}\mathbf{x}_r(t) + \underbrace{\mathbf{D}}_{:=\mathbf{D}_r}\mathbf{u}(t) \Leftrightarrow \mathbf{y}_r(t) = \mathbf{C}_r\mathbf{x}_r(t) + \mathbf{D}_r\mathbf{u}(t) \tag{7}$$

where **W** and **V** are the matrices that form the projector $\mathbf{\Pi} = \mathbf{V}\mathbf{W}^T;\ \mathbf{V}, \mathbf{W}^T \in \mathrm{R}^{n\times r}$ with $\mathbf{W}^T\mathbf{V} = \mathbf{I}_r$ where $\mathbf{I}_r$ is the identity matrix of size *r*, and $\mathbf{A}_r \in \mathrm{R}^{r\times r}$, $\mathbf{B}_r \in \mathrm{R}^{r\times m}$, $\mathbf{C}_r \in \mathrm{R}^{p\times r}$, $\mathbf{D}_r \in \mathrm{R}^{p\times m}$.

The aim is to achieve reduced-order controllers that are guaranteed to stabilize the closed-loop system when implemented in a closed-loop framework using the original building structure as the large-scale plant. To this end, one considers the feedback control loop as depicted in Fig. 1, and a stabilizing high-order controller **K**(s), with closed-loop performance index defined as I (**G**(s),**K**(s)), one seeks a low-order controller $\mathbf{K}_r$(s), with $r << n$, such that (**G**(s),$\mathbf{K}_r$(s)) is a stable closed-loop system and I (**G**(s),**K**(s)) ≈ I (**G**(s),$\mathbf{K}_r$(s)).

The unifying feature of all model and controller reduction techniques is that they are obtained by means of a projection. This process applies to controller and the closed-loop system as well.

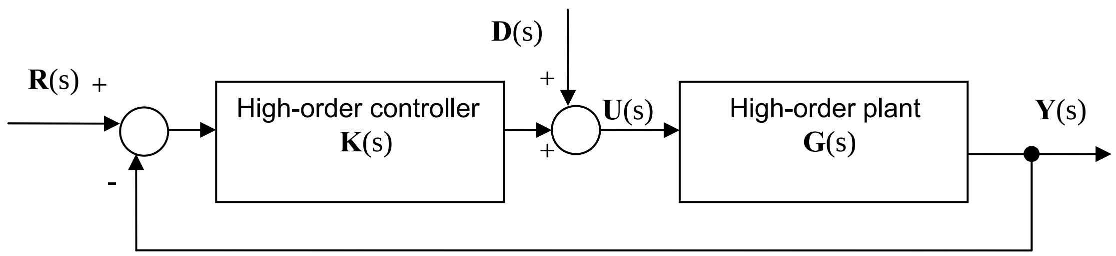

Figure 1 Feedback configuration.

2.2 Frequency weighted model reduction

Controller reduction problems are often formulated as special frequency weighted model reduction problems, where the frequency weights are chosen to enforce closed-loop stability and acceptable performance degradation, when the low-order controller is used in the original closed-loop system.

Let **G**(s) be the transfer of an n^{th} order time-invariant, continuous-time plant with state-space realization $\mathbf{G}(s)=\begin{bmatrix}\mathbf{A} & | & \mathbf{B}\\ - & - & -\\ \mathbf{C} & | & \mathbf{D}\end{bmatrix}$ and let **K**(s) be a stabilizing high-order (n_k^{th} order) controller with state-space realization $\mathbf{K}(s)=\begin{bmatrix}\mathbf{A}_{\mathrm{K}} & | & \mathbf{B}_{\mathrm{K}}\\ - & - & -\\ \mathbf{C}_{\mathrm{K}} & | & \mathbf{D}_{\mathrm{K}}\end{bmatrix}$.

The controller reduction problem is to seek a low order controller $\mathbf{K}_r(s)$ of order $r << n_k$ to replace **K**(s) such that the closed-loop stability and performance are preserved. The controller reduction problem can be recast as a frequency weighted model reduction if one regards the closed-loop system with $\mathbf{K}_r(s)$ replacing **K**(s) as being equivalent to that of Fig. 2. It is known from [1] that $\mathbf{K}_r(s)$ is a stabilizing controller if

- **K**(s) and $\mathbf{K}_r(s)$ have the same number of unstable poles and no poles on the imaginary axis; and
- Either

$$\left\|[\mathbf{K}(s)-\mathbf{K}_r(s)]\mathbf{G}(s)[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}\right\|_{H_\infty}<1$$

or

$$\left\|[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}\mathbf{G}(s)[\mathbf{K}(s)-\mathbf{K}_r(s)]\right\|_{H_\infty}<1$$

This can be thought as of a minimization of the weighted error given by

$$\|\mathbf{W}_0(s)(\mathbf{K}(s)-\mathbf{K}_r(s))\mathbf{W}_i(s)\|_{H_\infty}$$

where, to ensure closed-loop stability, one can choose the input and output weights as

$$\mathbf{W}_i(s)=\mathbf{I}\ ,\quad \mathbf{W}_0(s)=[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}\mathbf{G}(s)\ ,\ \text{or}$$

$$\mathbf{W}_0(s)=\mathbf{I}\quad \mathbf{W}_i(s)=\mathbf{G}(s)[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}.$$

On the other hand, to preserve closed-loop performance, one can use a two sided weighting of the form

$$\mathbf{W}_0(s)=[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}\mathbf{G}(s)$$
$$\mathbf{W}_i(s)=[\mathbf{I}+\mathbf{K}(s)\mathbf{G}(s)]^{-1}.$$

Following the same structure as the Enns's [2] frequency-weighted balanced reduction method, the reduced order controller can be obtained as in Proposition1 [8].

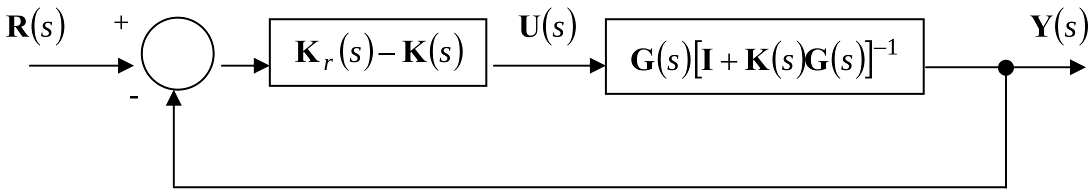

Figure 2 Modified Feedback Configuration

3. KRYLOV –BASED CONTROLLER ORDER REDUCTION

3.1. Moments matching method

Krylov techniques are based on moment matching, where one attempts to match the coefficients of a power series expansion of the transfer function for the original and reduced-order models [2].
If the transfer functions are expanded in a Laurent series around a given point in the complex plane, $\sigma \in C$ then

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \sum_{i=0}^{\infty} m_i(\sigma - s)^i \tag{8}$$

where the coefficients denoted by m_i are called moments of the system at a point σ.
Similarly, one obtains the moments of the reduced-order model as m_{ri} . The main idea of model-order reduction by moment matching is to match a given number of moments as $m_i = m_{ri}$, $i = 1,2,...,l$, $l << n$ of the original and reduced-order transfer functions.

A straightforward approach to constructing reduced-order models can be obtained by explicitly computing 2*r* moments of the original system, where *r* is the size of the reduced-order model. Then, the frequency response of the reduced-order system is forced to correspond to the selected moments. This can be viewed as a selection of the coefficients for the numerator and denominator of the reduced-order transfer function through the solution of a linear system involving Hankel matrices.
In [6] and [7], based by Krylov's subspace method and the moment expansion as in Eq. (8), we achieved two numerical algorithms for model reduction which were implemented in MATLAB.

In this paper we use these algorithms for obtain the controller order-reduced by matching a subset of the frequency and power moments. Reduced order models that do not preserve transfer function equivalence will be obtained.

Matching low frequency moments guarantee that the steady-state values will be preserved. For instance, the steady-state value to a step response is matched when the first low frequency moment is matched (see example). High and low frequency moments can be simultaneously matched.

3.2. Illustrative Example

The use of the model reduction procedures discussed so far will be illustrated by one example, where the dynamical controlled system is the third order stable SISO system with transfer function:

$$G(s) = \frac{-2(s-2.46)(s+4.46)}{(s+2)(s^2+5s+10)} \tag{9}$$

Notice that this transfer function has a zero on the right half of the complex plane, hence it is non-minimum phase. First, three different reduced order models of order one have been produced using the techniques introduced in this paper. The bode diagrams of the full order model and the reduced order models is depicted in Figure 3. The impulse response and the step response are given respectively in Figure 4 (a) and 4 (b).
All the obtained models and the H_2 and H_∞ norms of the model error have been obtained using Algorithm 1-3 [7].

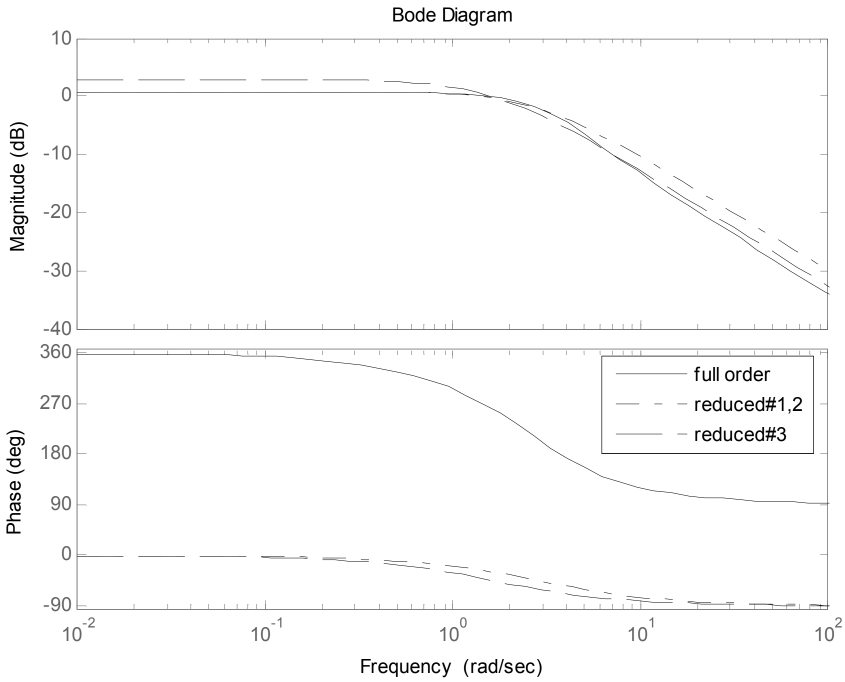

Figure 3 Bode plot: magnitude and phase

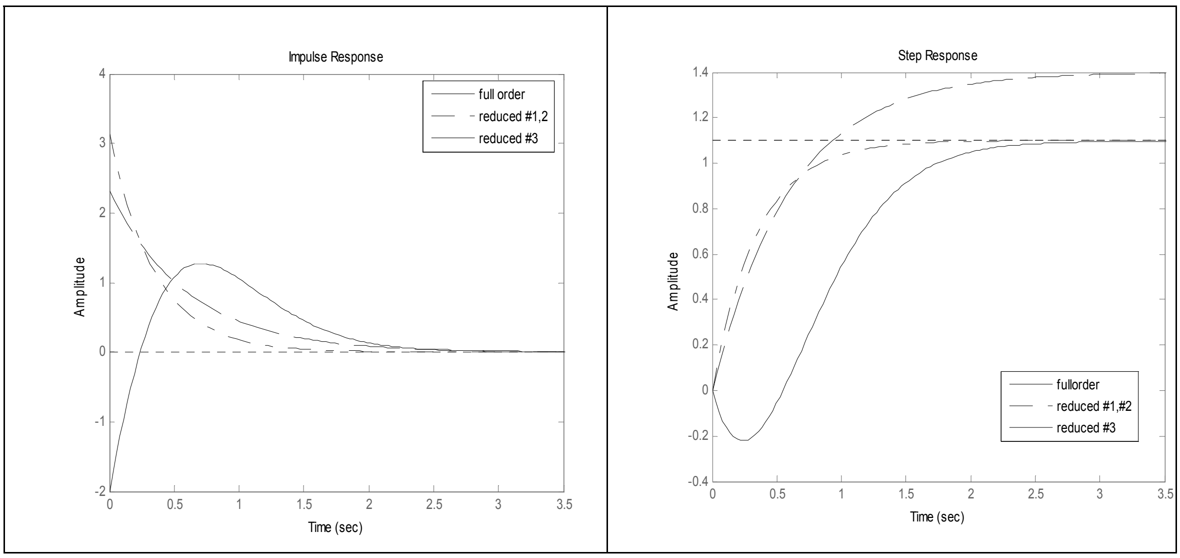

Figure 4 Reduced order models

5. CONCLUDING REMARKS

The problem of controller reduction in a closed-loop framework was presented using Krylov techniques. It was shown that one can obtain reduced-order controllers that approximate well the full-order controller and the full-order closed-loop systems in the neighborhood of specific frequencies or points in the complex plane.

Matching the first q Markov parameters guarantee that the first q time moments of the impulse response are matched. The preservation of this feature is especially important in non-minimum phase systems. For instance, the response of a non-minimum phase system to a positive step might present at time $t = 0$ a negative derivative. This behavior can be captured by matching high frequency moments.

Matching low frequency moments guarantee that the steady-state values will be preserved. For instance, the steady-state value to a step response is matched when the first low frequency moment is matched. High and low frequency moments can be simultaneously matched.

REFERENCES

[1] B.D.O Anderson and Y. Liu.: Controller reduction: Concepts and approaches. IEEE Trans. On Automatic Control, AC-34(8):802–812, 1989.
[2] Enns D.F.: Model reduction with balanced realizations: and error bound and a frequency weighted generalization. In Proceedings of the IEEE Conference on Decision and Control, pp. 127–132, (1984).
[3] Glover K.: All optimal Hankel-norm approximations of linear multivariable systems in their L_∞error bounds. International Journal of Control 39(6), 1115–1193, 1984.
[4] Nicoara D.: Large-Scale Dynamical Systems and Pseudospectra , Annals of the Oradea University, Volume V (XV), pag. 127-134, 2005.
[5] D.D. NICOARA, *Control Design and Minimal Realization of Dynamical Systems*, ANNALS of the ORADEA UNIVERSITY FASCICLE of MANAGEMENT and TECHNOLOGICAL ENGINEERING Volume VI (XVI), pag. 185-191, 2007.
[6] D. D. Nicoara: ON THE MODEL REDUCTION AND CONTROL OF DYNAMICAL SYSTEMS, International Workshop ″Advanced Researches in Computational Mechanics and Virtual Engineering pag.142-148, CD Edition, ISBN 973-635-823-2, 2006, Brasov, Romania.
[7] D. D. Nicoara: ON THE COMPUTATIONAL ASPECTS IN MODEL REDUCTION OF DYNAMICAL SYSTEMS, International Workshop ″Advanced Researches in Computational Mechanics and Virtual Engineering ″ pag.149-155, CD Edition, ISBN 973-635-823-2, 2006, Brasov, Romania.
[8] Dumitru Nicoara: On the Model and Controller Reduction for Large-Scale Dynamical Systems, ANNALS of the ORADEA UNIVERSITY FASCICLE of MANAGEMENT and TECHNOLOGICAL ENGINEERING Volume VII (XVII), 2008.
[9] Skelton R.E., Anderson B.D.O.: q-Markov covariance equivalent realizations, International Journal of Control 44(5), 1477–1490, 1986.